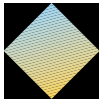


# Review of Algebra





## Review of Algebra

Here we review the basic rules and procedures of algebra that you need to know in order to be successful in calculus.

### Arithmetic Operations

The real numbers have the following properties:

$a + b = b + a$	$ab = ba$	(Commutative Law)
$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$	(Associative Law)
$a(b + c) = ab + ac$		(Distributive law)

In particular, putting  $a = -1$  in the Distributive Law, we get

$$-(b + c) = (-1)(b + c) = (-1)b + (-1)c$$

and so

$$-(b + c) = -b - c$$

#### EXAMPLE 1

(a)  $(3xy)(-4x) = 3(-4)x^2y = -12x^2y$

(b)  $2t(7x + 2tx - 11) = 14tx + 4t^2x - 22t$

(c)  $4 - 3(x - 2) = 4 - 3x + 6 = 10 - 3x$  ■

If we use the Distributive Law three times, we get

$$(a + b)(c + d) = (a + b)c + (a + b)d = ac + bc + ad + bd$$

This says that we multiply two factors by multiplying each term in one factor by each term in the other factor and adding the products. Schematically, we have

$$(a + b)(c + d)$$

In the case where  $c = a$  and  $d = b$ , we have

$$(a + b)^2 = a^2 + ba + ab + b^2$$

or

**1**

$$(a + b)^2 = a^2 + 2ab + b^2$$

Similarly, we obtain

**2**

$$(a - b)^2 = a^2 - 2ab + b^2$$

**EXAMPLE 2**

(a)  $(2x + 1)(3x - 5) = 6x^2 + 3x - 10x - 5 = 6x^2 - 7x - 5$

(b)  $(x + 6)^2 = x^2 + 12x + 36$

(c)  $3(x - 1)(4x + 3) - 2(x + 6) = 3(4x^2 - x - 3) - 2x - 12$   
 $= 12x^2 - 3x - 9 - 2x - 12$   
 $= 12x^2 - 5x - 21$

**Fractions**

To add two fractions with the same denominator, we use the Distributive Law:

$$\frac{a}{b} + \frac{c}{b} = \frac{1}{b} \times a + \frac{1}{b} \times c = \frac{1}{b}(a + c) = \frac{a + c}{b}$$

Thus, it is true that

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$$

But remember to avoid the following common error:

$$\frac{a}{b + c} \neq \frac{a}{b} + \frac{a}{c}$$

(For instance, take  $a = b = c = 1$  to see the error.)

To add two fractions with different denominators, we use a common denominator:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

We multiply such fractions as follows:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

In particular, it is true that

$$\frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}$$

To divide two fractions, we invert and multiply:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

**EXAMPLE 3**

$$\begin{aligned}
 \text{(a)} \quad \frac{x+3}{x} &= \frac{x}{x} + \frac{3}{x} = 1 + \frac{3}{x} \\
 \text{(b)} \quad \frac{3}{x-1} + \frac{x}{x+2} &= \frac{3(x+2) + x(x-1)}{(x-1)(x+2)} = \frac{3x+6+x^2-x}{x^2+x-2} \\
 &= \frac{x^2+2x+6}{x^2+x-2} \\
 \text{(c)} \quad \frac{s^2t}{u} \cdot \frac{ut}{-2} &= \frac{s^2t^2u}{-2u} = -\frac{s^2t^2}{2} \\
 \text{(d)} \quad \frac{\frac{x}{y} + 1}{1 - \frac{y}{x}} &= \frac{\frac{x+y}{y}}{\frac{x-y}{x}} = \frac{x+y}{y} \times \frac{x}{x-y} = \frac{x(x+y)}{y(x-y)} = \frac{x^2+xy}{xy-y^2}
 \end{aligned}$$

**▲ Factoring**

We have used the Distributive Law to expand certain algebraic expressions. We sometimes need to reverse this process (again using the Distributive Law) by factoring an expression as a product of simpler ones. The easiest situation occurs when the expression has a common factor as follows:

$$\begin{array}{c}
 \text{Expanding} \longrightarrow \\
 3x(x-2) = 3x^2 - 6x \\
 \longleftarrow \text{Factoring}
 \end{array}$$

To factor a quadratic of the form  $x^2 + bx + c$  we note that

$$(x+r)(x+s) = x^2 + (r+s)x + rs$$

so we need to choose numbers  $r$  and  $s$  so that  $r+s = b$  and  $rs = c$ .

**EXAMPLE 4** Factor  $x^2 + 5x - 24$ .

**SOLUTION** The two integers that add to give 5 and multiply to give  $-24$  are  $-3$  and 8. Therefore

$$x^2 + 5x - 24 = (x-3)(x+8)$$

**EXAMPLE 5** Factor  $2x^2 - 7x - 4$ .

**SOLUTION** Even though the coefficient of  $x^2$  is not 1, we can still look for factors of the form  $2x+r$  and  $x+s$ , where  $rs = -4$ . Experimentation reveals that

$$2x^2 - 7x - 4 = (2x+1)(x-4)$$

Some special quadratics can be factored by using Equations 1 or 2 (from right to left) or by using the formula for a difference of squares:

**3**

$$a^2 - b^2 = (a-b)(a+b)$$

The analogous formula for a difference of cubes is

$$\boxed{4} \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

which you can verify by expanding the right side. For a sum of cubes we have

$$\boxed{5} \quad a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

#### EXAMPLE 6

- (a)  $x^2 - 6x + 9 = (x - 3)^2$  (Equation 2;  $a = x, b = 3$ )  
 (b)  $4x^2 - 25 = (2x - 5)(2x + 5)$  (Equation 3;  $a = 2x, b = 5$ )  
 (c)  $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$  (Equation 5;  $a = x, b = 2$ )

**EXAMPLE 7** Simplify  $\frac{x^2 - 16}{x^2 - 2x - 8}$ .

**SOLUTION** Factoring numerator and denominator, we have

$$\frac{x^2 - 16}{x^2 - 2x - 8} = \frac{(x - 4)(x + 4)}{(x - 4)(x + 2)} = \frac{x + 4}{x + 2}$$

To factor polynomials of degree 3 or more, we sometimes use the following fact.

**6 The Factor Theorem** If  $P$  is a polynomial and  $P(b) = 0$ , then  $x - b$  is a factor of  $P(x)$ .

**EXAMPLE 8** Factor  $x^3 - 3x^2 - 10x + 24$ .

**SOLUTION** Let  $P(x) = x^3 - 3x^2 - 10x + 24$ . If  $P(b) = 0$ , where  $b$  is an integer, then  $b$  is a factor of 24. Thus, the possibilities for  $b$  are  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12$ , and  $\pm 24$ . We find that  $P(1) = 12, P(-1) = 30, P(2) = 0$ . By the Factor Theorem,  $x - 2$  is a factor. Instead of substituting further, we use long division as follows:

$$\begin{array}{r} x^2 - x - 12 \\ x - 2 \overline{) x^3 - 3x^2 - 10x + 24} \\ \underline{x^3 - 2x^2} \phantom{+ 24} \\ -x^2 - 10x \phantom{+ 24} \\ \underline{-x^2 + 2x} \phantom{+ 24} \\ -12x + 24 \\ \underline{-12x + 24} \\ 0 \end{array}$$

Therefore  $x^3 - 3x^2 - 10x + 24 = (x - 2)(x^2 - x - 12)$   
 $= (x - 2)(x + 3)(x - 4)$

## Completing the Square

Completing the square is a useful technique for graphing parabolas or integrating rational functions. Completing the square means rewriting a quadratic  $ax^2 + bx + c$

in the form  $a(x + p)^2 + q$  and can be accomplished by:

1. Factoring the number  $a$  from the terms involving  $x$ .
2. Adding and subtracting the square of half the coefficient of  $x$ .

In general, we have

$$\begin{aligned} ax^2 + bx + c &= a \left[ x^2 + \frac{b}{a}x \right] + c \\ &= a \left[ x^2 + \frac{b}{a}x + \left( \frac{b}{2a} \right)^2 - \left( \frac{b}{2a} \right)^2 \right] + c \\ &= a \left( x + \frac{b}{2a} \right)^2 + \left( c - \frac{b^2}{4a} \right) \end{aligned}$$

**EXAMPLE 9** Rewrite  $x^2 + x + 1$  by completing the square.

**SOLUTION** The square of half the coefficient of  $x$  is  $\frac{1}{4}$ . Thus

$$x^2 + x + 1 = x^2 + x + \frac{1}{4} - \frac{1}{4} + 1 = \left( x + \frac{1}{2} \right)^2 + \frac{3}{4}$$

**EXAMPLE 10**

$$\begin{aligned} 2x^2 - 12x + 11 &= 2[x^2 - 6x] + 11 = 2[x^2 - 6x + 9 - 9] + 11 \\ &= 2[(x - 3)^2 - 9] + 11 = 2(x - 3)^2 - 7 \end{aligned}$$

## Quadratic Formula

By completing the square as above we can obtain the following formula for the roots of a quadratic equation.

**7 The Quadratic Formula** The roots of the quadratic equation  $ax^2 + bx + c = 0$  are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**EXAMPLE 11** Solve the equation  $5x^2 + 3x - 3 = 0$ .

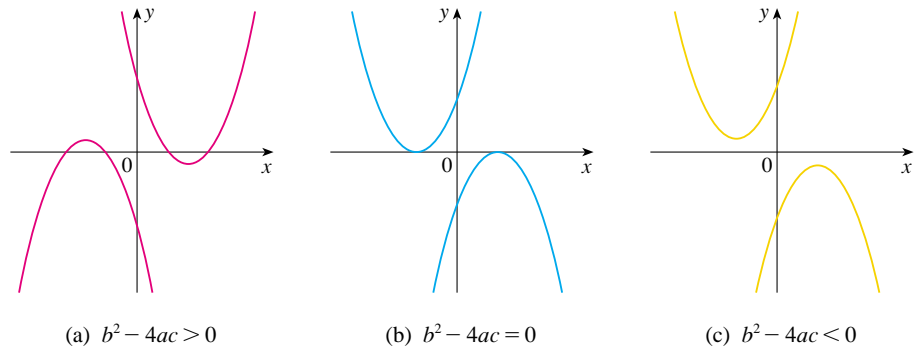
**SOLUTION** With  $a = 5$ ,  $b = 3$ ,  $c = -3$ , the quadratic formula gives the solutions

$$x = \frac{-3 \pm \sqrt{3^2 - 4(5)(-3)}}{2(5)} = \frac{-3 \pm \sqrt{69}}{10}$$

The quantity  $b^2 - 4ac$  that appears in the quadratic formula is called the **discriminant**. There are three possibilities:

1. If  $b^2 - 4ac > 0$ , the equation has two real roots.
2. If  $b^2 - 4ac = 0$ , the roots are equal.
3. If  $b^2 - 4ac < 0$ , the equation has no real root. (The roots are complex.)

These three cases correspond to the fact that the number of times the parabola  $y = ax^2 + bx + c$  crosses the  $x$ -axis is 2, 1, or 0 (see Figure 1). In case (3) the quadratic  $ax^2 + bx + c$  can't be factored and is called **irreducible**.



**FIGURE 1**

Possible graphs of  $y = ax^2 + bx + c$

**EXAMPLE 12** The quadratic  $x^2 + x + 2$  is irreducible because its discriminant is negative:

$$b^2 - 4ac = 1^2 - 4(1)(2) = -7 < 0$$

Therefore, it is impossible to factor  $x^2 + x + 2$ . ■

### The Binomial Theorem

Recall the binomial expression from Equation 1:

$$(a + b)^2 = a^2 + 2ab + b^2$$

If we multiply both sides by  $(a + b)$  and simplify, we get the binomial expansion

$$\boxed{8} \quad (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Repeating this procedure, we get

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

In general, we have the following formula.

**9 The Binomial Theorem** If  $k$  is a positive integer, then

$$\begin{aligned} (a + b)^k &= a^k + ka^{k-1}b + \frac{k(k-1)}{1 \cdot 2} a^{k-2}b^2 \\ &\quad + \frac{k(k-1)(k-2)}{1 \cdot 2 \cdot 3} a^{k-3}b^3 \\ &\quad + \cdots + \frac{k(k-1) \cdots (k-n+1)}{1 \cdot 2 \cdot 3 \cdots n} a^{k-n}b^n \\ &\quad + \cdots + kab^{k-1} + b^k \end{aligned}$$

**EXAMPLE 13** Expand  $(x - 2)^5$ .

**SOLUTION** Using the Binomial Theorem with  $a = x$ ,  $b = -2$ ,  $k = 5$ , we have

$$\begin{aligned}(x - 2)^5 &= x^5 + 5x^4(-2) + \frac{5 \cdot 4}{1 \cdot 2}x^3(-2)^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3}x^2(-2)^3 + 5x(-2)^4 + (-2)^5 \\ &= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32\end{aligned}$$

## Radicals

The most commonly occurring radicals are square roots. The symbol  $\sqrt{\quad}$  means “the positive square root of.” Thus

$$x = \sqrt{a} \quad \text{means} \quad x^2 = a \quad \text{and} \quad x \geq 0$$

Since  $a = x^2 \geq 0$ , the symbol  $\sqrt{a}$  makes sense only when  $a \geq 0$ . Here are two rules for working with square roots:

10

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

However, there is no similar rule for the square root of a sum. In fact, you should remember to avoid the following common error:

$$\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$$

(For instance, take  $a = 9$  and  $b = 16$  to see the error.)

**EXAMPLE 14**

$$(a) \frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

$$(b) \sqrt{x^2y} = \sqrt{x^2}\sqrt{y} = |x|\sqrt{y}$$

Notice that  $\sqrt{x^2} = |x|$  because  $\sqrt{\quad}$  indicates the positive square root. (See Appendix A.)

In general, if  $n$  is a positive integer,

$$\begin{aligned}x &= \sqrt[n]{a} \quad \text{means} \quad x^n = a \\ \text{If } n \text{ is even, then } a &\geq 0 \text{ and } x \geq 0.\end{aligned}$$

Thus  $\sqrt[3]{-8} = -2$  because  $(-2)^3 = -8$ , but  $\sqrt[4]{-8}$  and  $\sqrt[6]{-8}$  are not defined. The following rules are valid:

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

**EXAMPLE 15**  $\sqrt[3]{x^4} = \sqrt[3]{x^3x} = \sqrt[3]{x^3}\sqrt[3]{x} = x\sqrt[3]{x}$



To **rationalize** a numerator or denominator that contains an expression such as  $\sqrt{a} - \sqrt{b}$ , we multiply both the numerator and the denominator by the conjugate radical  $\sqrt{a} + \sqrt{b}$ . Then we can take advantage of the formula for a difference of squares:

$$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$$

**EXAMPLE 16** Rationalize the numerator in the expression  $\frac{\sqrt{x+4} - 2}{x}$ .

**SOLUTION** We multiply the numerator and the denominator by the conjugate radical  $\sqrt{x+4} + 2$ :

$$\begin{aligned} \frac{\sqrt{x+4} - 2}{x} &= \left( \frac{\sqrt{x+4} - 2}{x} \right) \left( \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2} \right) = \frac{(x+4) - 4}{x(\sqrt{x+4} + 2)} \\ &= \frac{x}{x(\sqrt{x+4} + 2)} = \frac{1}{\sqrt{x+4} + 2} \end{aligned}$$

## ▲ Exponents

Let  $a$  be any positive number and let  $n$  be a positive integer. Then, by definition,

1.  $a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$
2.  $a^0 = 1$
3.  $a^{-n} = \frac{1}{a^n}$
4.  $a^{1/n} = \sqrt[n]{a}$   
 $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$   $m$  is any integer

**11 Laws of Exponents** Let  $a$  and  $b$  be positive numbers and let  $r$  and  $s$  be any rational numbers (that is, ratios of integers). Then

1.  $a^r \times a^s = a^{r+s}$
2.  $\frac{a^r}{a^s} = a^{r-s}$
3.  $(a^r)^s = a^{rs}$
4.  $(ab)^r = a^r b^r$
5.  $\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$   $b \neq 0$

In words, these five laws can be stated as follows:

1. To multiply two powers of the same number, we add the exponents.
2. To divide two powers of the same number, we subtract the exponents.
3. To raise a power to a new power, we multiply the exponents.
4. To raise a product to a power, we raise each factor to the power.
5. To raise a quotient to a power, we raise both numerator and denominator to the power.

## EXAMPLE 17

(a)  $2^8 \times 8^2 = 2^8 \times (2^3)^2 = 2^8 \times 2^6 = 2^{14}$

(b) 
$$\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} = \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} + \frac{1}{y}} = \frac{\frac{y^2 - x^2}{x^2 y^2}}{\frac{y + x}{xy}} = \frac{y^2 - x^2}{x^2 y^2} \cdot \frac{xy}{y + x}$$
$$= \frac{(y - x)(y + x)}{xy(y + x)} = \frac{y - x}{xy}$$

(c)  $4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$     Alternative solution:  $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$

(d)  $\frac{1}{\sqrt[3]{x^4}} = \frac{1}{x^{4/3}} = x^{-4/3}$

(e)  $\left(\frac{x}{y}\right)^3 \left(\frac{y^2 x}{z}\right)^4 = \frac{x^3}{y^3} \cdot \frac{y^8 x^4}{z^4} = x^7 y^5 z^{-4}$



## Exercises

**A** [Click here for answers.](#)

1–16 ■ Expand and simplify.

- |  |                       |
|--|-----------------------|
| 1. $(-6ab)(0.5ac)$                     | 2. $-(2x^2y)(-xy^4)$  |
| 3. $2x(x - 5)$                         | 4. $(4 - 3x)x$        |
| 5. $-2(4 - 3a)$                        | 6. $8 - (4 + x)$      |
| 7. $4(x^2 - x + 2) - 5(x^2 - 2x + 1)$  |                       |
| 8. $5(3t - 4) - (t^2 + 2) - 2t(t - 3)$ |                       |
| 9. $(4x - 1)(3x + 7)$                  | 10. $x(x - 1)(x + 2)$ |
| 11. $(2x - 1)^2$                       | 12. $(2 + 3x)^2$      |
| 13. $y^4(6 - y)(5 + y)$                |                       |
| 14. $(t - 5)^2 - 2(t + 3)(8t - 1)$     |                       |
| 15. $(1 + 2x)(x^2 - 3x + 1)$           | 16. $(1 + x - x^2)^2$ |

17–28 ■ Perform the indicated operations and simplify.

- |  |  |
|--|--|
| 17. $\frac{2 + 8x}{2}$                                       | 18. $\frac{9b - 6}{3b}$                            |
| 19. $\frac{1}{x + 5} + \frac{2}{x - 3}$                      | 20. $\frac{1}{x + 1} + \frac{1}{x - 1}$            |
| 21. $u + 1 + \frac{u}{u + 1}$                                | 22. $\frac{2}{a^2} - \frac{3}{ab} + \frac{4}{b^2}$ |
| 23. $\frac{x/y}{z}$  | 24. $\frac{x}{y/z}$                                |
| 25. $\left(\frac{-2r}{s}\right)\left(\frac{s^2}{-6t}\right)$ | 26. $\frac{a}{bc} \div \frac{b}{ac}$               |

27. 
$$\frac{1 + \frac{1}{c - 1}}{1 - \frac{1}{c - 1}}$$

28. 
$$1 + \frac{1}{1 + \frac{1}{1 + x}}$$

29–48 ■ Factor the expression.

- |                            |                             |
|----------------------------|-----------------------------|
| 29. $2x + 12x^3$           | 30. $5ab - 8abc$            |
| 31. $x^2 + 7x + 6$         | 32. $x^2 - x - 6$           |
| 33. $x^2 - 2x - 8$         | 34. $2x^2 + 7x - 4$         |
| 35. $9x^2 - 36$            | 36. $8x^2 + 10x + 3$        |
| 37. $6x^2 - 5x - 6$        | 38. $x^2 + 10x + 25$        |
| 39. $t^3 + 1$              | 40. $4t^2 - 9s^2$           |
| 41. $4t^2 - 12t + 9$       | 42. $x^3 - 27$              |
| 43. $x^3 + 2x^2 + x$       | 44. $x^3 - 4x^2 + 5x - 2$   |
| 45. $x^3 + 3x^2 - x - 3$   | 46. $x^3 - 2x^2 - 23x + 60$ |
| 47. $x^3 + 5x^2 - 2x - 24$ | 48. $x^3 - 3x^2 - 4x + 12$  |

49–54 ■ Simplify the expression.

- |   |  |
|---|--|
| 49. $\frac{x^2 + x - 2}{x^2 - 3x + 2}$    | 50. $\frac{2x^2 - 3x - 2}{x^2 - 4}$        |
| 51. $\frac{x^2 - 1}{x^2 - 9x + 8}$        | 52. $\frac{x^3 + 5x^2 + 6x}{x^2 - x - 12}$ |
| 53. $\frac{1}{x + 3} + \frac{1}{x^2 - 9}$ |  |

54.  $\frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4}$

55–60 ■ Complete the square.

55.  $x^2 + 2x + 5$                       56.  $x^2 - 16x + 80$   
 57.  $x^2 - 5x + 10$                     58.  $x^2 + 3x + 1$   
 59.  $4x^2 + 4x - 2$                     60.  $3x^2 - 24x + 50$

61–68 ■ Solve the equation.

61.  $x^2 + 9x - 10 = 0$                 62.  $x^2 - 2x - 8 = 0$   
 63.  $x^2 + 9x - 1 = 0$                 64.  $x^2 - 2x - 7 = 0$   
 65.  $3x^2 + 5x + 1 = 0$                66.  $2x^2 + 7x + 2 = 0$   
 67.  $x^3 - 2x + 1 = 0$                 68.  $x^3 + 3x^2 + x - 1 = 0$

69–72 ■ Which of the quadratics are irreducible?

69.  $2x^2 + 3x + 4$                     70.  $2x^2 + 9x + 4$   
 71.  $3x^2 + x - 6$                     72.  $x^2 + 3x + 6$

73–76 ■ Use the Binomial Theorem to expand the expression.

73.  $(a + b)^6$                             74.  $(a + b)^7$   
 75.  $(x^2 - 1)^4$                         76.  $(3 + x^2)^5$

77–82 ■ Simplify the radicals.

77.  $\sqrt{32}\sqrt{2}$                     78.  $\frac{\sqrt[3]{-2}}{\sqrt[3]{54}}$                     79.  $\frac{\sqrt[4]{32x^4}}{\sqrt[4]{2}}$   
 80.  $\sqrt{xy}\sqrt{x^3y}$                 81.  $\sqrt{16a^4b^3}$                 82.  $\frac{\sqrt[5]{96a^6}}{\sqrt[5]{3a}}$

83–100 ■ Use the Laws of Exponents to rewrite and simplify the expression.

83.  $3^{10} \times 9^8$                             84.  $2^{16} \times 4^{10} \times 16^6$

85.  $\frac{x^9(2x)^4}{x^3}$

87.  $\frac{a^{-3}b^4}{a^{-5}b^5}$

89.  $3^{-1/2}$

91.  $125^{2/3}$

93.  $(2x^2y^4)^{3/2}$

95.  $\sqrt[5]{y^6}$

97.  $\frac{1}{(\sqrt{t})^5}$

99.  $\sqrt[4]{\frac{t^{1/2}\sqrt{st}}{s^{2/3}}}$

86.  $\frac{a^n \times a^{2n+1}}{a^{n-2}}$

88.  $\frac{x^{-1} + y^{-1}}{(x + y)^{-1}}$

90.  $96^{1/5}$

92.  $64^{-4/3}$

94.  $(x^{-5}y^3z^{10})^{-3/5}$

96.  $(\sqrt[4]{a})^3$

98.  $\frac{\sqrt[8]{x^5}}{\sqrt[4]{x^3}}$

100.  $\sqrt[4]{r^{2n+1}} \times \sqrt[4]{r^{-1}}$

101–108 ■ Rationalize the expression.

101.  $\frac{\sqrt{x} - 3}{x - 9}$

102.  $\frac{(1/\sqrt{x}) - 1}{x - 1}$

103.  $\frac{x\sqrt{x} - 8}{x - 4}$

104.  $\frac{\sqrt{2+h} + \sqrt{2-h}}{h}$

105.  $\frac{2}{3 - \sqrt{5}}$

106.  $\frac{1}{\sqrt{x} - \sqrt{y}}$

107.  $\sqrt{x^2 + 3x + 4} - x$

108.  $\sqrt{x^2 + x} - \sqrt{x^2 - x}$

109–116 ■ State whether or not the equation is true for all values of the variable.

109.  $\sqrt{x^2} = x$

110.  $\sqrt{x^2 + 4} = |x| + 2$

111.  $\frac{16 + a}{16} = 1 + \frac{a}{16}$

112.  $\frac{1}{x^{-1} + y^{-1}} = x + y$

113.  $\frac{x}{x + y} = \frac{1}{1 + y}$

114.  $\frac{2}{4 + x} = \frac{1}{2} + \frac{2}{x}$

115.  $(x^3)^4 = x^7$

116.  $6 - 4(x + a) = 6 - 4x - 4a$



## Answers

1.  $-3a^2bc$     2.  $2x^3y^5$     3.  $2x^2 - 10x$     4.  $4x - 3x^2$   
 5.  $-8 + 6a$     6.  $4 - x$     7.  $-x^2 + 6x + 3$   
 8.  $-3t^2 + 21t - 22$     9.  $12x^2 + 25x - 7$   
 10.  $x^3 + x^2 - 2x$     11.  $4x^2 - 4x + 1$   
 12.  $9x^2 + 12x + 4$     13.  $30y^4 + y^5 - y^6$   
 14.  $-15t^2 - 56t + 31$     15.  $2x^3 - 5x^2 - x + 1$   
 16.  $x^4 - 2x^3 - x^2 + 2x + 1$     17.  $1 + 4x$     18.  $3 - 2/b$   
 19.  $\frac{3x + 7}{x^2 + 2x - 15}$     20.  $\frac{2x}{x^2 - 1}$     21.  $\frac{u^2 + 3u + 1}{u + 1}$   
 22.  $\frac{2b^2 - 3ab + 4a^2}{a^2b^2}$     23.  $\frac{x}{yz}$     24.  $\frac{zx}{y}$     25.  $\frac{rs}{3t}$   
 26.  $\frac{a^2}{b^2}$     27.  $\frac{c}{c - 2}$     28.  $\frac{3 + 2x}{2 + x}$     29.  $2x(1 + 6x^2)$   
 30.  $ab(5 - 8c)$     31.  $(x + 6)(x + 1)$     32.  $(x - 3)(x + 2)$   
 33.  $(x - 4)(x + 2)$     34.  $(2x - 1)(x + 4)$   
 35.  $9(x - 2)(x + 2)$     36.  $(4x + 3)(2x + 1)$   
 37.  $(3x + 2)(2x - 3)$     38.  $(x + 5)^2$   
 39.  $(t + 1)(t^2 - t + 1)$     40.  $(2t - 3s)(2t + 3s)$   
 41.  $(2t - 3)^2$     42.  $(x - 3)(x^2 + 3x + 9)$   
 43.  $x(x + 1)^2$     44.  $(x - 1)^2(x - 2)$   
 45.  $(x - 1)(x + 1)(x + 3)$     46.  $(x - 3)(x + 5)(x - 4)$   
 47.  $(x - 2)(x + 3)(x + 4)$     48.  $(x - 2)(x - 3)(x + 2)$   
 49.  $\frac{x + 2}{x - 2}$     50.  $\frac{2x + 1}{x + 2}$     51.  $\frac{x + 1}{x - 8}$     52.  $\frac{x(x + 2)}{x - 4}$   
 53.  $\frac{x - 2}{x^2 - 9}$     54.  $\frac{x^2 - 6x - 4}{(x - 1)(x + 2)(x - 4)}$   
 55.  $(x + 1)^2 + 4$     56.  $(x - 8)^2 + 16$     57.  $(x - \frac{5}{2})^2 + \frac{15}{4}$   
 58.  $(x + \frac{3}{2})^2 - \frac{5}{4}$     59.  $(2x + 1)^2 - 3$   
 60.  $3(x - 4)^2 + 2$     61.  $1, -10$     62.  $-2, 4$
63.  $\frac{-9 \pm \sqrt{85}}{2}$     64.  $1 \pm 2\sqrt{2}$     65.  $\frac{-5 \pm \sqrt{13}}{6}$   
 66.  $\frac{-7 \pm \sqrt{33}}{4}$     67.  $1, \frac{-1 \pm \sqrt{5}}{2}$     68.  $-1, -1 \pm \sqrt{2}$   
 69. Irreducible    70. Not irreducible  
 71. Not irreducible (two real roots)    72. Irreducible  
 73.  $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$   
 74.  $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$   
 75.  $x^8 - 4x^6 + 6x^4 - 4x^2 + 1$   
 76.  $243 + 405x^2 + 270x^4 + 90x^6 + 15x^8 + x^{10}$   
 77.  $8$     78.  $-\frac{1}{3}$     79.  $2|x|$     80.  $x^2|y|$   
 81.  $4a^2b\sqrt{b}$     82.  $2a$     83.  $3^{26}$     84.  $2^{60}$     85.  $16x^{10}$   
 86.  $a^{2n+3}$     87.  $\frac{a^2}{b}$     88.  $\frac{(x + y)^2}{xy}$     89.  $\frac{1}{\sqrt{3}}$   
 90.  $2^5\sqrt{3}$     91.  $25$     92.  $\frac{1}{256}$     93.  $2\sqrt{2}|x|^3y^6$   
 94.  $\frac{x^3}{y^{9/5}z^6}$     95.  $y^{6/5}$     96.  $a^{3/4}$     97.  $t^{-5/2}$     98.  $\frac{1}{x^{1/8}}$   
 99.  $\frac{t^{1/4}}{s^{1/24}}$     100.  $r^{n/2}$     101.  $\frac{1}{\sqrt{x + 3}}$     102.  $\frac{-1}{\sqrt{x + x}}$   
 103.  $\frac{x^2 + 4x + 16}{x\sqrt{x + 8}}$     104.  $\frac{2}{\sqrt{2 + h} - \sqrt{2 - h}}$   
 105.  $\frac{3 + \sqrt{5}}{2}$     106.  $\frac{\sqrt{x} + \sqrt{y}}{x - y}$   
 107.  $\frac{3x + 4}{\sqrt{x^2 + 3x + 4} + x}$     108.  $\frac{2x}{\sqrt{x^2 + x} + \sqrt{x^2 - x}}$   
 109. False    110. False    111. True    112. False  
 113. False    114. False    115. False    116. True