Summer assignment for AP Calculus AB for 2022-2023 – Mrs. Purtell, J3

Welcome to AP Calculus AB. This course is a full year class, taught at the level of a college or university calculus course, where Calculus AB covers 1 semester of college calculus content. Successful completion of this course will provide preparation for the AP exam, which may qualify you to receive credit for the equivalent college course (check with colleges you are interested in to see what their policy is for AB Calculus). Students entering this course should have **mastered** the equivalent of 4 years of high school mathematics (successful completion of Precalculus). **NO REVIEW OF ALGEBRA TOPICS WILL BE INCLUDED IN THE CLASS CONTENT.**

The Calculus AB course is designed to develop students' understanding of concepts, methods, and applications through emphasizing multiple approaches to representing solutions to problems (graphically, numerically, analytically, and verbally). The textbook is Calculus of a Single Variable, AP Edition, Larson and Edwards. You will be given a textbook for use at home, and there will be a class set that you may use at school (lost fee for this textbook is $100).

The course will use the TI-89 graphing calculator. Students may purchase their own calculator (TI-89 Titanium – check Staples, eBay, Costco, Amazon.com), otherwise the student will be given a TI-89 on the 1st day of class (if lost or damaged during the year, the fee is $130). You may use other calculators, however no instruction will be provided on anything other than the TI-89.

You will get the detailed course overview and grading guidelines on the 1st day of class, however, you should know that unlike other high school math classes, you will not be allowed to use notes on any tests, there will be no team tests, no test revisions, daily homework is mandatory, and late work is not accepted.

Summer Assignment – 90 Review Problems

The summer assignment is designed to provide review and assessment of precalculus topics required to be successful in calculus. The summer assignment is designed to help you identify the skills you may need to practice **before** the school year begins. A test will be given during the **first week** of school on this material. The summer assignment will be graded and is due on the **first** full day of class – NO EXCEPTIONS. The assignment appears longer than it really is because there are valuable notes and examples embedded between the exercises. Be sure to show ALL work, organized and labeled with answers boxed.

Google Classroom – code to join: **qvez5s3**
Remind – code to join – text **81010** – message **@0periodcal**

If you are struggling with any of the topics, please utilize your resources to LEARN the material on your own. There are many decent resources available online (website, videos, lessons), but you are responsible for your own learning and understanding of course prerequisite concepts and material. Next to each type of problem will be the TOPICS NOTES that are referenced from Mrs. Rivara. Please save your notes from Precalculus and use them for this assignment!

Good luck with this assignment, and I look forward to working with you in the fall!

Mrs. Purtell

Summer Review Packet for Students Entering AP Calculus AB

Complex Fractions (TOPIC 45)

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction. Use the LCD to clear each fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

2. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

3. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Negative & Fractional Exponents (TOPIC 4)

Simplify each of the following. Be sure to rewrite without a negative or fractional exponent.

4. a. $2x^{-7}$ b. $5x^{\frac{1}{3}}y^{-2}(3z)^{-3}$ c. $(25x^8y^{-6})^{-\frac{1}{2}}$

Functions (TOPIC 8 and TOPIC 3)

To evaluate a function for a given value, simply plug the value into the function for x.

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$, find $f(g(x))$.

$f(g(x)) = f(x - 4)$
$= 2(x - 4)^2 + 1$
$= 2(x^2 - 8x + 16) + 1$
$= 2x^2 - 16x + 33$

note: $(f \circ g)(x) = f(g(x))$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each of the following:

5. $f(2) =$ ____________ 6. $g(-3) =$ ____________ 7. $f[g(-2)] =$ ______________

8. $\dfrac{f(x+h) - f(x)}{h} =$ __________________________ **(TOPIC 109)**

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each of the following:

9. $h[f(-2)] =$ ___________________________

10. $f[g(x - 1)] =$ _________________________

11. $g[h(x^3)] =$ ____________________________

Let $f(x) = \sin x$ and find each using EXACT values. No calculator allowed and DO NOT use your unit circle. Note: the unit circle will need to be memorized for this course. (TOPIC 56)

12. $f\left(\frac{\pi}{2}\right) =$ _________________ 13. $f\left(\frac{3\pi}{2}\right) =$ ___________________

Average Rate of Change (AROC) is basically the formula for slope: **(TOPIC 109)**

$$\frac{f(x+h)-f(x)}{(x+h)-(x)}$$

Instantaneous Rate of Change (IROC) is taking the limit as $h \to 0$ of AROC: **(TOPIC 109)**

$$\lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{(x+h)-(x)}\right)$$

There are three types of functions that can be cleaned up algebraically in order to find IROC. Those functions and their algebraic technique are listed below:

- POLYNOMIALS: use a FOIL-ing technique
- RADICALS: use a technique of rationalizing the numerator with its conjugate
- RATIONALS: use a common denominators technique

Other than that, IROC for EXPONENTIALS, TRIG FUNCTIONS, ABSOLUTE VALUE, HYPERBOLAS, etc. can only be found using an approximation for h. For these cases, use $h = 0.001$ as a way to estimate.

Note: the vocabulary words are all interchangeable

IROC **slope of the tangent** **derivative** **instantaneous rate of change**

Note: we will study IROC at the beginning of the year, so you need to be able to correctly compute AROC.

Find AROC for the given functions below. Use algebraic techniques for each type of equation:

14. $f(x) = 9x + 3$

$f(x+h) =$

15. $f(x) = 5 - 2x^2$

$f(x+h) =$

16. $f(x) = 3\sqrt{x-2}$

$f(x+h) =$

17. $f(x) = \frac{-5}{x+1}$

$f(x+h) =$

Special Technique for Factoring (TOPIC 21)

Factor the expression below completely. Look for a common factor to determine where to begin.

Example: $\mathbf{3(2x-1)^2 + (4x-5)(2x-1)}$ **common factor = $\mathbf{2x-1}$**

$\mathbf{(2x-1)[3(2x-1) + 4x - 5)]}$

$\mathbf{(2x-1)[6x - 3 + 4x - 5]}$

$\mathbf{(2x-1)[10x-8]}$

$\mathbf{2(2x-1)(5x-4)}$

Factor completely.

18. $4(x+7)^2 + (x+7)(3x-5)$

19. $(3x+1)(2x-5) - (2x-5)^2$

Manipulating Exponentials (TOPIC 40, TOPIC 43, TOPIC 44)
Use exponent properties to rewrite and solve.

20. Solve: $\left(\frac{2}{64^x}\right) = \sqrt[3]{32^x}$

21. Solve for A: $27 \cdot 3^{x-2} = A \cdot 3^x$

22. Rewrite in $y = ab^x$ form: $y = 5(3)^{2x+4} - 7$

Sigma Notation for Area Under a Curve (TOPIC 31, TOPIC 32)

Area Under a Curve
Area under a curve $f(x)$ can be approximated using n rectangles with width Δx. This area can be computed by hand or translated into sigma notation.

Example: Given the equation $f(x) = -2(x-3)^2$, use the interval $4 \leq x \leq 7$ and 12 left-endpoint rectangles to approximate the area under the curve.

sigma notation: $0.25\sum_{k=0}^{11} -2\left((0.25k+4)-3\right)^2 = -38.3125$

where $\Delta x = \frac{7-4}{12} = 0.25$ and the argument $= 0.25k + 4$

23. Given the equation $f(x) = -2(x-3)^2$, use the interval $4 \leq x \leq 7$ and 12 right-endpoint rectangles to approximate the area under the curve.

24. Find the trapezoidal approximation for the area under the curve of $f(x) = -2(x-3)^2$ on the interval $4 \le x \le 7$ using 12 partitions.

25. Given the equation $f(x) = 2^x$, use the interval $2 \le x \le 4$ and 5 left-endpoint rectangles to approximate the area under the curve.

26. Given the equation $f(x) = 2^x + 18$ on the interval $2 \le x \le 4$ and 5 left-endpoint rectangles to approximate the area under the curve. Use your work from the problem above to help.

Domain and Range (TOPIC 3, TOPIC 93)

Find the domain and range of each function. Write your answer in INTERVAL notation.

27. $f(x) = -\sqrt{x+3}$	28. $f(x) = 3\sin x$	29. $f(x) = \frac{2}{x-1}$
domain:	domain:	domain:
range:	range:	range:

Equation of a line (TOPIC 9)

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y = m(x - x_1) + y_1$

Horizontal line: $y = c$ (slope is 0)

30. Find the equation of a line passing through the points (-3, 6) and (1, 2). State your equation in point-slope form.	31. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).
32. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$. State your equation in point-slope form.	33. Find the equation of a line perpendicular to the y-axis passing through the point (4, 7). State your equation in point-slope form.

Use the "switch and solve" method to find the inverse for each function. (TOPIC 7 and TOPIC 47)

34. $f(x) = \frac{2x}{x+3}$

35. $f(x) = \frac{x-5}{3x}$

PROVE that $f(x)$ and $g(x)$ are inverses of each other. Note: recall that to PROVE one function is an inverse of another function, you need to show that $f(g(x)) = g(f(x)) = x$. **(TOPIC 7)**

36. $f(x) = \frac{x^3}{2}$ $g(x) = \sqrt[3]{2x}$

<u>Polynomial Division (TOPIC 92)</u>

Use Polydoku or Long Division to divide. Note: if you know Synthetic Division you may also use that method.

37. $\frac{x^3-3x^2+x-3}{x-3}$

38. $(-2x^3 - 5x^2 + 8x + 15) \div (x + 3)$

Common Logs & Natural Logs (TOPIC 48, TOPIC 51, TOPIC 99, and TOPIC 100)

39. Rewrite from log to exponential form: $\log_x 4 = 9$

40. Solve: $2.21^x = 8$

41. Evaluate using change of base or by rewriting: $\log_5 20$

42. Evaluate by hand (no calculator): $\ln e^5$

43. Solve: $2e^x + 3 = 40$

44. Solve: $5 \ln x + 6 = 10$

Radians and Degrees (TOPIC 18)

Radians → Degrees: multiply by $\frac{180°}{\pi}$ to convert to degrees.	Degrees → Radians: multiply by $\frac{\pi}{180°}$ to convert to radians.

45. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

46. Convert to radians: a. 45° b. –17° c. 237°

Angles in Standard Position (TOPIC 20)

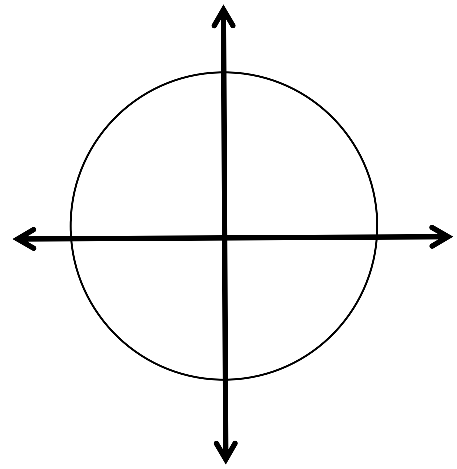
47. Sketch the angle.

   a. $\frac{11\pi}{6}$

   b. 230°

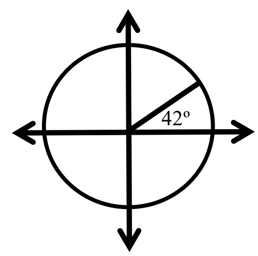
   c. $-\frac{5\pi}{3}$

   d. 1.8 radians



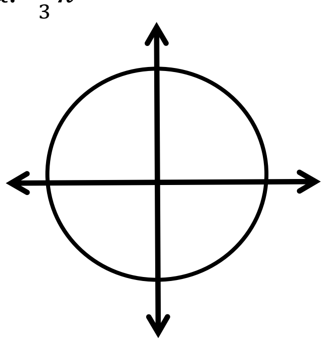
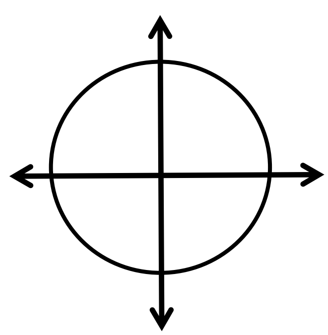
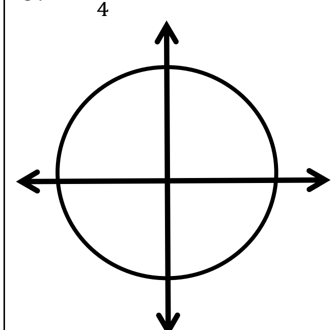
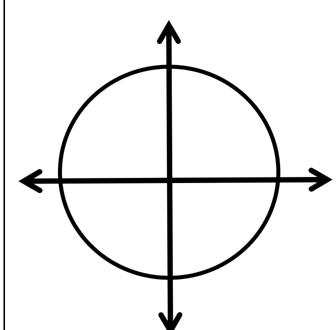
A Reference Angle is the angle between the ray and the x-axis. Below is the set of angles that all have the same reference angle.

1^{st} quadrant: 42°
2^{nd} quadrant: 138°
3^{rd} quadrant: 222°
4^{th} quadrant: 318°



Reference Angles (TOPIC 55, TOPIC 60, and TOPIC 62)

48. Sketch the angle and list the set of angles that all have the same reference angle.

a. $\frac{2}{3}\pi$	b. 225°	c. $-\frac{\pi}{4}$	d. 30°
			
set of angles:	set of angles:	set of angles:	set of angles:

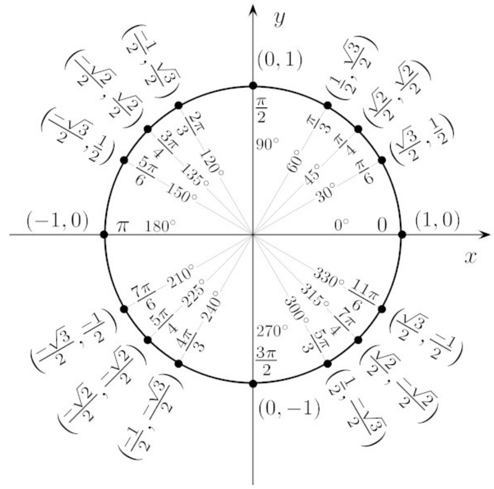
Unit Circle (TOPIC 56, TOPIC 63)

You can determine trig values of special angles by using the unit circle.

$\sin\theta =$ the y-coordinate

$\cos\theta =$ the x-coordinate

$\tan\theta =$ the slope or $\frac{\sin\theta}{\cos\theta}$



Examples: $\sin 45° = \frac{\sqrt{2}}{2}$

$\cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

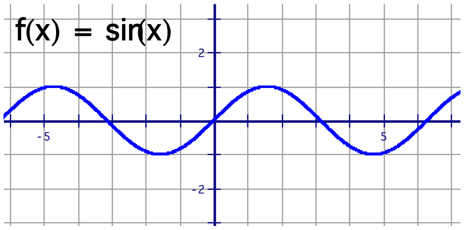
$\tan\frac{5\pi}{6} = \frac{1/2}{-\sqrt{3}/2} = -\frac{\sqrt{3}}{3}$

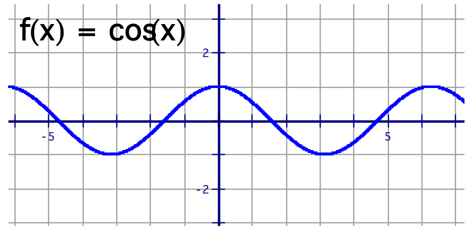
NOTE: The Unit Circle <u>must</u> be memorized for this course.

49. Find each value in EXACT form. Try to cover up the unit circle above and try these from memory.

a. $\sin 180°$	b. $\cos\frac{5\pi}{4}$	c. $\sin\left(-\frac{\pi}{2}\right)$
d. $\tan\frac{7\pi}{4}$	e. $\cos\frac{4\pi}{3}$	f. $\tan\left(-\frac{\pi}{3}\right)$

<u>**Graphing Trig Functions (TOPIC 66)**</u>





The graphs of $y = \sin x$ and $y = \cos x$ are very closely related to one another, in that they both have a period of 2π and an amplitude of 1. In fact, the cosine graph is really just a shift of the sine graph by 90° or $\frac{\pi}{2}$ radians to the left.

The parent graph for each of the equations is as follows:

$y = a\sin\big(b(x-h)\big) + k$ $\qquad$ $y = a\cos\big(b(x-h)\big) + k$

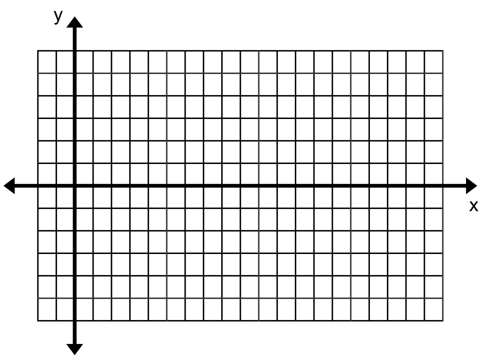
For both graphs:

- a = the amplitude (distance above or below the midline)
- b = the angular frequency (how many complete cycles of the graph fit into 2π, also $Pb = 2\pi$)
- h = the horizontal shift
- k = the vertical shift

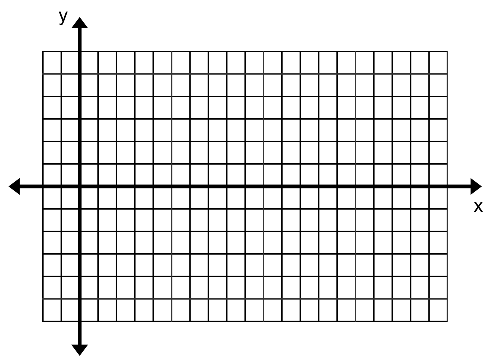
Remember, the **sine** graph starts on its locator point. The **cosine** graph starts above its locator point (if the amplitude is positive) or below its locator point (if the amplitude is negative).

Graph the functions below.

50. $f(x) = -\cos\left(x - \frac{\pi}{2}\right)$



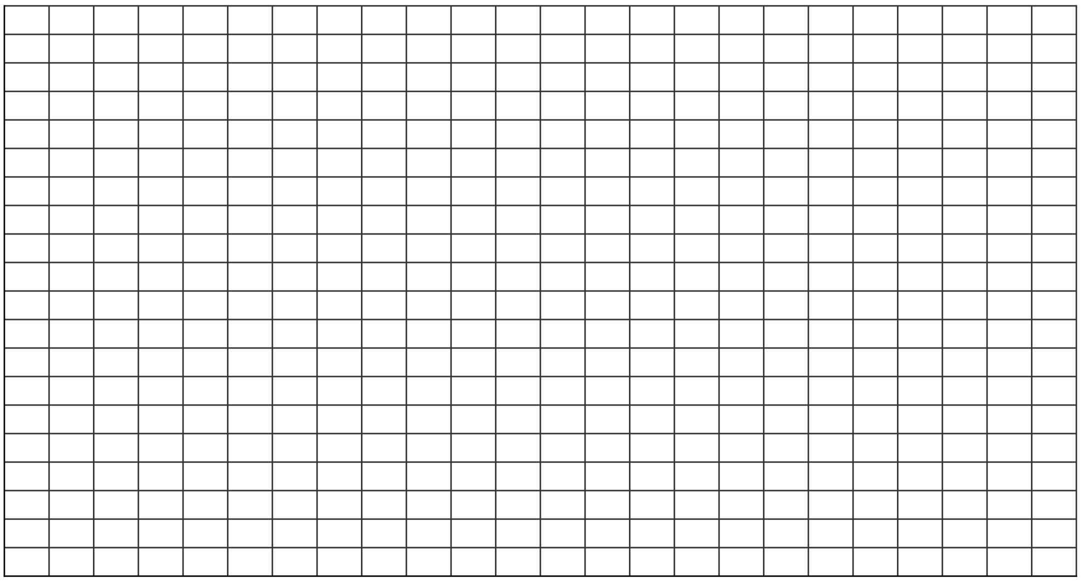
51. $f(x) = 3\cos(x + \pi) - 1$



Trigonometric Equations in Context (TOPIC 67, TOPIC 86)

Use what you know about graphing trig equations to write the equation of the trig function that models the situation below. Then answer the questions that follow.

52. The yearly temperature in Sonoma can be approximated by a sinusoidal function. In January the temperature was at a low of 45˚F. In July the temperature is at a high of 105˚F (assume these are the yearly high and low temperatures for Sonoma).



a. Write a sinusoidal equation that will model the temperature t months after January.

b. Find the temperature in April.

c. Find the first two times when the temperature reaches 98° F. Make sure to solve algebraically and show your work.

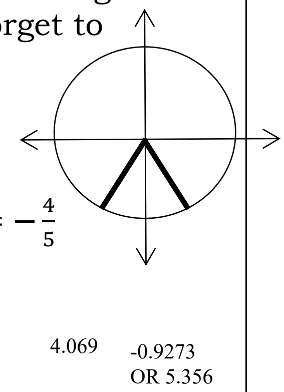
Solving Trig Equations (with an argument other than x):

When working with trig equations that have an argument other than x, consider using a substitution of θ. You will need to solve the problem to find ALL solutions in the given domain, then substitute back in to solve for your original variable. Don't forget to add/subtract the period to find any additional solutions needed.

Example:

$5\sin(2x-1)+3=-1$

$\sin(2x-1)=-\frac{4}{5}$, let $\theta=2x-1$, find two primary angles where $\sin\theta=-\frac{4}{5}$



$\theta=-0.9273$ (which is coterminal to 5.356) and $\theta=4.069$

$\theta=-0.9273$ (which is coterminal to 5.356)	$\theta=4.069$
$2x-1=5.356$	$2x-1=4.069$
$x=3.178$ (subtract π to get previous solution)	$x=2.535$ (add π to get next solution)
$x=0.037$	$x=5.677$

Solving Trigonometric Equations (TOPIC 76, TOPIC 78, TOPIC 80, TOPIC 85, and TOPIC 88)

Solve each of the equations for $0\le x\le 2\pi$. Isolate the variable, sketch a reference triangle, find ALL the solutions within the given domain $0\le x\le 2\pi$. Remember to check for more/less than two solutions when the b-value is other than 1 and the period is other than 2π. Use trig identities as needed to rewrite the trig functions. (See formulas at the end of the packet.) Note: also be sure to utilize factoring strategies as needed.

53. $\sin x=-\frac{1}{2}$	54. $2\cos x=\sqrt{3}$	55. $\sin^2\alpha=\frac{1}{2}$

56. $\sin(2x)=-\frac{\sqrt{3}}{2}$	57. $2\cos^2\theta-\cos\theta-1=0$

58. $4\sin^2\theta + 7\sin\theta = 2$	59. $\sin^2\beta + \cos(2\beta) - \cos\beta = 0$

Reciprocal Trigonometric Functions (TOPIC 68)

Use what you know about the unit circle to complete the problems below.

60. Find the EXACT value (no decimal): $\cot\left(\frac{5\pi}{3}\right)$

61. Use the calculator to approximate: $\sec\left(\frac{3\pi}{8}\right)$

62. Solve for $[0, 2\pi]$ in EXACT form: $\csc\theta = \sqrt{2}$

(TOPIC 61 and TOPIC 68)

63. Given $\csc A = -\frac{\sqrt{58}}{3}$ in quadrant 4, draw a sketch of the angle and find the following in simple radical form.

$\sin A$ = ______________ $\cos A$ = ______________

$\tan A$ = ______________ $\sec A$ = ______________

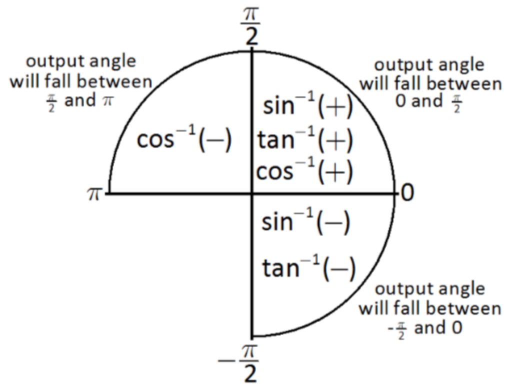
$\cot A$ = ______________

Inverse Trigonometric Functions (TOPIC 77, TOPIC 81)

Recall: Both of the notations below denote the same thing, the "inverse of sine":

$\arcsin(x)$ AND $\sin^{-1}(x)$

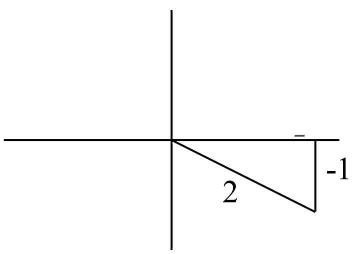
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:

Express the value of "y" in radians.

$y = arctan\left(\frac{-1}{\sqrt{3}}\right)$ hint: draw a reference triangle



This means the reference angle is 30° or $\frac{\pi}{6}$.
So, $y = -\frac{\pi}{6}$ so that it falls in the interval from $-\frac{\pi}{2} < y < \frac{\pi}{2}$. Answer: $y = -\frac{\pi}{6}$

For each of the following, express the EXACT value for "y" in radians.

64. $y = arcsin\left(\frac{-\sqrt{3}}{2}\right)$

65. $y = arccos(-1)$

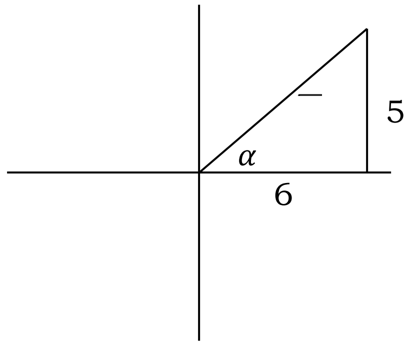
66. $y = arctan(-1)$

Drawing Triangles and Finding Values (TOPIC 77, TOPIC 81)

Angles in the unit circle provide EXACT values, but sometimes you need additional angles that are NOT in the unit circle. That's when double angle and half angle formulas can be useful.

Given angle α in quadrant 1 such that $\sin\alpha = \frac{5}{\sqrt{61}}$, find the EXACT value of each angle below.

Draw the reference triangle in the correct quadrant first.
Find the missing side using Pythagorean Theorem.



$\sin\alpha = \frac{5}{\sqrt{61}}$ $\quad \cos\alpha = \frac{6}{\sqrt{61}}$ $\quad \tan\alpha = \frac{5}{6}$

$\csc\alpha = \frac{\sqrt{61}}{5}$ $\quad \sec\alpha = \frac{\sqrt{61}}{6}$ $\quad \cot\alpha = \frac{6}{5}$

Use the angle formulas to find each value below.

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha = 2\left(\frac{5}{\sqrt{61}}\right)\left(\frac{6}{\sqrt{61}}\right) = \frac{60}{61}$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = (\cos\alpha)^2 - (\sin\alpha)^2 = \left(\frac{6}{\sqrt{61}}\right)^2 - \left(\frac{5}{\sqrt{61}}\right)^2 = \frac{36}{61} - \frac{25}{61} = \frac{11}{61}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\alpha}{1+\cos\alpha} = \frac{\frac{5}{\sqrt{61}}}{1+\frac{6}{\sqrt{61}}} = \frac{\sqrt{61}-6}{5}$$

Given $\tan\alpha = \frac{4}{5}$, draw a diagram then give the value without a calculator.

67.

$\sin\alpha =$ __________ $\quad \cos\alpha =$ __________

$\csc\alpha =$ __________ $\quad \sec\alpha =$ __________

$\cot\alpha =$ __________

$\sin(2\alpha) =$ $\quad \cos(2\alpha) =$ $\quad \tan(2\alpha) =$

$\sin\left(\frac{\alpha}{2}\right) =$ $\quad \cos\left(\frac{\alpha}{2}\right) =$

U-Substitution (TOPIC 89)

Solve each using U-Substitution.

68. $\begin{cases} \sqrt{x} + 2y^2 = 22 \\ 3\sqrt{x} - y^2 = 3 \end{cases}$

69. $(x^2 - 5x + 4)^2 - 3(x^2 - 5x + 4) - 4 = 0$

Limits (TOPIC 74)

Finding limits numerically.
Complete the table and use the result to estimate the limit.

70. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4} =$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$				…			

71. $\lim_{x \to -5} \frac{\sqrt{4-x}-3}{x+5} =$

x	-5.1	-5.01	-5.001	-5	-4.999	-4.99	-4.9
$f(x)$				…			

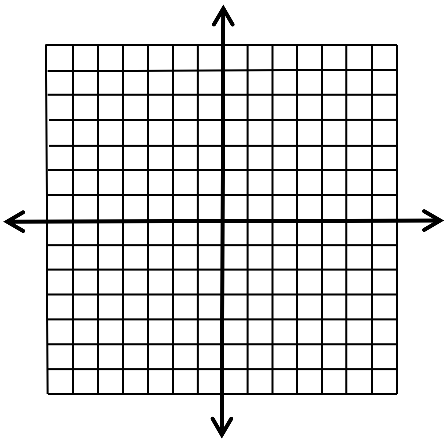
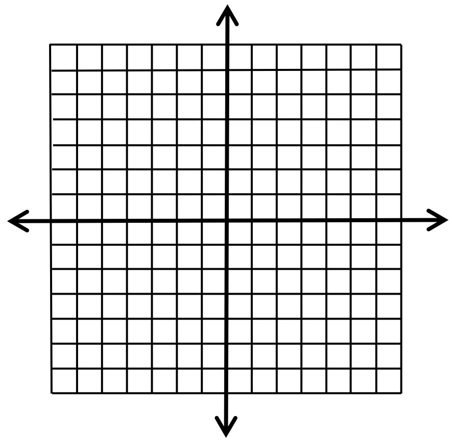
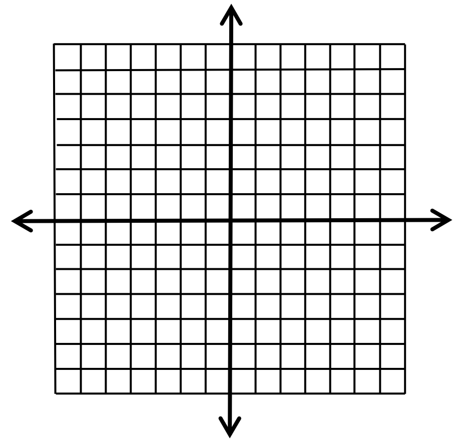
Finding limits graphically (TOPIC 74)

Find each limit graphically. Use your calculator to make a sketch and find each limit.

72. $\lim_{x\to 0} \cos x =$

73. $\lim_{x\to 5} \frac{2}{x-5} =$

74. $\lim_{x\to 1} \begin{cases} x^2+3 & x \neq 1 \\ 2 & x = 1 \end{cases} =$

Evaluating Limits Analytically (TOPIC 74)

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution. Hint: use algebraic strategies like factoring, rationalizing the numerator, and working with an LCD.

75. $\lim_{x\to 2}(4x^2+3)$

76. $\lim_{x\to 1} \frac{x^2+x+2}{x+1}$

77. $\lim_{x\to 1} \left(\frac{x^2-1}{x-1}\right)$

78. $\lim_{x\to -3} \frac{x^2+x-6}{x+3}$

One-Sided Limits (TOPIC 74)

Find the limit if it exists. First, try to solve for the overall limit. If an overall limit exists, then the one-sided limit will be the same as the overall limit. If not, use the graph and/or a table of values to evaluate one-sided limits.

79. $\lim_{x\to 5^+} \frac{x-5}{x^2-25}$

80. $\lim_{x\to -3^-} \frac{x}{\sqrt{x^2-9}}$

Vertical Asymptotes (TOPIC 98)

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

81. $f(x) = \frac{x^2}{x^2-4}$

82. $f(x) = \frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes (TOPIC 97, TOPIC 98)

Determine the horizontal asymptotes using the three situations below.

$\frac{small}{big}$	$\frac{big}{small}$	$\frac{same}{same}$
Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.	Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound.	Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Note: If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.

Determine all Horizontal Asymptotes.

83. $f(x) = \frac{x^2-2x+1}{x^3+x-7}$

84. $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$

Determine each limit as x goes to infinity. (TOPIC 96, TOPIC 97)

RECALL: This is the same process you used to find Horizontal Asymptotes for a rational function.

$\frac{small}{big}$	$\frac{big}{small}$	$\frac{same}{same}$
Degree of the numerator is less than the degree of the denominator. The limit as $x \to \infty$ is zero.	Degree of the numerator is greater than the degree of the denominator. The limit as $x \to \infty$ is ∞.	Degree of the numerator is the same as the degree of the denominator. The limit as $x \to \infty$ is the ratio of coefficients.

85. $\lim_{x\to\infty}\left(\frac{\sqrt{16x^4-3x^2+100}}{(3x+4)^2}\right)$

86. $\lim_{x\to\infty}\left(\frac{7x+6-2x^3}{3+14x+x^2}\right)$

Limits to Infinity (TOPIC 96 and TOPIC 97)

Technically, a rational function does not have a limit if it goes to $\pm\infty$, however, you can state the direction the limit is headed if both the left and right hand side go in the same direction. We reserve that the limit does not exist (DNE) only for when the left hand limit is not equal to the right hand limit.

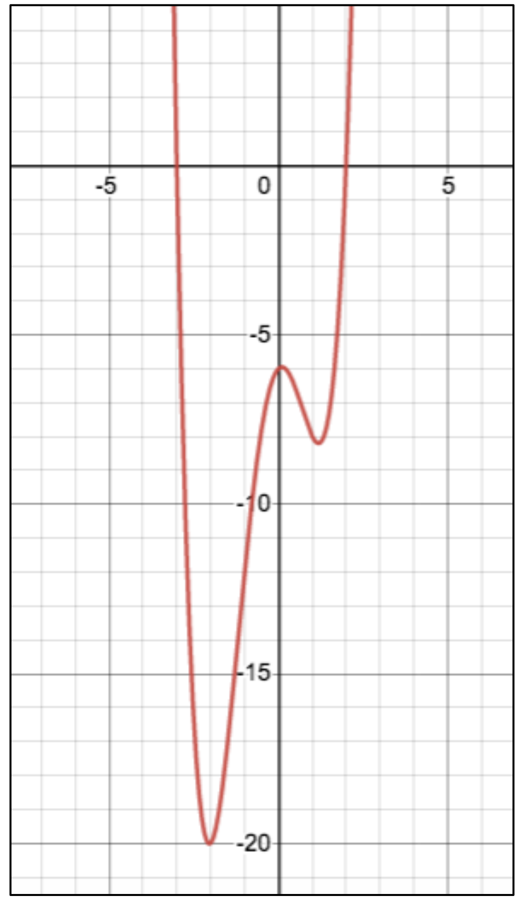
Determine each limit if it exists. If the limit approaches ∞ or $-\infty$, please state which one the limit approaches.

87. $\lim_{x\to 1^+}\frac{2+x}{1-x}$

88. $\lim_{x\to 0}\frac{2}{\sin x}$

Positive/Negative/Increasing/Decreasing/Concave Up/Concave Down (TOPIC 93 and TOPIC 94)

89. Given the function below, describe the (approximate) open intervals where the graph is:



Positive: ____________________________

Negative: ____________________________

Increasing: __________________________

Decreasing: __________________________

Concave Up: ________________________

Concave Down: ______________________

90. Sketch a *possible* graph with the following:
- decreasing on $(-\infty, -2)$ and $(2, 4)$
- increasing on $(-2, 2)$ and $(4, \infty)$
- concave up on $(-\infty, 0)$ and $(3, \infty)$
- negative on $(-3, 0)$ only

Remember, this summer assignment is designed to provide review and assessment of precalculus topics required to be successful in calculus. The packet is due on the first full day of school and you will be tested on this review material the first week of school. You will need to have the Unit Circle memorized next year, so please spend some extra time practicing it over the summer if needed.

If there's something that you don't understand, please ask for help so that you can figure it out on your own. You are responsible for knowing ALL of the content in this packet before taking calculus. Feel free to contact me through Remind or email at cpurtell@sonomaschools.org if you have questions.

Good luck with this assignment and thanks in advance for putting in all of the hard work!
~ Mrs. Purtell

Formulas

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2 \sin x \cos x$ $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ $\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

Half Angle Formula: $\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$ $\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$ $\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

Angle Sum/Difference Formula: $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Logarithms: $y = \log_b x$ is equivalent to $x = b^y$

Product property: $\log_b m + \log_b n = \log_b(mn)$

Quotient property: $\log_b m - \log_b n = \log_b\left(\frac{m}{n}\right)$

Power property: $\log_b m^p = p\log_b m$

Property of equality: If $\log_b m = \log_b n$, then $m = n$

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Derivative of a Function:

Slope of a tangent line to a curve OR the derivative $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$ OR $y = m(x - x_1) + y_1$

Standard form of a line: $Ax + By = C$

Graphing form of a parabola: $y = a(x-h)^2 + k$

Standard form of a parabola: $y = ax^2 + bx + c$

Sinusoidal forms: $y = a\sin b(x-h) + k$ OR $y = a\cos b(x-h) + k$
Where a = amplitude, $\frac{2\pi}{b}$ = period, (h, k) = locator point