Accelerated Algebra II with Statistics and Precalculus
Overview

Accelerated Algebra II with Statistics and Precalculus is a course that combines the courses of study for the advanced levels of both the newly-designed Algebra II with Statistics course and the Precalculus course. This course can be taken in lieu of Algebra II with Statistics and following the successful completion of Geometry with Data Analysis and either Algebra I with Probability or the middle school accelerated sequence. It is the culmination of the three years of required mathematics content and is intended for students who wish to pursue careers in science, technology, engineering, or mathematics (STEM) that require the study of calculus. It prepares students for the AP Calculus BC/Calculus III/Introduction to Linear Algebra track.

Accelerated Algebra II with Statistics and Precalculus builds essential concepts necessary for students to meet their postsecondary goals (whether they pursue additional study or enter the workforce), function as effective citizens, and recognize the wonder, joy, and beauty of mathematics (NCTM, 2018). In particular, it builds foundational knowledge of algebra and functions needed for students to take the specialized courses which follow it. This course also focuses on inferential statistics, which allows students to draw conclusions about populations and cause-and-effect based on random samples and controlled experiments.

In *Accelerated Algebra II with Statistics and Precalculus*, students incorporate knowledge and skills from several mathematics content areas, leading to a deeper understanding of fundamental relationships within the discipline and building a solid foundation for further study. In the content area of Algebra and Functions, students explore an expanded range of functions, including polynomial, rational, trigonometric, logarithmic, radical, and general piecewise-defined functions. Students also solve equations associated with these classes of functions. In addition to focusing on the families of functions, a deeper look at functions as a system will be investigated, including composition of functions and inverses. Additionally, vectors and their operations will be considered in this course. In the content area of Data Analysis, Statistics, and Probability, students learn how to make inferences about a population from a random sample drawn from the population and how to analyze cause-and-effect by conducting randomized experiments. Students are introduced to the study of matrices in the Number and Quantity content area.

A focus on mathematical modeling and real-world statistical problem-solving is included across the course. It is essential for students to use technology and other mathematical tools such as graphing calculators, online graphing software, and spreadsheets to explore functions, equations, and analyze data.

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both within the classroom and in life. **The Student Mathematical Practices are standards to be incorporated across all grades.**

Student Mathematical Practices	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

The standards indicating what students should know or be able to do by the end of the course are listed in the right columns of the content area tables. The essential concepts are described in the left columns of the content area tables. In some cases, focus areas are indicated within the tables. Only those focus areas which are appropriate for this course are included.

Statements in **bold print** indicate the scope of the standard and align the standard to related content in other courses. The full scope of every standard should be addressed during instruction.

Rather than teach the contents in sequential order (Algebra 2 in the first semester, Precalculus in the second semester), the curricula have been merged for greater breadth and depth within individual topics over that of the individual courses. The topics have also been resequenced from the individual courses for better flow regarding prerequisite knowledge.

Students in the *Accelerated Algebra II with Statistics and Precalculus* class will have a different learning experience than students in either regular- or advanced-level courses. There is an expectation that comprehension and proficiency will be more profound, and students will study concepts in greater breadth and depth, including enrichment and extensions into applications. Students will use higher-level thinking skills as they explore the content and more abstract thinking will be necessary. Students will be required to complete more work outside of class than in a regular- or advanced-level course. Assessments will be more complex and will require that students organize thoughts, make connections, and analyze more efficiently. Greater emphasis will be placed on theory and process as well.

Algebra II with Statistics Content Standards

Each content standard completes the stem *"Students will…"*

Number and Quantity	
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	1. Identify numbers written in the form $a + bi$, where a and b are real numbers and $i^2 = -1$, as complex numbers. a. Add, subtract, multiply, and divide complex numbers using the commutative, associative, and distributive properties. b. Represent complex numbers on the complex plane in rectangular form. **c. Extend to using synthetic division with complex roots.**
Matrices are a useful way to represent information.	2. Use matrices to represent and manipulate data. 3. Multiply matrices by scalars to produce new matrices. 4. Add, subtract, and multiply matrices of appropriate dimensions. 5. Describe the roles that zero and identity matrices play in matrix addition and multiplication, recognizing that they are similar to the roles of 0 and 1 in the real numbers. a. Find the additive and multiplicative inverses of square matrices, using technology as appropriate. b. Explain the role of the determinant in determining if a square matrix has a multiplicative inverse.

Algebra and Functions

Focus 1: Algebra	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	6. Factor polynomials using common factoring techniques, and use the factored form of a polynomial to reveal the zeros of the function it defines. 7. Prove polynomial identities and use them to describe numerical relationships. *Example: The polynomial identity* $1 - x^n = (1 - x)(1 + x + x^2 + x^3 + \ldots + x^{n-1} + x^n)$ *can be used to find the sum of the first* n *terms of a geometric sequence with common ratio* x *by dividing both sides of the identity by* $(1 - x)$.
Use polynomial identities to solve problems.	8. **Additional Standard: Know and apply the Binomial Theorem for the expansion of** $(x + y)^n$ **in powers of x and y for a positive integer, n, where x and y are any numbers.**
Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.	9. Explain why extraneous solutions to an equation may arise and how to check to be sure that a candidate solution satisfies an equation**. Extend to radical equations.**
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	10. For exponential models, express as a logarithm the solution to $ab^{ct} = d$, where a, c, and d are real numbers and the base b is 2 or 10; evaluate the logarithm using technology to solve an exponential equation. Extend to additional bases.

Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	**10.** Create equations and inequalities in one variable and use them to solve problems**. Extend to equations arising from polynomial, trigonometric (sine and cosine and tangent (and their reciprocals)), logarithmic, radical, and general piecewise functions.** 11. Solve quadratic equations with real coefficients that have complex solutions. 12. Solve simple equations involving exponential, radical, logarithmic, and trigonometric functions using inverse functions.
Rewrite rational expressions.	13. Additional Standard: Add, subtract, multiply, and divide rational expressions. **Extend to partial fraction decomposition.**

	14. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales and use them to make predictions. **Extend to polynomial, trigonometric (sine and cosine** and tangent (**and their reciprocals)), logarithmic, reciprocal, radical, and general piecewise functions.**
Focus 2: Connecting Algebra to Functions	
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	15. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. **a.** Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. **Extend to cases where *f(x)* and/or *g(x)* are polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.**
Focus 3: Functions	
Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., $f(x) = x^2$), recursive definitions, tables, and graphs.	16. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Extend to polynomial, trigonometric (sine and cosine** and tangent (**and their reciprocals)), logarithmic, radical, and general piecewise functions**.
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	17. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k \cdot f(x)$, $f(k \cdot x)$, and $(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.**

Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	18. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries (including even and odd); end behavior; and periodicity.* **Extend to polynomial, trigonometric (sine and cosine** and tangent (**and their reciprocals)), logarithmic, reciprocal, radical, and general piecewise functions.** 19. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **Extend to polynomial, trigonometric (sine and cosine** and tangent (**and their reciprocals)), logarithmic, reciprocal, radical, and general piecewise functions.** Include compositions of functions. 20. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Extend to polynomial, trigonometric (sine and cosine** and tangent (**and their reciprocals)), logarithmic, reciprocal, radical, and general piecewise functions.** 21. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **Extend to polynomial, trigonometric (sine and cosine** and tangent (**and their reciprocals)), logarithmic, reciprocal, radical, and general piecewise functions.** a. Graph polynomial functions expressed symbolically, identifying zeros when suitable factorizations are available, and showing end behavior. b. Graph sine and cosine and tangent (**and their reciprocals)** functions expressed symbolically, showing period, midline, and amplitude. c. Graph logarithmic functions expressed symbolically, showing intercepts and end behavior. d. Graph reciprocal and other rational functions expressed symbolically, identifying horizontal and vertical asymptotes. e. Graph square root and cube root functions expressed symbolically. f. Compare the graphs of inverse functions and the relationships between their key features, including but not limited to quadratic, square root, exponential, and logarithmic functions.

	22. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle, building on work with non-right triangle trigonometry.
Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.	23. Use the mathematical modeling cycle to solve real-world problems involving polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions, from the simplification of the problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility.
Build new functions from existing functions.	24. Additional Standard: Graph conic sections from second-degree equations, including circles and parabolas **as well as ellipses and hyperbolas.**

Data Analysis, Statistics, and Probability	
Focus 1: Quantitative Literacy	
Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	25. Use mathematical and statistical reasoning about normal distributions to draw conclusions and assess risk; limit to informal arguments. *Example: If candidate A is leading candidate B by 2% in a poll which has a margin of error of less than 3%, should we be surprised if candidate B wins the election?*
Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.	26. Design and carry out an experiment or survey to answer a question of interest, and write an informal persuasive argument based on the results. *Example: Use the statistical problem-solving cycle to answer the question, "Is there an association between playing a musical instrument and doing well in mathematics?"*

Focus 2: Visualizing and Summarizing Data	
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	27. From a normal distribution, use technology to find the mean and standard deviation and estimate population percentages by applying the empirical rule. a. Use technology to determine if a given set of data is normal by applying the empirical rule. b. Estimate areas under a normal curve to solve problems in context, using calculators, spreadsheets, and tables as appropriate.
Focus 3: Statistical Inference	
Study designs are of three main types: sample survey, experiment, and observational study.	28. Describe the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. *Examples: random assignment in experiments, random selection in surveys and observational studies*
The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups.	29. Distinguish between a statistic and a parameter and use statistical processes to make inferences about population parameters based on statistics from random samples from that population. 30. Describe differences between randomly selecting samples and randomly assigning subjects to experimental treatment groups in terms of inferences drawn regarding a population versus regarding cause and effect. *Example: Data from a group of plants randomly selected from a field allows inference regarding the rest of the plants in the field, while randomly assigning each plant to one of two treatments allows inference regarding differences in the effects of the two treatments. If the plants were both randomly selected and randomly assigned, we can infer that the difference in effects of the two treatments would also be observed when applied to the rest of the plants in the field.*

The scope and validity of statistical inferences are dependent on the role of randomization in the study design.	31. Explain the consequences, due to uncontrolled variables, of non-randomized assignment of subjects to groups in experiments. *Example: Students are studying whether or not listening to music while completing mathematics homework improves their quiz scores. Rather than assigning students to either listen to music or not at random, they simply observe what the students do on their own and find that the music-listening group has a higher mean quiz score. Can they conclude that listening to music while studying is likely to raise the quiz scores of students who do not already listen to music? What other factors may have been responsible for the observed difference in mean quiz scores?*
Bias, such as sampling, response, or nonresponse bias, may occur in surveys, yielding results that are not representative of the population of interest.	32. Evaluate where bias, including sampling, response, or nonresponse bias, may occur in surveys, and whether results are representative of the population of interest. *Example: Selecting students eating lunch in the cafeteria to participate in a survey may not accurately represent the student body, as students who do not eat in the cafeteria may not be accounted for and may have different opinions, or students may not respond honestly to questions that may be embarrassing, such as how much time they spend on homework.*
The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic.	33. Evaluate the effect of sample size on the expected variability in the sampling distribution of a sample statistic. a. Simulate a sampling distribution of sample means from a population with a known distribution, observing the effect of the sample size on the variability. b. Demonstrate that the standard deviation of each simulated sampling distribution is the known standard deviation of the population divided by the square root of the sample size.
The sampling distribution of a sample statistic formed from repeated samples for a given sample size drawn from a population can be used to identify typical behavior for that statistic. Examining several such sampling distributions leads to estimating a set of plausible values for the population parameter, using the margin of error as a measure that describes the sampling variability.	34. Produce a sampling distribution by repeatedly selecting samples of the same size from a given population or from a population simulated by bootstrapping (resampling with replacement from an observed sample). Do initial examples by hand, then use technology to generate a large number of samples. a. Verify that a sampling distribution is centered at the population mean and approximately normal if the sample size is large enough. b. Verify that 95% of sample means are within two standard deviations of the sampling distribution from the population mean. c. Create and interpret a 95% confidence interval based on an observed mean from a sampling distribution.

	35. Use data from a randomized experiment to compare two treatments; limit to informal use of simulations to decide if an observed difference in the responses of the two treatment groups is unlikely to have occurred due to randomization alone, thus implying that the difference between the treatment groups is meaningful. *Example: Fifteen students are randomly assigned to a treatment group that listens to music while completing mathematics homework and another 15 are assigned to a control group that does not, and their means on the next quiz are found to be different. To test whether the differences seem significant, all the scores from the two groups are placed on index cards and repeatedly shuffled into two new groups of 15 each, each time recording the difference in the means of the two groups. The differences in means of the treatment and control groups are then compared to the differences in means of the mixed groups to see how likely it is to occur.*

Geometry and Measurement

Focus 1: Measurement	
When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.	36. Define the radian measure of an angle as the constant of proportionality of the length of an arc it intercepts to the radius of the circle; in particular, it is the length of the arc intercepted on the unit circle.
Focus 2: Transformations (Note: There are no *Algebra II with Statistics* standards in Focus 2)	
Focus 3: Geometric Argument, Reasoning, and Proof (Note: There are no *Algebra II with Statistics* standards in Focus 3)	
Focus 4: Solving Applied Problems and Modeling in Geometry	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	37. Choose trigonometric functions (sine and cosine) to model periodic phenomena with specified amplitude, frequency, and midline. 38. Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to calculate trigonometric ratios. Extend to additional Pythagorean, **sum, difference, double, and half angle** identities. 39. Derive and apply the formula $A = ½ \cdot ab \cdot sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side, extending the domain of sine to include right and obtuse angles.

	40. Derive and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. **Extend the domain of sine and cosine to include right and obtuse angles.** *Examples: surveying problems, resultant forces*

Precalculus Content Standards

Each numbered standard completes the stem *"Students will…"*

Number and Quantity	
The Complex Number System	
Perform arithmetic operations with complex numbers.	1. Define the constant *e* in a variety of contexts. *Example: the total interest earned if a 100% annual rate is continuously compounded.* a. Explore the behavior of the function $y=e^x$ and its applications. b. Explore the behavior of $ln(x)$, the logarithmic function with base *e*, and its applications. 2. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Represent complex numbers and their operations on the complex plane.	3. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. 4. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *Example*: $(-1+\sqrt{3}i)^3 = 8$ *because* $(-1+\sqrt{3}i)$ *has modulus 2 and argument 120°.* 5. **Use Demoivre's Theorem to compute powers of complex numbers.** 6. **Reverse-engineer Demoivre's Theorem to find roots of complex numbers.** 7. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.	8. Analyze possible zeros for a polynomial function over the complex numbers by applying the Fundamental Theorem of Algebra, using a graph of the function, or factoring with algebraic identities. **Use additional techniques of Descartes's Rule of Signs, Intermediate Value Theorem, and bounds on real zeros for further zero analysis.**
Limits	
Understand limits of functions.	9. Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity. a. Apply limits of functions at specific values and at infinity in problems involving convergence and divergence.
Vector and Matrix Quantities	
Represent and model with vector quantities.	10. Explain that vector quantities have both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. *Examples:* **v**, |**v**|, ||**v**||, v. 11. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. 12. Solve problems involving velocity and other quantities that can be represented by vectors. 13. Find the scalar (dot) product of two vectors as the sum of the products of corresponding components and explain its relationship to the cosine of the angle formed by two vectors. **Use dot products to show orthogonality of two vectors.**

Perform operations on vectors.	14. Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule, understanding that the magnitude of a sum of two vectors is not always the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Explain vector subtraction, $\boldsymbol{v} - \boldsymbol{w}$, as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. 15. Multiply a vector by a scalar. *d.* Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. *Example*: $c(v_x, v_y) = (cv_x, cv_y)$ e. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = \|c\|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\| v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$). 16. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. 17. **Find cross products of three-dimensional vectors using determinants.** 18. **Use cross products to find vectors orthogonal to two vectors. Tie cross products to normal vectors of planes and to equations of planes.**

Algebra	
Seeing Structure in Expressions	
Write expressions in equivalent forms to solve problems.	19. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems, extending to infinite geometric series. *Examples: calculate mortgage payments; determine the long-term level of medication if a patient takes 50 mg of a medication every 4 hours, while 70% of the medication is filtered out of the patient's blood.*
Arithmetic With Polynomials and Rational Expressions	
Understand the relationship between zeros and factors of polynomials.	20. Derive and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
Use polynomial identities to solve problems.	21. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer, n, where x and y are any numbers.
Rewrite rational expressions.	22. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection; long division; **synthetic division, including divisors of degree 2 or more;** or, for the more complicated cases, a computer algebra system. 23. Add, subtract, multiply, and divide rational expressions. a. Explain why rational expressions form a system analogous to the rational numbers, which is closed under addition, subtraction, multiplication, and division by a non-zero rational expression. 24. **Use partial fraction decomposition to write rational expressions as a sum or difference of simple rational expressions.** 25. **Use Heaviside Cover-Up Method to achieve partial fraction decomposition for certain rational expressions, and identify situations where Heaviside Cover-Up Method is advantageous/not advantageous.**

Reasoning With Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	26. Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a clear-cut solution. Construct a viable argument to justify a solution method. **Include equations that may involve linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, and trigonometric functions, and their inverses**. 27. Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise.
Solve systems of equations.	28. Represent a system of linear equations as a single matrix equation in a vector variable. 29. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Functions	
Interpreting Functions	
Interpret functions that arise in applications in terms of the context.	30. Compare and contrast families of functions and their representations algebraically, graphically, numerically, and verbally in terms of their key features. *Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries (including even and odd); end behavior; asymptotes; and periodicity*. **Families of functions include but are not limited to linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, trigonometric, and their inverses.** 31. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Extend from polynomial, exponential, logarithmic, and radical to rational and all trigonometric functions.** a. Find the difference quotient $\frac{f(x+\Delta x)-f(x)}{\Delta x}$ of a function and use it to evaluate the average rate of change at a point. b. Explore how the average rate of change of a function over an interval (presented symbolically or as a table) can be used to approximate the instantaneous rate of change at a point as the interval decreases. **Tie this concept to derivatives.**
Analyze functions using different representations.	32. Graph functions expressed symbolically and show key features of the graph, by hand and using technology. Use the equation of functions to identify key features in order to generate a graph. a. Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior. b. Graph trigonometric functions and their inverses, showing period, midline, amplitude, and phase shift.

Building Functions	
Build a function that models a relationship between two quantities.	33. Compose functions. **Extend to polynomial, trigonometric, radical, and rational functions.** *Example: If* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then T(*h(t)*) is the temperature at the location of the weather balloon as a function of time.*
Build new functions from existing functions.	34. Find inverse functions. a. **Introduce concept of one-to-one functions.** b. Given that a function has an inverse, write an expression for the inverse of the function. *Example*: *Given* $f(x) = 2x^3$ *or* $f(x) = (x + 1)/(x – 1)$ *for* $x \neq 1$ *find* $f^{-1}(x)$. c. Verify by composition that one function is the inverse of another. d. Read values of an inverse function from a graph or a table, given that the function has an inverse. e. Produce an invertible function from a non-invertible function by restricting the domain **for functions that are not one-to-one**. 35. Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. **Extend from logarithms with base 2 and 10 to a base of *e*.** Extend to additional bases. 36. Identify the effect on the graph of replacing (x) by $(x)+k$, $k\cdot(x)$, $(k\cdot x)$, and $(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. **Extend the analysis to include all trigonometric, rational, and general piecewise-defined functions with and without technology.** *Example: Describe the sequence of transformations that will relate* y=sin(x) and y=2sin(3x).

	37. Graph conic sections from second-degree equations, extending from circles and parabolas to ellipses and hyperbolas, using technology to discover patterns. a. Graph conic sections given their standard form. *Example: The graph of* $\frac{x^2}{9}+\frac{(y-3)^2}{4}=1$ *will be an ellipse centered at (0,3) with major axis 3 and minor* axis *2, while the graph of* $\frac{x^2}{9}-\frac{(y-3)^2}{4}=1$ *will be a hyperbola centered at (0,3) with asymptotes with slope ±3/2.* b. Identify the conic section that will be formed, given its equation in general form. *Example*: $5y^2-25x^2=-25$ *will be a hyperbola*. 38. **Solve applications involving modeling with equations of conic sections.**
Trigonometric Functions	
Recognize attributes of trigonometric functions and solve problems involving trigonometry.	39. Solve application-based problems involving parametric and polar equations. a. Graph parametric and polar equations. b. Convert parametric and polar equations to rectangular form **and vice versa**.
Extend the domain of trigonometric functions using the unit circle.	40. Use special triangles to determine geometrically the values of sine, cosine, and tangent for π/3, π/4, and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π – *x*, π + *x*, and 2π – *x* in terms of their values for *x*, where *x* is any real number. 41. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Model periodic phenomena with trigonometric functions.	42. Demonstrate that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **Tie to one-to-one functions.** 43. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.	44. Use trigonometric identities to solve problems. a. Use the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ to derive the other forms of the identity. *Example:* $1 + \cot^2(\theta) = \csc^2(\theta)$ b. Use the angle sum formulas for sine, cosine, and tangent to derive the double angle formulas. c. Use the Pythagorean and double angle identities to prove other simple identities. **d. Use double angle identities to prove half-angle identities.**