Key

Math 4

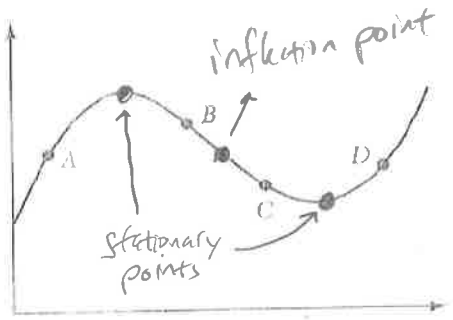
Name_____

Date_____

5-6 Practice

1. The graph of a function f is shown below. Several points are labeled.

Graph of f



(a) At which labeled points is f' positive? Explain.
(b) Between which pairs of labeled points does f have a stationary point?
(c) Between which pair of labeled points does f have an inflection point?
(d) Between which pair of labeled points is f' increasing?
(e) Between which pair of labeled points does f' achieve its minimum value?

a) A + D: Positive slope

b) Between A + B and C + D

c) Between B + C

d) Concave up .... Between C + D

e) Minimum where graph goes from Concave down to concave up (inflection point)
Between B + C

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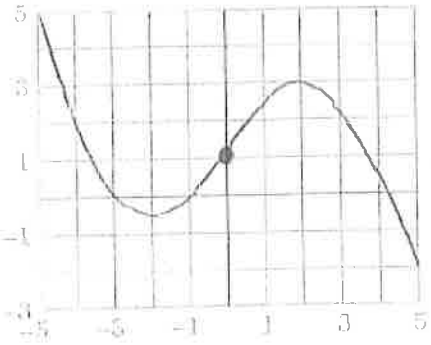
2. The graph of a function f is shown below.

Graph of f



(a) On which intervals is f' negative? Positive?
(b) On which intervals is f' increasing? Decreasing?
(c) Where does f' achieve its maximum value? Estimate this value of f'.
(d) Where does f' achieve its minimum value? Estimate this value of f'.
(e) Sketch a graph of f'. [NOTE: Your sketch should be consistent with your answers to parts (a)–(d).]

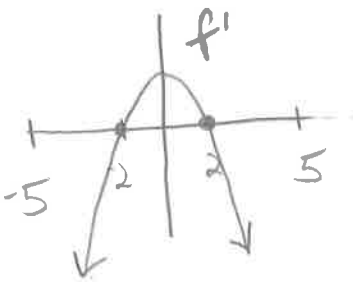
a) f' negative: (-5, -2) + (2, 5)
positive: (-2, 2)

b) increasing: (-5, 0)
decreasing: (0, 5)

c) Inflection point: X = 0

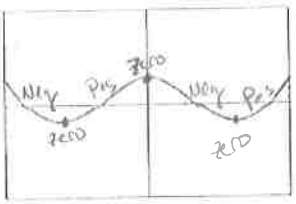
d) Minimum when f goes from concave down to concave up.
No minimum

e)



3. The graph of a function f is shown below

Graph of f

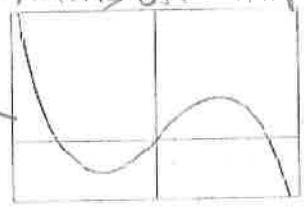


Graph III

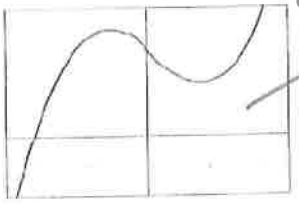
Which of the following figures could be the graph of f'? For each graph that you did *not* select, explain why it can't be the graph of f'.

Starts off with positive y-values → should be negative since f decreases at first.

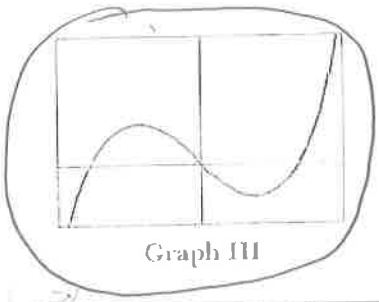
Only one zero (should be 3)



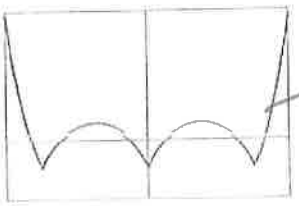
Graph I



Graph II



Graph III



Graph IV

More than 3 zeroes. And it just looks funny :-P

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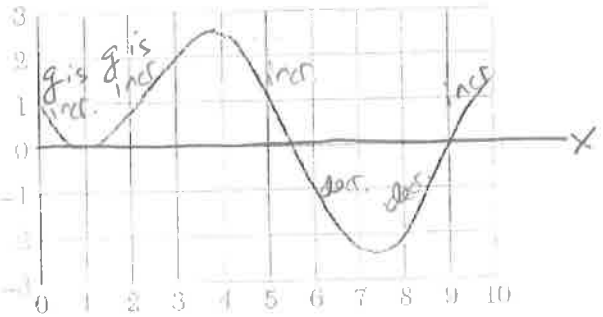
4. The graph of the derivative of a function g is shown below. Use the graph of g' to answer the following questions about g. [NOTE: The graph of g is not shown.]

Graph of g'



(a) Where does g have stationary points?

(b) Where does g have local maxima? Local minima?

(c) The graph of g' has a local maximum at $x = 3.8$ and a local minimum at $x = 7.4$. What do these facts say about the graph of g?

(d) Is g concave up or concave down at $x = 5$? At $x = 8$? Justify your answers.

a) Stationary points @ zeros, so $x = 1, 5.5$ & 9

b) Max: $x = 5.5$ incr to decr.

Min: $x = 9$ decr. to incr.

c) g has inflection points at $x = 3.8$ & $x = 7.4$

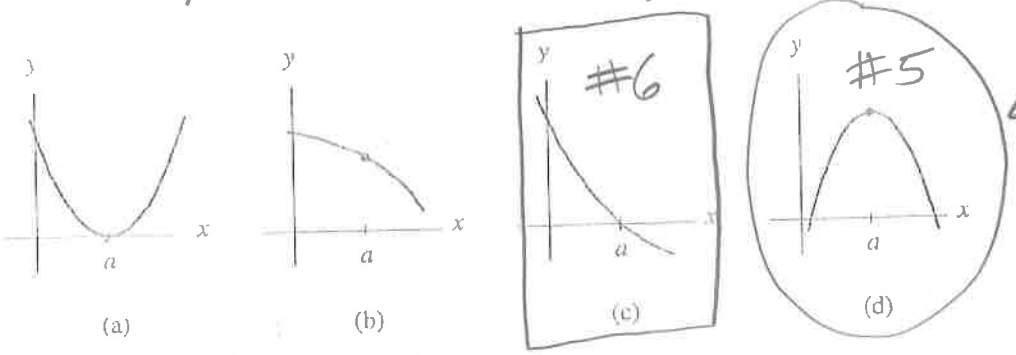
d) $x = 5$: g' is decreasing, so g is concave down.

$x = 8$: g' is increasing, so g' is concave up.

In problems 5 and 6, be sure to explain why your choice is correct.

5. Which one of the graphs in Figure 12 could represent a function $f(x)$ for which $f(a) > 0$, $f'(a) = 0$, and $f''(a) < 0$? At $x=a$, f is positive, max/min + concave down

6. Which one of the graphs in Figure 12 could represent a function $f(x)$ for which $f(a) = 0$, $f'(a) < 0$, and $f''(a) > 0$?
At $x=a$, f has a zero, is decreasing + concave up

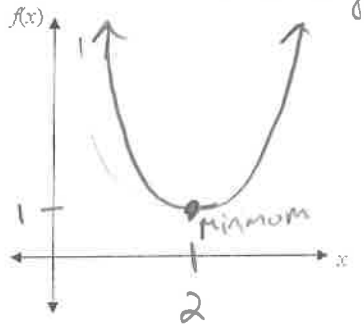


For problems 7 and 8, sketch a possible graph of $f(x)$ given the following characteristics.

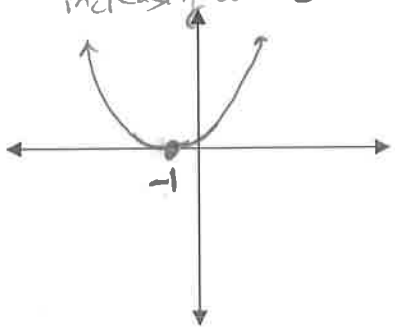
7. $f(2) = 1$; $f'(2) = 0$; concave up for all x. min/max @ $x=2$

8. $f(-1) = 0$; $f'(x) < 0$ for $x < -1$; $f'(-1) = 0$; and $f'(x) > 0$ for $x > -1$.

7) Decreasing when ↑

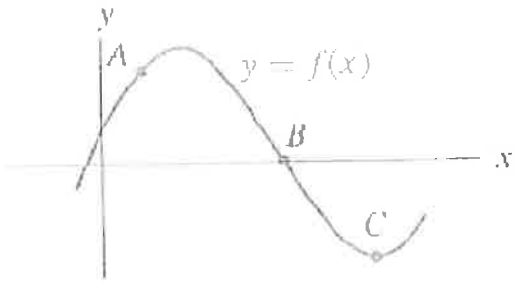


8) min/max ↑ increasing when ↗



9. Refer to the graph in Figure 13. Fill in each entry of the grid with POS, NEG, or 0. Graph is neg/pos f decr/incr f' Concave down/up f

A	Pos	Pos	Neg
B	0	Neg	0
C	Neg	0	Pos



10. $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 5$ *Show all work for the following. NO CALCULATOR UNTIL PART e!*

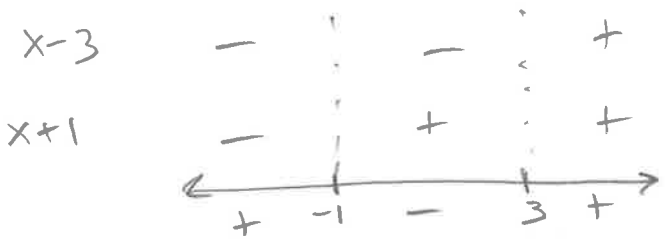
a. Find f'. Determine the intervals on which f is increasing & decreasing. *Hint: NLA!*

slope!

$f'(x) = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3 \quad x = -1$



Increasing: $x < -1$ & $x > 3$
$(-\infty, -1)$ & $(3, \infty)$

Decreasing: $-1 < x < 3$
$(-1, 3)$

b. Use your *NLA* to determine the coordinates of the relative maximum and relative minimum.

Max: $x = -1$
$f(-1) = 6.\overline{6}$
$(-1, 6.\overline{6})$

Min: $x = 3$
$f(3) = -4$
$(3, -4)$

c. Find f''. Determine the intervals on which f is concave up & concave down. *Hint: NLA!*

$f''(x) = 2x - 2$

$0 = 2x - 2$

$x = 1$

Concave down: $x < 1$
$(-\infty, 1)$

Concave up: $x > 1$
$(1, \infty)$

d. Use your *NLA* to determine the coordinates of the point(s) of inflection.

$x = 1$

$f(1) = \frac{4}{3}$

$(1, \frac{4}{3})$

e. Check your answers to parts a – d using your calculator. (You do not have to sketch the graph.)

Yay, I was right!