Essential Skills for Pre-Calculus Honors Placement

1. Simplify, leaving only positive exponents.

$$\left(81x^{-40}\right)^{-\frac{1}{4}}\left(9^{-1}x^{4}\right)^{-\frac{3}{2}}$$

2. Solve over the complex numbers <u>BY COMPLETING THE SQUARE</u>.

$$3x^2-7x+18=0$$

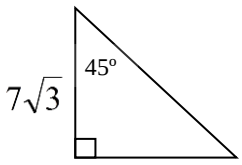
3. Solve the rational equation. <u>STATE ANY RESTRICTIONS</u>.

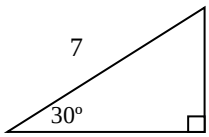
$$\frac{1}{x^2+11x+30}=\frac{1}{x+5}-4$$

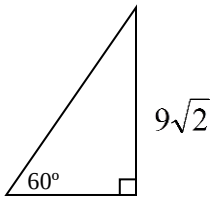
4. Write the equation of the line containing the points below. Your answer should be in slope-intercept form.

$\left(\frac{24}{5},\frac{24}{5}\right)$ and $\left(-\frac{24}{5},\frac{36}{5}\right)$

5. Determine the length of all missing sides.
   - Leave all answers in simple radical form.
   - Rationalize denominators when appropriate.
   - NO CREDIT FOR DECIMAL ANSWERS.

A. 

B. 

C. 

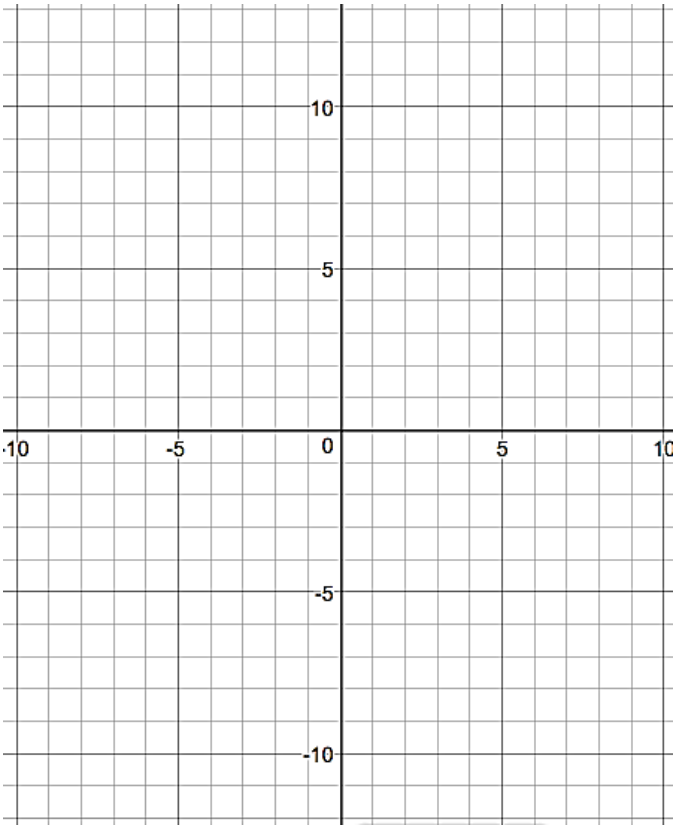
6. Determine the following for the quadratic function $y = -2x^2 + 8x - 2$. Then graph ***showing at least 5 points***. Show all work below.

Axis of Symmetry: _____

Vertex: _____

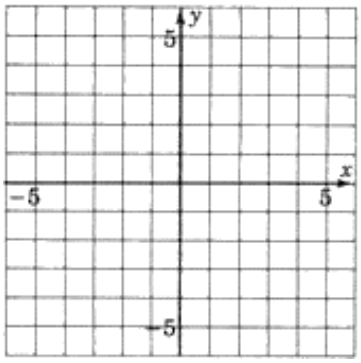
x-intercept(s): _____

y-intercept: _____

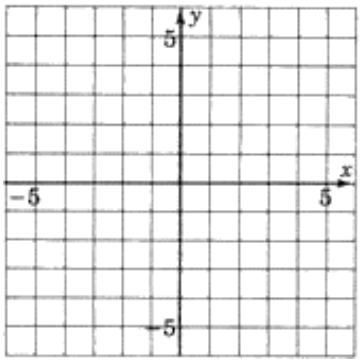


#1 - 9: Graph the following.

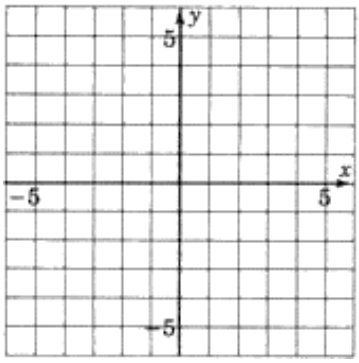
1. $y = -\frac{1}{4}x + 1$



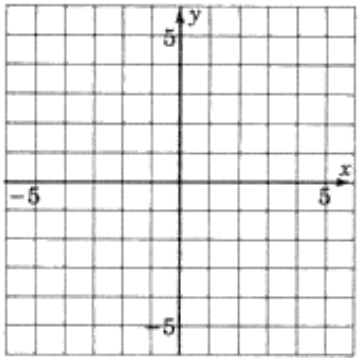
2. $4x + y = 3$



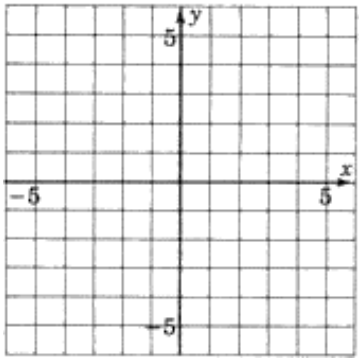
3. $-2y = -3x + 8$



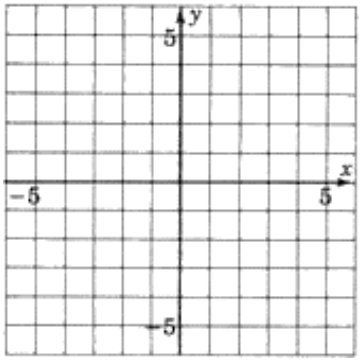
4. $-\frac{1}{3}x = y$



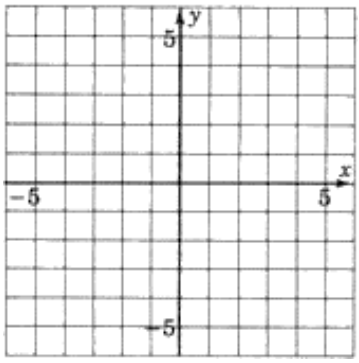
5. $\frac{y}{4} = -x + 1$



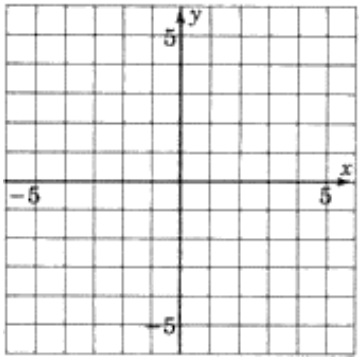
6. $-x = y$



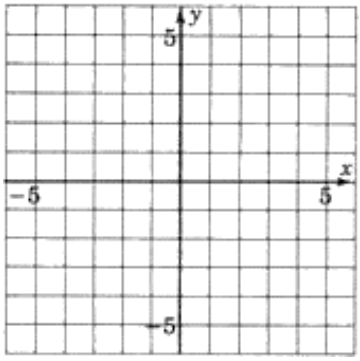
7. $\frac{3}{4}x + y = -5$



8. $y = 3$



9. $x = -4$



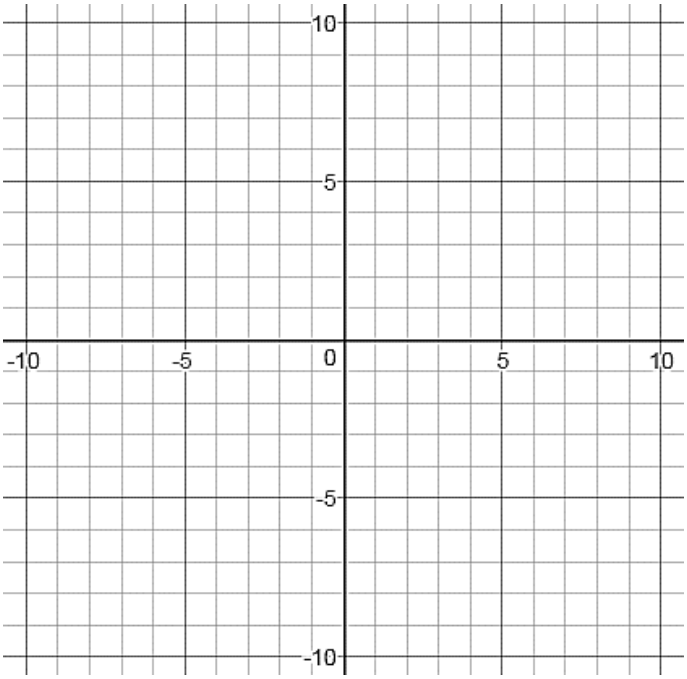
10. a) Find the *x*- and *y*-intercepts of the graph of the following :

$$\frac{4}{3}x - y = -8$$

x-intercept: _____

y-intercept: _____

b) Now graph this line **using those intercepts**.



More Practice. Many of these come from the Honors Pre-Calculus Summer Assignment. You might want to save your work!

I. Literal Equations: SOLVE ALL EQUATIONS FOR *x*. **State any restrictions**.

1. $r^2x - 5x = 7$

2. $2B = \dfrac{y-2x}{y-m}$

II. Linear Equations: Solve each equation.

2. $(x+1)^2 + 2(x-2) = (x+1)(x-2)$

3. $\dfrac{7}{3}(2x+3) + \dfrac{3}{4}\left(\dfrac{x}{5} - \dfrac{15}{2}\right) = \dfrac{11}{2}$

III. Rational Equations: Solve each equation, **stating restrictions**.

3. $\dfrac{3x-2}{3} + \dfrac{x-3}{2} = \dfrac{5}{6}$

4. $\dfrac{x-2}{x^2-x-6} = \dfrac{1}{x^2-4} + \dfrac{3}{2x+4}$

5. $\dfrac{x}{x+1} + 3 = \dfrac{1}{x^2+x}$

IV. Radical/Rational Exponent Equations: Solve each equation, **checking for extraneous solutions**.

6. $(x^2 - x + 10)^{\frac{1}{4}} = 2$

7. $(x-5)^{\frac{2}{3}} = 16$

8. $\sqrt{3x+4} = 1 + \sqrt{x+5}$

9. $(x^2 - x - 22)^{\frac{4}{3}} = 16$

V. Fraction Review: Simplify completely (but leave answers exact). #17 is invisible so you don't have to do it.

10. $\left(-\frac{3}{7}\right) \div \frac{5}{21}\left(-\frac{1}{30}\right)$

11. $\frac{5p}{7} - \frac{p}{7}$

12. $-\frac{3}{4} - \left(-\frac{2}{9}\right)$

13. $-5\frac{1}{4} + \frac{4}{9}$

14. $\left(\frac{3}{8}\right)(7)$

15. $\frac{3}{14} \div \frac{15}{4} \div \frac{12}{7}$

16. $\frac{4}{9} \div 5$

17. $45\left(\frac{\pi}{180}\right)$

18. $\left(\frac{3\pi}{2}\right)\left(\frac{180}{\pi}\right)$

19. $-\dfrac{3 - \frac{4}{3}}{\frac{2}{5}}$

VI. Linear Equations:

20. Write the slope-intercept form of the equation of the line passing through (5, –8) and perpendicular to $5y = 2x - 11$.

21. Write the equation of the line with undefined slope and passing through (59, 30).

22. Write the equation of the line such that passes through $f(3) = 11$ and $f(6) = 4$.

23. Find the value of m such that $2x - 3my = 6$ will be perpendicular to the line whose equation is $2x + 5y = 5$.

24. Determine the x-intercept, y-intercept and slope of the line $7x + 6y - 30 = 0$.

VII. Linear Systems of Equations: Solve each system any way you'd like.

25. $y - x = -2$
$-3y = -3x + 4$

26. $2x + 4y + 3z = 6$
$x - 2y + z = -5$
$x - 3y - 2z = -7$

VIII. Combining Functions: Given $f(x) = 3x - 1$, $g(x) = x + 3$, $h(x) = x^2 - 2x + 1$, find:

29. $h(f(g(x))$

30. $(g \circ f)(2)$

31. $f^{-1}(h(x))$

32. $h(h(-3))$

33. $(h \cdot f)(x)$

34. $(f+g+h)(5)$

IX. Absolute Value Equations and Inequalities: Find and graph the solution set.

35. $|3x+2|=5$

36. $|p-1|=-9$

37. $|x-3|=x+5$

38. $|x+1| \geq 7$

39. $|5x-3|-2<10$

X. Linear and Quadratic Inequalities: Solve.

40. $-8 \leq 1-3(x-2)<13$

41. $4>\frac{2x-3}{-3}>-4$

41. $\left(x-\frac{1}{2}\right)(x+2)>0$

42. $(x+6)^2 \leq 8$

43. $x^2 - 4x < 5$

44. $x^2+3x+2 > -2$

XI. Inverses: In each instance determine the equation of $f^{-1}(x)$.

45. $f(x)=-11x+9$

46. $f(x)=2\sqrt{x}-1$

XII. Complex Numbers: Simplify.

48. $\sqrt{-49}$

49. $\sqrt{-18}$

50. $(3i)^2$

51. $\frac{5}{3-i}$

52. $\sqrt{3}\sqrt{-3}$

53. i^{19}

54. $i^7 - i^{22}+i+i^{72}$

XIII. Exponents: Simplify each, leaving only positive exponents.

55. $8x^{\frac{1}{6}} \cdot x^{\frac{1}{6}} \cdot 7x^{\frac{5}{6}}$

56. $\left(\frac{x^{\frac{1}{4}}}{y^{-\frac{3}{4}}}\right)^{24}$

57. $\sqrt[5]{\frac{x}{4x^3}}$

58. $\sqrt[3]{-54x^8y^9} \cdot \sqrt[3]{16x^5y^2}$

59. $\frac{12x^{-3}}{y^4} \cdot \frac{(y^{-2}x^2)^{-1}}{15x^{-2}}$

60. $32^{\frac{3}{5}}$

61. $(27m^9n^{-3})^{-\frac{2}{3}}$

XIV. Factoring Quadratics (and one cubic & quartic): Factor completely.

62. $16x^3 - 28xy$ 63. $x^2 - 64$ 64. $4x^2 - 19x - 5$

65. $81 - m^4$ 66. $5x^2 - 8x + 3$ 67. $8x^3 - 27$

XV. Solving Quadratic Equations (and one cubic):

a. Use the **Zero Product Property** or the **Square Root Property** to solve each equation over the complex numbers.

68. $6x^2 + 19x - 20 = 0$ 69. $(x-12)^2 = 18$

70. $3x(x-1)^{\frac{1}{2}} + 2(x-1)^{\frac{3}{2}} = 0$ 71. $x^2 - 7 = 14 - 2x^2$ 72. $-4x^2 - 20 = 0$

73. $(9x+5)^2 = -182$ 74. $5x^3 - 35x^2 = 0$

b. Solve over the complex numbers using the **Quadratic Formula**.

75. $x^2 - 2x = -4$ 76. $5x^2 - 2 = 4x$ 77. $x^3 - 64 = 0$

c. Solve over the Real Numbers by **Completing the Square**.

78. $x^2 + 10x - 1 = 0$ 79. $3x^2 - 12x + 9 = 0$ 80. $x^2 + 7x = 4$

XVI. Graphing Quadratics:

81. State the vertex; then transform into standard form: $y = \frac{1}{2}(x+6)^2 - 1$.

82. State the y-intercept, then transform into vertex form: $y = 2x^2 - 12x + 36$

83. Transform into vertex form: $4y - 2 = x^2 + 6x$

#84 – 86: Determine the equation of the parabola with these characteristics. Your answer can be in standard or vertex form.

84. There is one double root at $(-2,0)$ and the y-intercept is $(0,8)$.
85. The *x*-intercepts are $(-4,0)$ and $(2,0)$, it is reflected over the *x*-axis, and stretched by a factor of 2.
86. Vertex is $(-2,-5)$ and it passes through the point $(-6,-9)$.

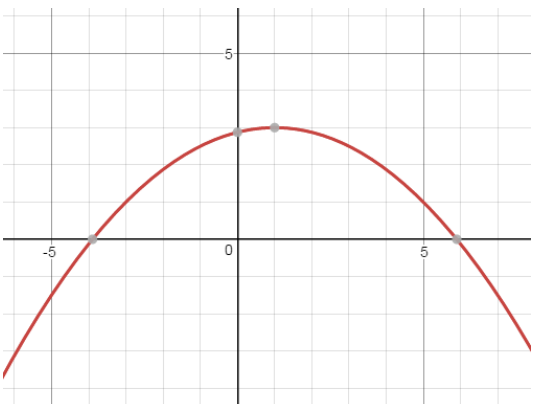
#87 – 90: For each of the following, state the

- Axis of symmetry
- Vertex
- *x*-intercept(s) and
- *y*-intercept.
- Then graph

87. $y = \frac{1}{4}(x+4)^2 + 1$

88. $y = -2x^2 - 12x - 6$

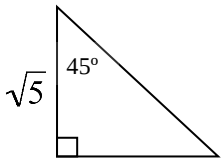
91. Write the equation of this parabola:

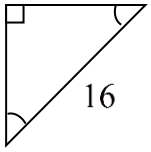


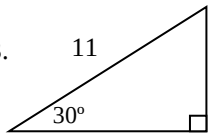
89. $y = 3(x-4)^2 - 6$

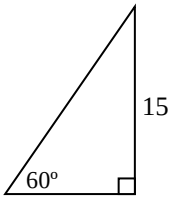
90. $y = -x^2 + 8x - 15$

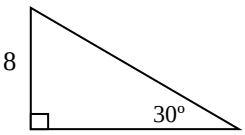
XVII. Special Right Triangles: Determine the lengths of all missing sides. Leave all answers in simple radical form. Rationalize when appropriate. NO DECIMALS!

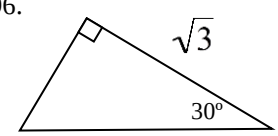
91. 

92. 

93. 

94. 

95. 

96. 

XVIII. Distance and midpoint

97. Find the length and midpoint of the segment with endpoints A $\left(\frac{1}{2}, 1\right)$ and B $\left(-\frac{5}{2}, \frac{4}{3}\right)$

XIX. Logarithmic and Exponential Form:

99. Write in logarithmic form: $81^{\frac{1}{2}} = 9$

100. Write in exponential form: $\log 0.001 = -3$

101. Expand: $\log_a \frac{x^2\sqrt{x}}{y^3 z}$

102. Condense: $2\log_a x - \frac{1}{2}(\log_a + 3\log_a x)$

103. Simplify

a) $\log_x x^2$ b) $9^{\log_3 4}$ c) $3\log_7 \frac{1}{49}$ d) $\log_5(-25)$

XX. Exponential Equations: Solve, leaving answers exact unless told otherwise.

104. $4^{3\log_2 4 - \log_2 32} = x$

105. $9^{x-1} = 27^{3x}$

106. Solve, round answer to the thousandths place: $9^x = 6$

XXI. Logarithmic Equations: Solve, leaving answers exact unless told otherwise.

107. $\log_2(2x - 6) = 4$

108. $\log_4(4x - 5) = \log_4(3x + 7)$

109. $\log_{(x+2)} 16 = 2$

110. $\log(x - 5) + \log(x + 2) = \log 8$

111. Solve, round answer to the thousandths place: $\ln 5x = 2$

XXII. Graphing: Sketch the graph of the equation or inequality:

112. $y < -x$

113. $y = \log_3 x$

114. $y = 1 - \frac{1}{4}x^2$

115. $y > -4\left|\frac{1}{2}x + 3\right| + 5$

116. $y = \sqrt{x-3}$

117. $y = \sqrt[3]{x}$

118. $y = x^3 + 2$

119. $y = \left(\frac{1}{2}\right)^x$

120. $\begin{cases} 3x + 2 < y \\ 3y > -x - 6 \\ y \le 1 \\ x \le 3 \end{cases}$

121. $\begin{cases} 3x - 2y < -6 \\ y > \frac{3}{5}|x-2| - 3 \end{cases}$

122. $4(x-3)^2 + 4y^2 = 36$

123. $9x^2 + 16y^2 - 18x + 64y - 71 = 0$

124. $x = -2(y-5)^2 + 1$

125. $-\frac{(x-3)^2}{25} + \frac{y^2}{16} = 1$