Indicator 40 Class Notes by Mrs. Joshi

Ratios and Rates

Accelerated Math Class

3.1 Lesson

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Key Vocabulary
ratio, *p. 100*
rate, *p. 100*
unit rate, *p. 100*

A **ratio** is a comparison of two quantities using division.

$\frac{3}{4}$, 3 to 4, 3 : 4

A **rate** is a ratio of two quantities with different units.

$\frac{60 \text{ miles}}{2 \text{ hours}}$

A rate with a denominator of 1 is called a **unit rate**.

$\frac{30 \text{ miles}}{1 \text{ hour}}$

EXAMPLE 1 Finding Ratios and Rates

There are 45 males and 60 females in a subway car. The subway car travels 2.5 miles in 5 minutes.

a. Find the ratio of males to females.

b. Find the speed of the subway car.

a. $\frac{\text{males}}{\text{females}} = \frac{45}{60} = \frac{3}{4}$

∴ The ratio of males to females is $\frac{3}{4}$.

b. 2.5 miles in 5 minutes $= \frac{2.5 \text{ mi}}{5 \text{ min}} = \frac{2.5 \text{ mi} \div 5}{5 \text{ min} \div 5} = \frac{0.5 \text{ mi}}{1 \text{ min}}$

∴ The speed is 0.5 mile per minute.

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EXAMPLE 2 Finding a Rate from a Table

The table shows the amount of money you can raise by walking for a charity. Find your unit rate in dollars per mile.

		+2	+2	+2
Distance (miles)	2	4	6	8
Money (dollars)	24	48	72	96
		+24	+24	+24

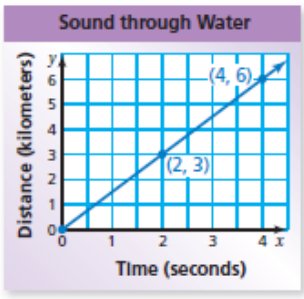
Use the table to find the unit rate.

$$\frac{\text{change in money}}{\text{change in distance}} = \frac{\$24}{2 \text{ mi}}$$ The money raised increases by \$24 every 2 miles.

$$= \frac{\$12}{1 \text{ mi}}$$ Simplify.

∴ Your unit rate is \$12 per mile.

EXAMPLE 3 Finding a Rate from a Line Graph



The graph shows the distance that sound travels through water. Find the speed of sound in kilometers per second.

Step 1: Choose a point on the line.

The point (2, 3) shows you that sound travels 3 kilometers in 2 seconds.

Step 2: Find the speed.

$$\frac{\text{distance traveled}}{\text{elapsed time}} = \frac{3}{2}$$ ← kilometers, ← seconds

$$= \frac{1.5 \text{ km}}{1 \text{ sec}}$$ Simplify.

∴ The speed is 1.5 kilometers per second.

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1.5 Lesson

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Key Vocabulary
conversion factor, *p. 32*

To convert between customary and metric units, multiply by one or more *conversion factors*.

Key Idea

Conversion Factor

A **conversion factor** is a rate that equals 1.

	Relationship	*Conversion factors*
Example	$1 \text{ m} \approx 3.28 \text{ ft}$	$\frac{1 \text{ m}}{3.28 \text{ ft}}$ and $\frac{3.28 \text{ ft}}{1 \text{ m}}$

EXAMPLE 1 Converting Between Systems

Convert 20 centimeters to inches.

Method 1: Use a conversion factor.

1 in. ≈ 2.54 cm

$$20 \text{ cm} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}} \approx 7.87 \text{ in.}$$

∴ So, 20 centimeters is about 7.87 inches.

Method 2: Use a proportion.

Let x be the number of inches equivalent to 20 centimeters.

inches → $\frac{1}{2.54} \approx \frac{x}{20}$ ← inches
centimeters → centimeters

Write a proportion.

$20 \cdot \frac{1}{2.54} \approx 20 \cdot \frac{x}{20}$ Multiply each side by 20.

$7.87 \approx x$ Simplify.

∴ So, 20 centimeters is about 7.87 inches.

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On Your Own

Copy and complete the statement.

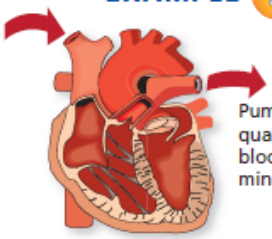
1. 10 qt ≈ ____ L
2. 4 km ≈ ____ mi
3. 18 in. ≈ ____ cm
4. 84 lb ≈ ____ kg

On Your Own

1. 9.5
2. 2.5
3. 45.72
4. 37.8

EXAMPLE 2 Converting a Rate: Changing One Unit

Convert the pumping rate of the human heart to liters per minute.



Pumps 5 quarts of blood per minute

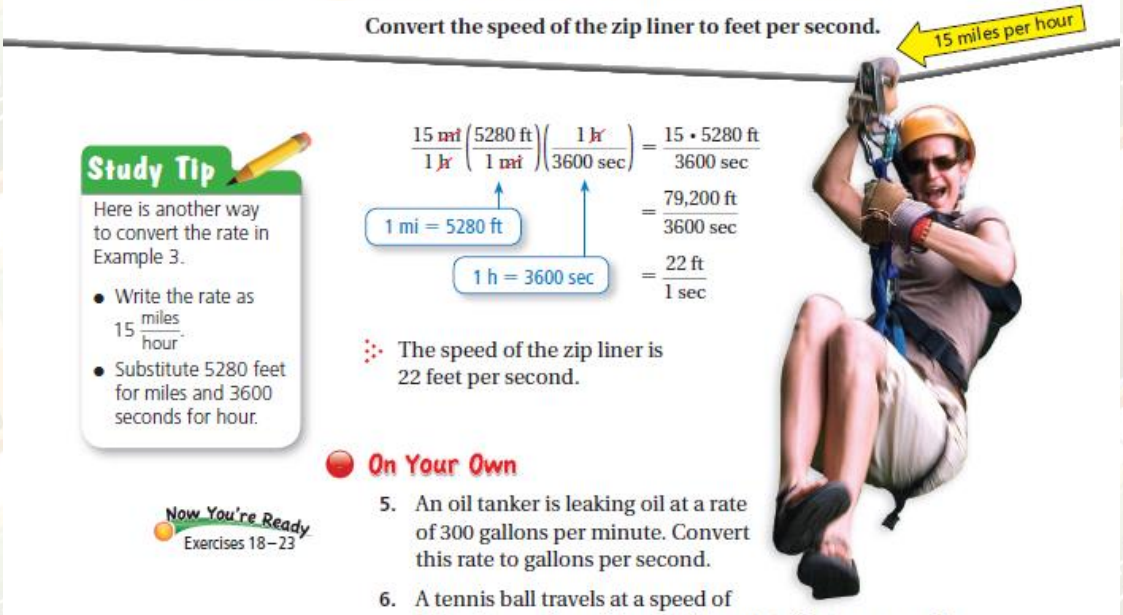
$$\frac{5\ \text{qt}}{1\ \text{min}} \cdot \frac{0.95\ \text{L}}{1\ \text{qt}} \approx \frac{4.75\ \text{L}}{1\ \text{min}}$$

(1 qt ≈ 0.95 L)

∴ The rate of 5 quarts per minute is about 4.75 liters per minute.

EXAMPLE 3 Converting a Speed: Changing Both Units

Convert the speed of the zip liner to feet per second.



$$\frac{15\ \text{mi}}{1\ \text{hr}}\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{sec}}\right) = \frac{15 \cdot 5280\ \text{ft}}{3600\ \text{sec}}$$

$$= \frac{79{,}200\ \text{ft}}{3600\ \text{sec}}$$

$$= \frac{22\ \text{ft}}{1\ \text{sec}}$$

(1 mi = 5280 ft; 1 h = 3600 sec)

∴ The speed of the zip liner is 22 feet per second.

Study Tip

Here is another way to convert the rate in Example 3.

- Write the rate as $15\ \frac{\text{miles}}{\text{hour}}$.
- Substitute 5280 feet for miles and 3600 seconds for hour.

Now You're Ready
Exercises 18–23

On Your Own

5. An oil tanker is leaking oil at a rate of 300 gallons per minute. Convert this rate to gallons per second.
6. A tennis ball travels at a speed of 120 miles per hour. Convert this rate to feet per second.
7. A kite boarder travels at a speed of 10 meters per second. Convert this rate to kilometers per minute.

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On Your Own

5. 5 gallons per second
6. 176 feet per second
7. 0.6 kilometer per minute

EXAMPLE 4 Converting Units for Area

Remember

Area is measured in *square units*. Volume is measured in *cubic units*.

The painting *Fracture* by Benedict Gibson has an area of 2880 square inches. What is the area of the painting in square feet?

1 ft = 12 in.

$$2880 \text{ in.}^2 = 2880 \text{ in.}^2 \cdot \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 2880 \text{ in.}^2 \cdot \frac{1 \text{ ft}^2}{144 \text{ in.}^2}$$

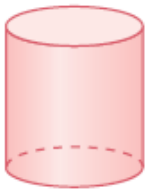
$$= \frac{2880}{144} \text{ ft}^2$$

$$= 20 \text{ ft}^2$$

∴ The area of the painting is 20 square feet.

EXAMPLE 5 Converting Units for Volume

What is the volume of the cylinder in cubic centimeters?



Volume = 80 m^3

1 m = 100 cm

$$80 \text{ m}^3 = 80 \text{ m}^3 \cdot \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3$$

$$= 80 \text{ m}^3 \cdot \frac{1{,}000{,}000 \text{ cm}^3}{1 \text{ m}^3}$$

$$= 80{,}000{,}000 \text{ cm}^3$$

∴ The volume is 80,000,000 cubic centimeters.

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On Your Own

8. The painting *Busy Market* by Haitian painter Frantz Petion has an area of 6 square feet. What is the area of the painting in square inches?

9. The volume of a pyramid is 50 cubic centimeters. What is the volume of the pyramid in cubic millimeters?

On Your Own

8. 864 $in.^2$
9. 50,000 mm^3

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4.4b Comparing Rates

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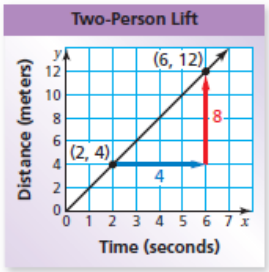
EXAMPLE 1 Comparing Proportional Relationships



The distance y (in meters) traveled by a four-person ski lift in x seconds is represented by the equation $y = 2.5x$. The graph shows the distance traveled by a two-person ski lift.

a. **Which ski lift is faster?**

Four-Person Lift

The equation is written in slope-intercept form.

$y = 2.5x$

The slope is 2.5.

The four-person lift travels 2.5 meters per second.

Two-Person Lift

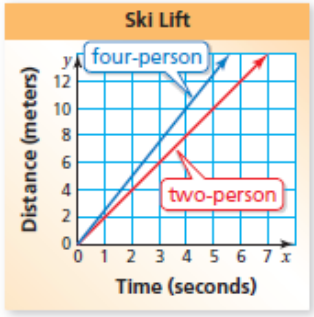
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{8}{4} = 2$$

The two-person lift travels 2 meters per second.

∴ So, the four-person lift is faster than the two-person lift.

b. **Graph the equation that represents the four-person lift in the same coordinate plane as the two-person lift. Compare the steepness of the graphs. What does this mean in the context of the problem?**

∴ The graph that represents the four-person lift is steeper than the graph that represents the two-person lift. So, the four-person lift is faster.



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EXAMPLE 2 Comparing Functions

The earnings y (in dollars) of a nighttime employee working x hours is represented by the function $y = 7.5x + 30$. The table shows the earnings of a daytime employee.

		+1	+1	+1
Time (hours)	1	2	3	4
Earnings (dollars)	12.50	25.00	37.50	50.00
		+12.50	+12.50	+12.50

a. Which employee has a higher hourly wage?

Nighttime Employee

$y = 7.5x + 30$

The slope is 7.5.

The nighttime employee earns \$7.50 per hour.

Daytime Employee

$$\frac{\text{change in earnings}}{\text{change in time}} = \frac{\$12.50}{1 \text{ hour}}$$

The daytime employee earns \$12.50 per hour.

∴ So, the daytime employee has a higher hourly wage.

b. Write a function that relates the daytime employee's earnings to the number of hours worked. Graph the functions that represent the earnings of the two employees in the same coordinate plane. Interpret the graphs.

Use a verbal model to write a function that represents the earnings of the daytime employee.

$$\text{Earnings} = \text{Hourly wage} \cdot \text{Hours worked}$$

$$y = 12.5x$$

∴ The graph shows that the daytime employee has a higher hourly wage, but does not earn more money than the nighttime employee until each person has worked more than 6 hours.