Orcutt Union School District
August 5-6, 2019

Understanding and Teaching Number Sense

What is Number Sense?

Assessing Our Number Sense

Use number sense to answer each of the following questions.

1. What strategies will you use to find the sum of these values?

13
22
15
17
20
8
+ 5

2. $2{,}768 \times 7 - 2{,}768 \times 6 =$

3. Subtract each of the following using mental math. Record the thinking you used to find the difference.

a) $325 - 75 =$ b) $208 - 46 =$ c) $256 - 38 =$

Visualizing Quantity

1.
2.
3.
4.
5.
6.
7.
8.

- ❑ Subitizing is "instantly seeing how many." From a Latin word meaning suddenly, subitizing is the direct perceptual apprehension of the numerosity of a group.
- ❑ Young children may use perceptual subitizing to make units for counting and to build their initial ideas of cardinality.
- ❑ The spatial arrangement of sets influences how difficult they are to subitize. Children usually find rectangular arrangements easiest, followed by linear, circular, and scrambled arrangements.
- ❑ A first graders' limits for subitizing scrambled arrangements is about four or five items, and most adults is not much greater.

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NUMBER SENSE boils down to this.

- number meanings (Counting, quantity recognition, writing and reading numbers, one-to-one correspondence, number conservation),
- number relationships (special relationship, more or less, anchor values, part/part/whole, comparing and ordering),
- number magnitude (place value, estimation and relative size to another number),
- and the skill of using operations (addition, subtraction, multiplication and division including understanding the meaning of the operation as well as strategies for using the operations).

BUILDING NUMBER SENSE

- It is not a one-time event, but builds over time. How are you continuing to build a sense of numbers in learners?
- It is built through graphical representations, daily routines, and games.

Number Meaning

Kindergarten has many concepts related to number meaning. It begins with early number development of these 5 concepts.

- One to one correspondence (each item gets only one count tag)
- Stable order (the count words are consistently used in a stable order)
- Cardinality (the last word in the count represents the cardinality of the set)
- Order irrelevance (the items may be counted in any order)
- Item irrelevance (anything can be counted, whether it be a concrete object or an abstract concept, whether the set is homogeneous or heterogeneous)

My Notes:

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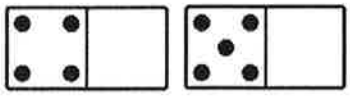
Examples of Visualizing and writing numbers:

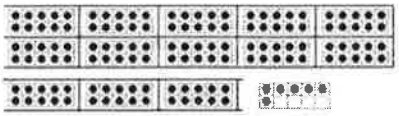
Using a ten frame, place this many counters on your ten frame.



Using a ten frame, place this many counters on your ten frame.

13

What does 136 look like?



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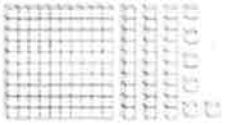
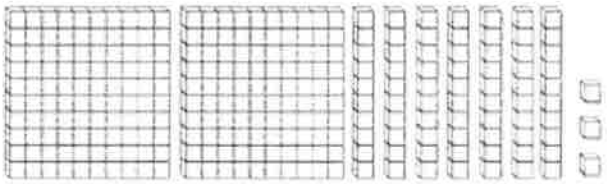


YOUR TURN

1. Write the number that is shown in this illustration.



2. Use base ten blocks to draw a picture and then write this as a number.

1 hundred, 4 tens, 5 ones

My Notes:

Examples of Counting:

Count from 8 by 1s

Count from 15 by 10s

Count from 25 by 5s? 100s? by 1/3s

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

YOUR TURN

1. What is the new set of numbers at your grade level?

2. How will you have student count these new values? When? What supports will they need?

My Notes:

Fraction Meaning

Models: Area, Line or Set

Definition: A whole is partitioned into three equal sized pieces. When we want to reference only 1 of the 3 pieces, we call the size of one-third of the whole. We write one-third as $\frac{1}{3}$.

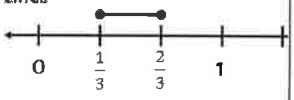
Examples of Fraction Meaning:

Area Model:

What does 1/3 of this rectang look like?

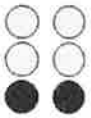
Linear Model:

Number Lines



Cuisenaire Rods

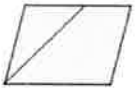
Set Model:



Set models are tricky for students. It is easier for many student to see the ratio of to 2 than it is to see the fraction 2/6.

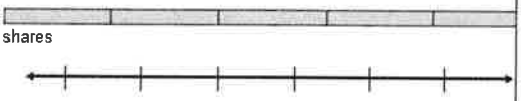
Equal Share Examples

Non-Examples of Equal Shares



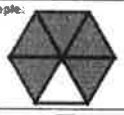
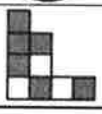
Length Models and Equal Shares

Yes
Equal shares



Non-examples of Equal shares

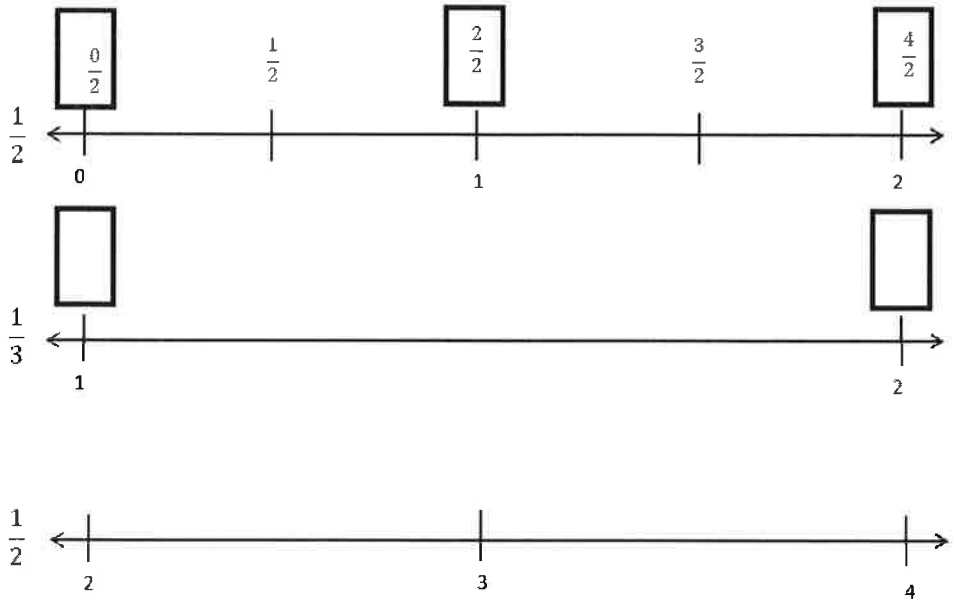
YOUR TURN

1.	Total Number of Equal Parts	Total Number of Shaped Equal Parts	Unit Fraction	Fraction describing Shaded Parts
Sample: 	6	5	$\frac{1}{6}$	$\frac{5}{6}$
a. 				
b. 				

2.What are three ways to show 2 fifths?

3. How will you have student count these new values? When? What supports will they need?

4.Estimate to equally partition and label the unit fractions on a number line. Label the wholes as fractions and box them.



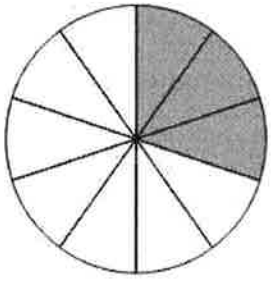
Decimal Meaning

How do we help students name decimals and make sense of the digits as they relate to drawings and representations of decimals?

Examples of Decimal Meaning:

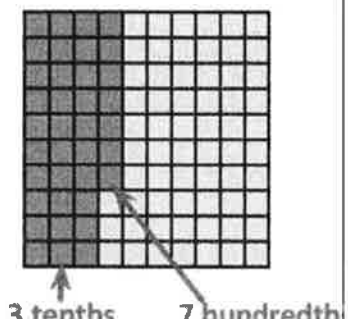
Ones	Tenths	Hundredths
0	3	

$\frac{3}{10}$ =



= three-tenth
= 0.3

$\frac{3}{10} + \frac{7}{100} =$ 0.37



YOUR TURN

1. How would you model each problem below? How do you write each one in a number format?

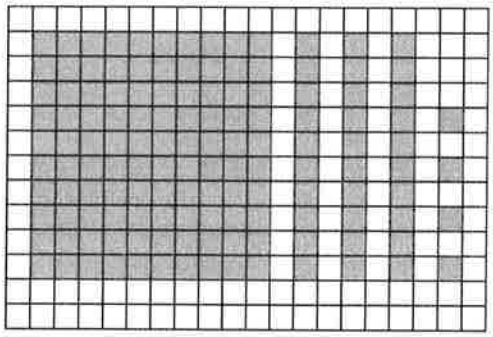
_____ = 4 ones, 2 tenths, 5 hundredths

_____ = 4 ones, 1 tenth, 15 hundredths

_____ = 4 ones, 25 hundredths

_____ = 3 ones, 12 tenths, 5 hundredths

2. Jesse said that this picture represents 134. Qe'Shawn says it represents 13.4. Who is correct? Why?



Planning for Number Meaning

My number set includes:

I already address:

I need to include:

This is how I will improve students' number meaning of my set of new values.

When?

With what activities/problems/materials?

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Number Relationships

Numbers are related to other numbers. As students learn a new set of numbers, they must understand how that new number is related to other numbers that they already know as well as the new ones they are learning. You might ask how the numbers are the same? How are they different? How is is the number related to a benchmark value such as 10 or 100?

- How is 8 like 10 and how is it different from 10?
- What is 108 related to 10? To 100? To 200?
- What is 3 fourths related to 0? To 1? To ½?
- What is 3 tenths related to 0? To 1? To 0.5?

Examples of Building Number Relationships More Or Less:

What is **one more** than this value?	What is **one less** than this value?	What is **ten less** than this value?
7	13	128

YOUR TURN

1. Use the hundreds chart to find
   a) 30 more than the given value. _____
   b) 50 more than the given value _____
   c) 100 more then the given value _____

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

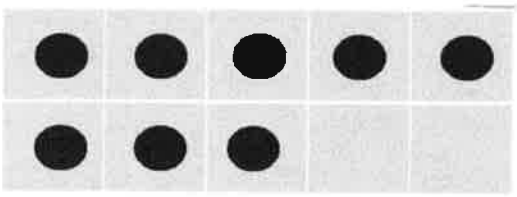
2. What would be the value of 1 tenth more?
   (a) 3 tenths + 1 tenth =
   (b) 0.11 + 1 tenth =
   (c) 0.2083 + 1 tenth =
   (d) 0.0998 + 1 tenth =

3. What would be the value of 1 hundredth more?
   (a) 3 tenths becomes =
   (b) 0.11 becomes =
   (c) 0.2083 becomes =
   (d) 0.0998 becomes =

My Notes:

Examples of Building Number Relationships Benchmark Values:

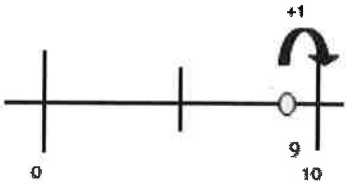
Ten is an important benchmark.
How many more counters are needed to fill this ten frame?

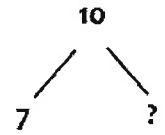


Scaffolding includes:
How many counters are in the top row?
How many counters are in the bottom row?
How can we count on from the top row?
How many total are in the frame?
How many more counters are needed to fill the frame?

Bridge from ten frames into less scaffolding and the use of other models.

Use a number line or use a number mountain to find a partner of 10?

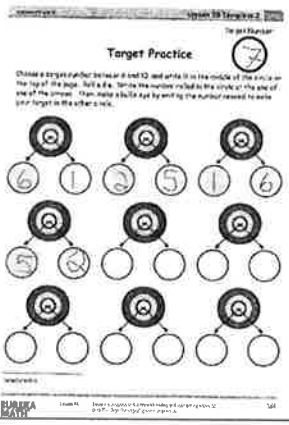


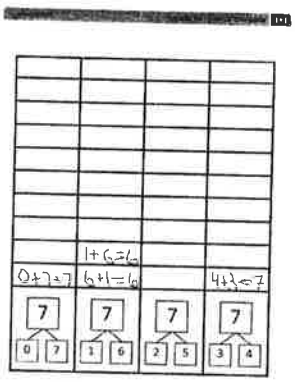


The last step is asking students for partners of 10 given only a value.

7

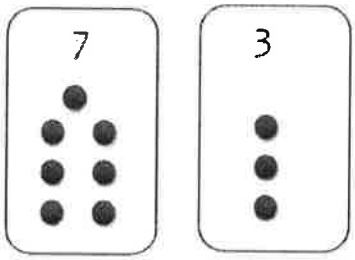
My Notes:





Examples of Building Number Relationships Comparisons:

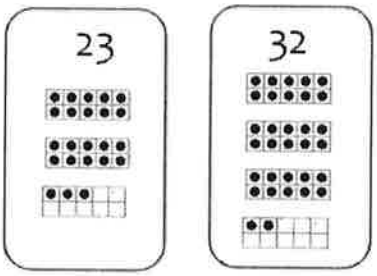
Try this first:



Now students can use symbols to complete this sentence.

7 ◯ 3

Don't forget that students need to see the visual representation of a new value to compare.



YOUR TURN

1. Partner Task: Write in symbols and compare

a)Partner A: 16 ones and 2 tens
Partner B: 4 tens

b)Partner A: 4 hundreds 12 tens 4 ones
Partner B: 2 hundreds and 34 ones

Hundreds	Tens	Ones

2. Partner Task: Compare these values.

Partner A: 9 hundreds 5 ten thousands, 9 ones
Partner B: 6 ten thousands 5 hundreds 9 ones

9 hundreds 5 ten thousands 9 ones ◯ 6 ten thousands 5 hundreds 9 ones

3.Use a number line to compare.
Construct a number line on your white board.

- Partition it into equal segments
- Begin the number line at 20 and count by 10s until you get to 130
- Place each of these values on the number line

120 34 102 14 41

4. Partner Task: Write in symbols and compare

a)Partner A: 36 and 9 thousandths
Partner B: 4 tens

b)Partner A: 202 hundredths
Partner B: 2 hundreds and 2 thousandths

5.Use a number line to compare.
Construct a number line on your white board.

- Partition it into 10 equal segments
- Begin the number line at 3 and count by tenths until you get to 4
- Place each of these values on the number line

3.12 3.4 3.04 3.14 3.41

My Notes:

Planning for Number Relationships

My number set includes:

I already address:

I need to include:

This is how I will improve students' number relationships of my set of new values.

When?

With what activities/problems/materials?

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Number Magnitude

Once students have meaning and relationships of a set of numbers, they can then learn and understand magnitude. This is how the digits of a number value relate to one another. For example, how is the 7 of 47 different from the 7 in 735? along with place value, number magnitude includes estimations.

Steps to Building Place Value.:

1. Count by 10s.
2. Counting by groups of 10 with left-over

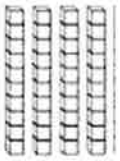
	18	19	
	28		
		39	
47			50

   a. Using 23 cubes, count them in groups of 10 to record how many groups of 10 and how many are left over. (Record as 23 cubes is ___ groups of 10 cubes and ____ left over.)

   b. Using chips and a ten frame, count out 37 chips. Record how many groups of 10 are complete and how many chips are left over.

   c. Begin with 3 groups of 10 object and 6 more. What number is this? (Written work to support would be 3 groups of 10 objects and 6 more make the number _____.)

3. Numbers made of 10s



   a. Connecting 4 tens is the number 40. Show 4 groups of tens (linking cubes) to connect to the number 40.

   b. Use the ten frames to show 7 tens. Write the number of chips this represents.

4. Making numbers with 10s and 1s

   a. Formalize language of digits, tens place, and ones place.

   b. Represent a value with a manipulative, in place value by number of tens and ones, and as a two-digit number. Flexibility to move among and between these forms.

   c. Given the number 56, describe this value in terms of tens and ones and be able to justify with a picture.

5. Ways of Making Numbers

   a. Given 34, show different ways to make this number using 10s and 1s (ie. 3 tens and 4 ones or 2 tens and 14 ones, or 34 ones).

   b. Show these equivalents in pictures as well as in number sentences.

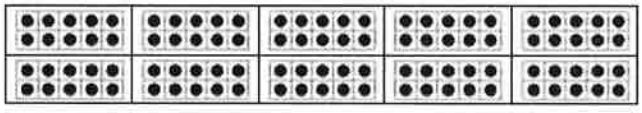
6. Tens and Ones on a Hundreds chart

   a. Using the hundreds chart, color in 4. Count by 10s and color in each number. Notice the pattern on the chart.

   b. Color in the numbers that have a 5 in the ones place. What do you notice about all the numbers?

   c. Color in the numbers that have an 8 in the tens place. What do you notice about all the numbers?

7. Counting and writing to 100



   a. Build 100. (It is 10 tens)

   b. Count in patterns of ones, tens, or hundreds from decades and from any given number (i.e. 300, 400, 500, ... and 310, 320, 330, ... and 312, 322, 332, ..., and 312, 412, 512, ...).

   c. Use manipulatives to count in the hundreds to represent the value of the numbers you are counting.

   d. From a picture of a value, write the number (ten frames and place value blocks).

8. Numbers by Hundreds and Tens
   a. Beginning with a representation of a value, represent a value that is 10 more or 10 less, 100 more or 100 less than the value. Show in pictures and also as a number sentence (see next page).
   b. Using chips and a ten frame, count out 37 chips. Record how many groups of 10 are complete and how many chips are left over
   c. Begin with 3 groups of 10 object and 6 more. What number is this? (Written work to support student knowledge would be 3 groups of 10 objects and 6 more make the number _____.)
9. Patterns on the 100s Chart
   a. Make observations about the numbers on the 100s chart. What do you notice about the numbers as you move to the right? What do you notice about the numbers as you move down? What do you notice about the digit in a column? What changes? In a row? What changes?
   b. Given a section of a 100s chart where some numbers are present, identify the values that should be in the empty boxes.
   c. Describe what happens to 468 when it becomes 668. Describe what happens to 743 when it becomes 783.

	450	460	
		660	
740			770

10. Ways of Making Numbers
    a. Use a variety of games and puzzles to make numbers
       1. What is the number that you would write if there are 12 ones and 3 tens);
       2. Make 500 (see center)
    b. Decompose numbers into multiple representations of place value (538 is 5 hundreds, 3 tens and 8 ones and it is also 4 hundreds 13 tens and 8 ones, etc).

YOUR TURN

Complete this table by representing each value in all formats. Yes this connects to number meaning.

Representation	Place Value	Expended Notation	Word Number	Number
	2 hundreds, 4 tens, 3 ones	**200 + 40 + 3**		
			Three hundred fifty-one	
				35

My Notes:

Operations of Numbers

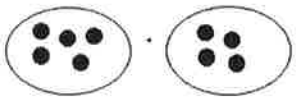
We finally are going to look at the operations of numbers. Notice how you are thinking about the values already when we haven't yet addressed how to add, subtract, multiply or divide. There are two main ideas that will reoccur with each operation: the meaning of the operation and strategies to use with different sets of numbers with the operations. As we use various strategies, you will also use different tools.

Addition Strategies	Subtraction Strategies
• Making Ten • Doubles • Doubles Plus 1 • Decomposing Numbers(Combinations) • Place value • Compensation • Friendly Numbers	• Count Backwards • Count Up • Take From Ten • Take to Ten • Doubles • Decomposing Numbers(Combinations) • Place value • Compensation • Friendly Numbers
Tools • Ten Frame • Number Bonds • Place Value Disks/Chips • Number Lines	• Tape Diagrams • Hidden Zero Cards • Algorithms
Multiplication and Division Strategies	**Tools:**
• Area • Place value • Properties • Decomposing Values	• Place Value Disks/Chips • Number Lines • Area • Algorithms

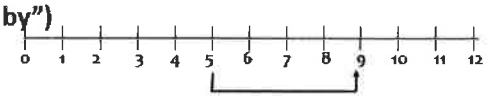
Addition

Addition is about combining sets together or adding onto an existing set.

❑**Aggregation (combining)**

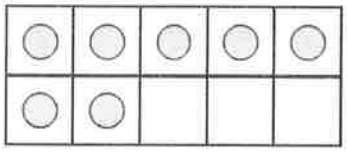


❑**Augmentation ("increased by" or "goes up by")**

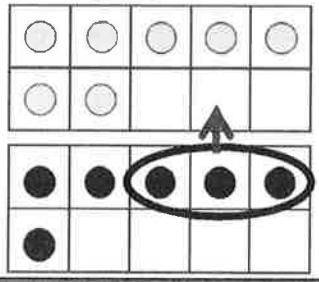


Addition: Making Ten Stratgy

7 + ___ = 10



7 + 6 = ____



7 + 6
= 7 + 3 + 3
= 10 + 3
=13

YOUR TURN

1. Use a double ten frame to model each problem.

- 5 + 5

- 5 + 5 + 8

3. Use a double ten frame to model each problem.

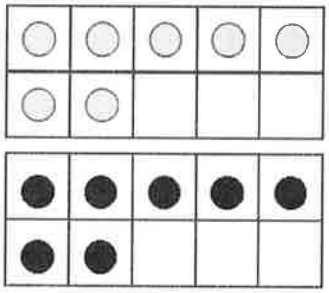
8 + 2

8 + 5

8 + 9

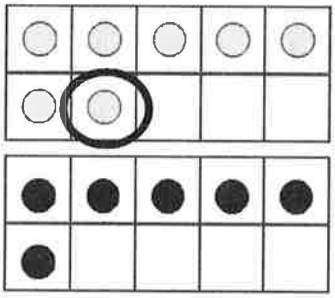
Addition: Doubles and Doubles + 1

7 + 7 = ____



7 + 7
= 5 + 2 + 5 + 2
= 10 + 4
=14

7 + 6 = ____

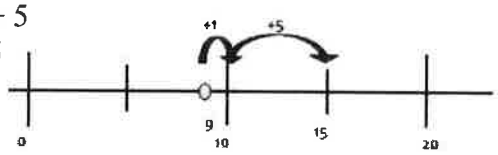


7 + 6
= 6 + 1 + 6
= 6 + 6 + 1
=12 + 1
= 13

Addition: Making Tens

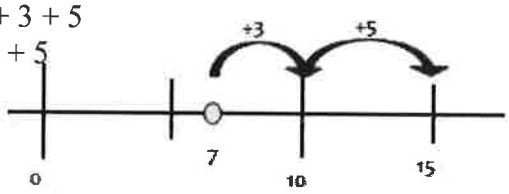
9 + 6 = ____

9 + 6
= 9 + 1 + 5
= 10 + 5
=15



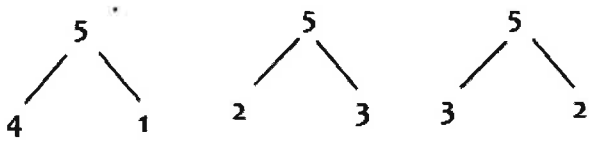
7 + 8 = ____

7 + 8
= 7 + 3 + 5
= 10 + 5
=15



Addition: Number Combinations (Decomposing Numbers)

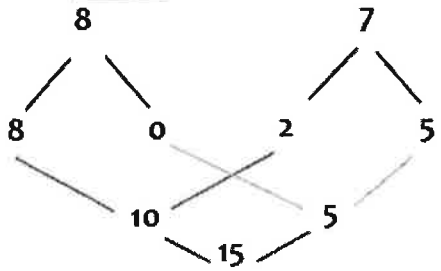
What are the combinations of 5?



What are the combinations of 5?

Addition: Making Tens

8 + 7 = _____



$8 + 7$
$= (8 + 0) + (2 + 5)$
$= (8 + 2) + (0 + 5)$
$= 10 + 5$
$= 15$

YOUR TURN

1. Partner Task:
- ❑ Partner A: Show me 8 on your ten frame
- ❑ Partner B: Show me 5 on your ten frame

What is the sum of 8 and 5?

2.Partner Task:
- ❑ Partner A: Show me 5 on your ten frame
- ❑ Partner B: Show me 6 on your ten frame

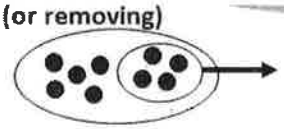
What is the sum of 5 and 6?

Subtraction

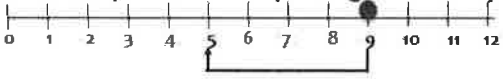
Subtraction is about removing from a set, reducing a set, or comparing two sets.

Many students "think addition" for solving all subtraction problems. They are inverses of one another and this strategy works as long as we continue to support students in using this strategy with larger values.

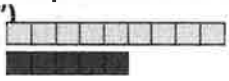
❑**Partitioning (or removing)**



❑**Reduction ("decreased by" or "goes down")**



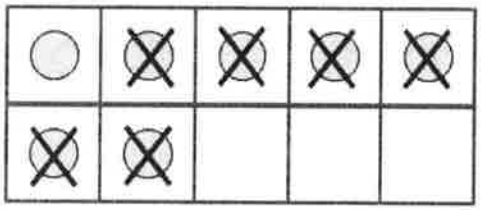
❑**Comparison (difference or "how much greater?")**



Subtraction: Counting Back and Counting Up

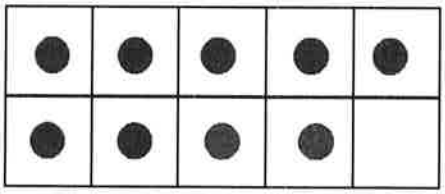
7 – 6 = ____

Students count out 7 and then count backwards six.



9 – 7 = ____

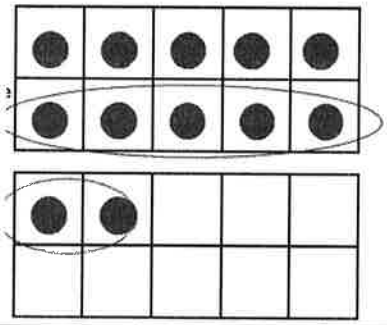
Students think 7 + ??? makes 9. Starting with 7, they count on until they get to 9. It is important to use color to keep track of what is being added to make 9.



Subtraction: Back to 10

12 – 7 = ____

Students count back to 10. This removes 2. of the 7. then remove the remaining 5 from the 10. You are left with 5. This is great for students who can compose numbers to ten easily and have their combinations to 10.



12 – 4 = ____

Students count back to 10. This removes 2. of the 4. Remove 2 more from the 10. The difference is 8.

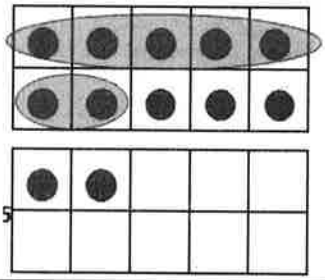
14 – 6 = ____

Students count back to 10. This removes 4. of the 4. Remove 4 more from the 10. The difference is 6.

Subtraction: Take from 10

12 – 7 = ____

This strategy decomposed the 12 into 2 and 10. then you can remove the 7 from the 10. The different is 3. Combine this with the 2 that is also left and the difference is 5.



12 – 4 = ____

12 is 2 and 10. 10 – 4 = 6 and 6 and the 2 remaining is 8.

14 – 6 = ____

14 is 4 and 10. 10 – 6 = 4. and 4 and the remaining 4 is 8.

Number Lines

Place Value Chart (Addition) 27 + 65

Tens	Ones

Place Value Chart (Addition) 604 – 238 =

Hundreds	Tens	Ones

YOUR TURN

Choose a strategy and a tool for each problem. Choose at least 2 problems to do with different strategies using the same tool and 2 problems with the same strategy but different tools.

1. 12 + 20 =

2. 12 + 23 =

3. 34 + 45 =

4. 34 + 49 + 32=

5. 146 + 389 =

6. 5820 + 327 =

7. 12 – 3 =

8. 22 – 3 =

9. 122 – 13 =

10. 156 – 31 =

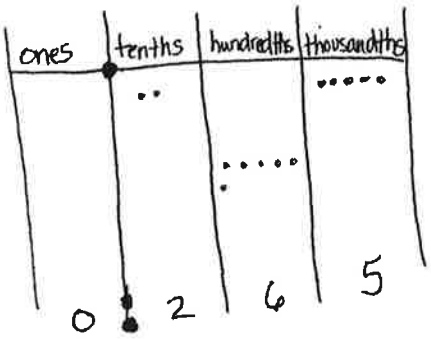
11. 156 – 39 =

12. 258 – 79 =

Addition and Subtraction of Decimals

Add the following using a place value chart.

1. 3 tenths + 5 tenths =



0.205
+ 0.060
0.265

2. 12 tenths + 9 tenths =

3. 3 hundredths + 4 hundredths =

4. 3 hundredths + 4 tenths =

5. 1.8 - 9 tenths

6. 1hundred 8 hundredths + 2one 4 hundredths

Planning for Addition and Subtraction

I already address:

I need to include:

What tools will I include that I don't already?

When?

With what activities/problems/materials?

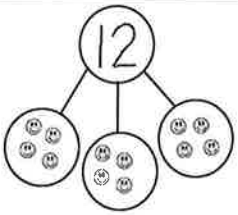
Multiplication as an Operation

Multiplication as an operation has several meanings. It is about repeated groups or equal groups, it is about area, it is about multiples and it is about comparisons. When students start learning multiplication it is critical to read the problem that is written as 3 x 4 as "3 groups of 4 objects" so that students build a mental model of repeated groups. When they just say 3 x 4 as "3 times 4" they are not building any mental image of groups.

Examples: Meaning of Multiplication

_____ x _____ = _____

(# of groups) (how many items in each group) (Total # of items)



❑3 x 4

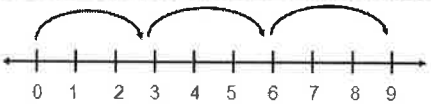
❑3 groups of 4 objects in each group

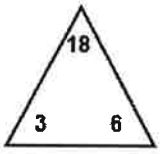
Examples: Strategies for Learning Multiplication Facts

Skip Counting Patterns

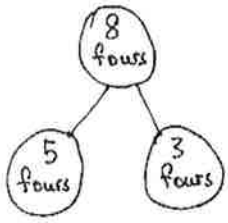
Let's count by threes. (Direct students to count forward and backward to 30. Whisper the numbers between threes and speak each three out loud. For example, whisper 1, whisper 2, say 3, whisper 4, whisper 5, say 6, and so on.) When they are better, count forward and backwards by 3s.

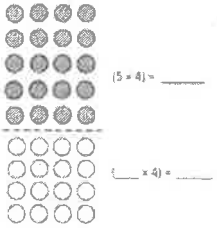
Use of Number Line or Triangle Fact Families





Distributive Property





Patterns

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Visual Supports

Visualizing 5 Facts

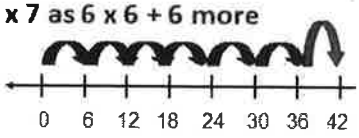
Visualizing 7 Facts

May

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

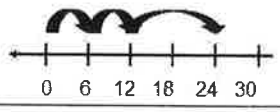
Helping Facts

6 x 7 as 6 x 6 + 6 more



6 x 4

6 x 2 x 2



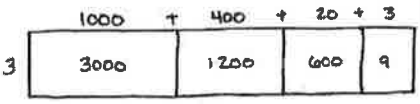
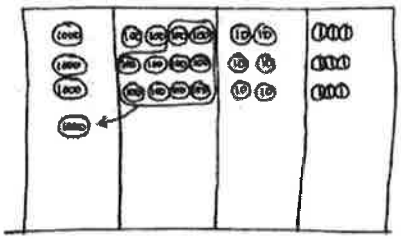
Division as an Operation

Division as an operation asks two questions. How many are in each group? or How many groups? When division is linked to the multiplication statement below, it is easy to see how these questions come directly from divisions inverse relationship to multiplication.

_____ (# of groups) x _____ (how many items in each group) = _____ (Total # of items)

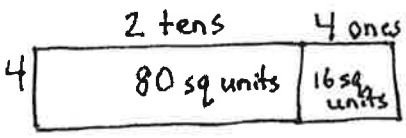
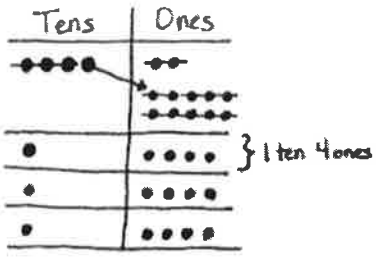
Examples: Meaning of Divison	
❑ If 24 marbles are put into 4 groups, how many will be in each group?	❑ We take 24 marbles and put 6 marbles in each group. How many groups will be formed?

Context for Multiplication: Mr. Meyers wants to have 3 times the amount of money he currently has in his bank account so he can take a trip. He currently has $1432 in his account. How much does he need in the account to take the trip?

Area Model	Place Value Chart/Coins	Algorithm
1423 x 3 = 	1423 x 3 = 	1423 x 3 9 ← 3 x 3 ones 60 ← 3 x 2 tens 1200 ← 3 x 4 hundreds 3000 ← 3 x 1 thousand 4269
23 x 5 =	23 x 5 =	23 x 5 =

Hundreds	Tens	Ones

Division Algorithms

Area Model	Place Value Chart/Coins
$96 \div 4 =$ 	$42 \div 3 =$ 

Practice These:

1. $36 \div 3$
2. $42 \div 3$
3. $84 \div 3$
4. $655 \div 5$
5. $726 \div 3$
6. $297 \div 4$
7. $678 \div 4$
8. $704 \div 3$
9. $407 \div 2$
10. $627 \div 3$
11. $3{,}070 \div 5$
12. $8{,}313 \div 4$

Hundreds	Tens	Ones

Gaining Fluency

Timed tests do not lead to fluency. Students gain fluency by first learning small sets of facts using the strategies that were described above. They must practice each set of new facts in a distributed manner so they recognize the new facts without confusing them or not practicing the old facts. when the facts are learned, students should be given timed practice.

- ❑ Present Strategies in Small Sets "7 +/- 2"
- ❑ Provide Distributed Practice
- ❑ Provide Timed Practice
  - 50% new set, 50% review
  - The latter insures high levels of success

Teacher Material										
	Dot Cards	**Dot Plates**	**Five Frame (blank)**	**Ten frame (blank)**	**Ten Frame (printed)**	**Number Cards**	**Number words**	**Dice**	**Cubes**	**Number Lines**
Dot cards										
Dot Plates										
Five Frame (blank)										
Ten frame (blank)										
Ten Frame (printed)										
Number Cards										
Number words										
Dice										
Cubes										
Number line										