

Indicator 4 Class Notes by Mrs. Joshi


Dividing Fractions

4. Interpret and compute quotients of fractions using visual models and equations to represent problems.

a. Use quotients of fractions to analyze and solve problems.

2.5 Lesson



Key Vocabulary 
reciprocals, p. 72

Two numbers whose product is 1 are **reciprocals**. To write a reciprocal of a number, write the number as a fraction. Then invert the fraction.

The Meaning of a Word ● Invert

When you **invert** a glass, you turn it over.



EXAMPLE 1 Writing Reciprocals

Study Tip

When any number is multiplied by 0, the product is 0. So, the number 0 does not have a reciprocal.

	<i>Original Number</i>	<i>Fraction</i>	<i>Reciprocal</i>	<i>Check</i>
a.	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{3}$	$\frac{3}{5} \times \frac{5}{3} = 1$
b.	$\frac{9}{5}$	$\frac{9}{5}$	$\frac{5}{9}$	$\frac{9}{5} \times \frac{5}{9} = 1$
c.	2	$\frac{2}{1}$	$\frac{1}{2}$	$\frac{2}{1} \times \frac{1}{2} = 1$

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Key Idea

Dividing Fractions

Words To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Numbers $\frac{1}{5} \div \frac{3}{4} = \frac{1}{5} \times \frac{4}{3}$

Algebra $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$, where $b, c,$ and $d \neq 0$

EXAMPLE 2 Dividing a Fraction by a Fraction

Find $\frac{1}{6} \div \frac{2}{3}$.

$$\frac{1}{6} \div \frac{2}{3} = \frac{1}{6} \times \frac{3}{2}$$

Multiply by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$.

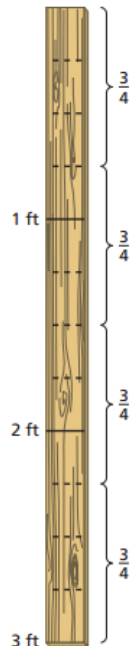
$$= \frac{1 \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \times 2}$$

Multiply fractions. Divide out the common factor 3.

$$= \frac{1}{4}$$

Simplify.

EXAMPLE 3 Dividing a Whole Number by a Fraction



A piece of wood is 3 feet long. How many $\frac{3}{4}$ -foot pieces can be cut from the piece of wood?

Method 1: Draw a diagram. Mark each foot on the diagram. Then divide each foot into $\frac{1}{4}$ -foot sections.

Count the number of $\frac{3}{4}$ -foot pieces of wood. There are four.

∴ So, four $\frac{3}{4}$ -foot pieces can be cut from the piece of wood.

Method 2: Divide 3 by $\frac{3}{4}$ to find the number of $\frac{3}{4}$ -foot pieces.

$$3 \div \frac{3}{4} = 3 \times \frac{4}{3}$$

Multiply by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$$= \frac{\overset{1}{\cancel{3}} \times 4}{\underset{1}{\cancel{3}}}$$

Multiply. Divide out the common factor 3.

$$= 4$$

Simplify.

∴ So, four $\frac{3}{4}$ -foot pieces can be cut from the piece of wood.

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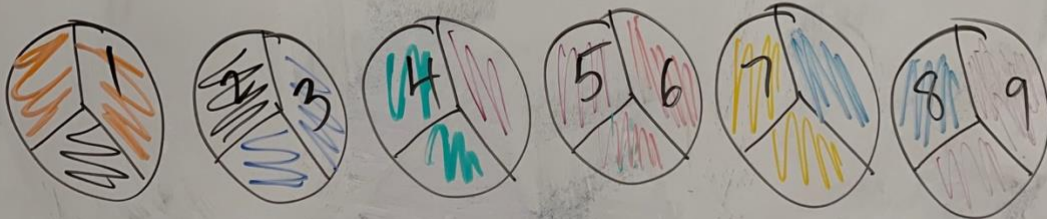
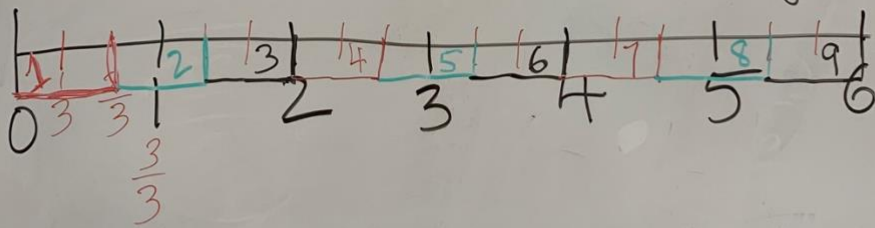
Visual Models of Dividing Fractions

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Dividing Fractions

$$6 \div \frac{2}{3} \qquad \overset{3}{\cancel{6}} \times \frac{3}{\cancel{2}} = 9$$

9 groups



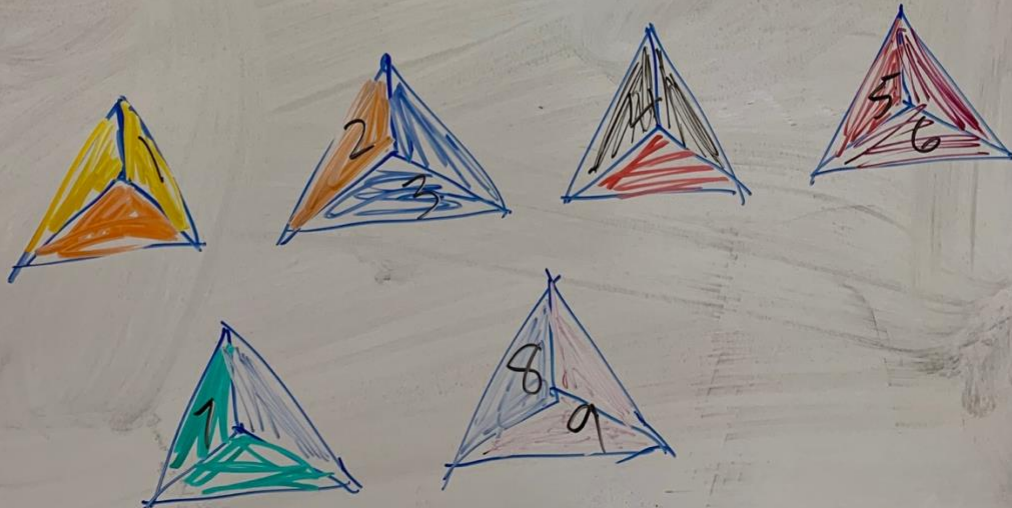
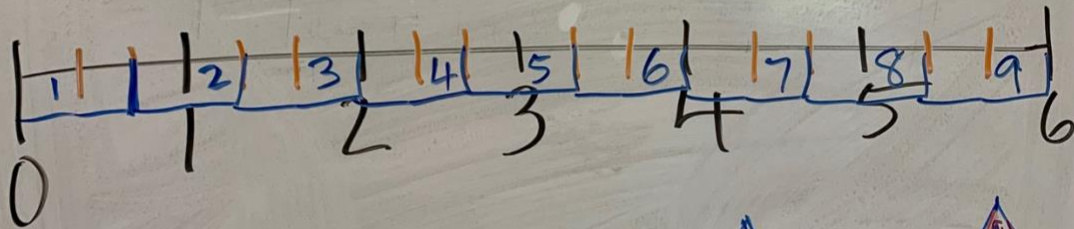
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Dividing Fractions

$$6 \div \frac{2}{3} = 9$$



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Visual Models of Dividing Fractions



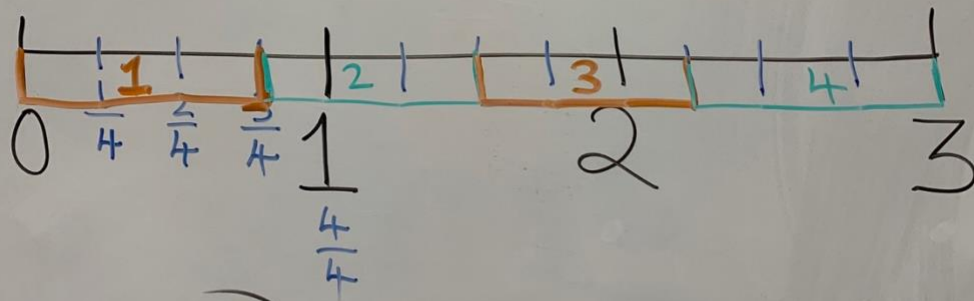
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$$3 \div \frac{3}{4}$$

One Way

$$\frac{\cancel{3} \times 4}{\cancel{3}} = \textcircled{4}$$

Fraction Model



4

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Visual Models of Dividing Fractions

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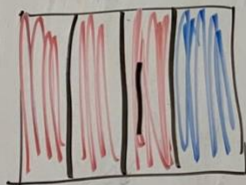
Another Way

Fraction Model

$$3 \div \frac{3}{4}$$



OR



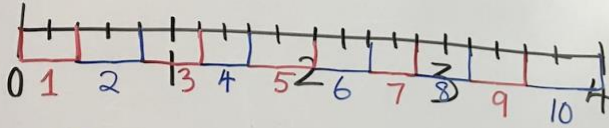
$$3 \div \frac{3}{4} = 4$$

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Visual Models of Dividing Fractions

Visual Model

Check: $4 \div \frac{2}{5}$


$$\frac{2\cancel{4}}{1} \times \frac{5}{\cancel{2}_1} = 10$$


There are 10 groups of $\frac{2}{5}$ ths in 4.

Indicator 1

Dividing Fractions

How many groups of $\frac{2}{5}$ do I have in 4?

$$4 \div \frac{2}{5}$$


10

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Visual Models of Dividing Fractions

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$$12 \div \frac{3}{4}$$

Lizzy



$$12 \div \frac{3}{4} = 16$$

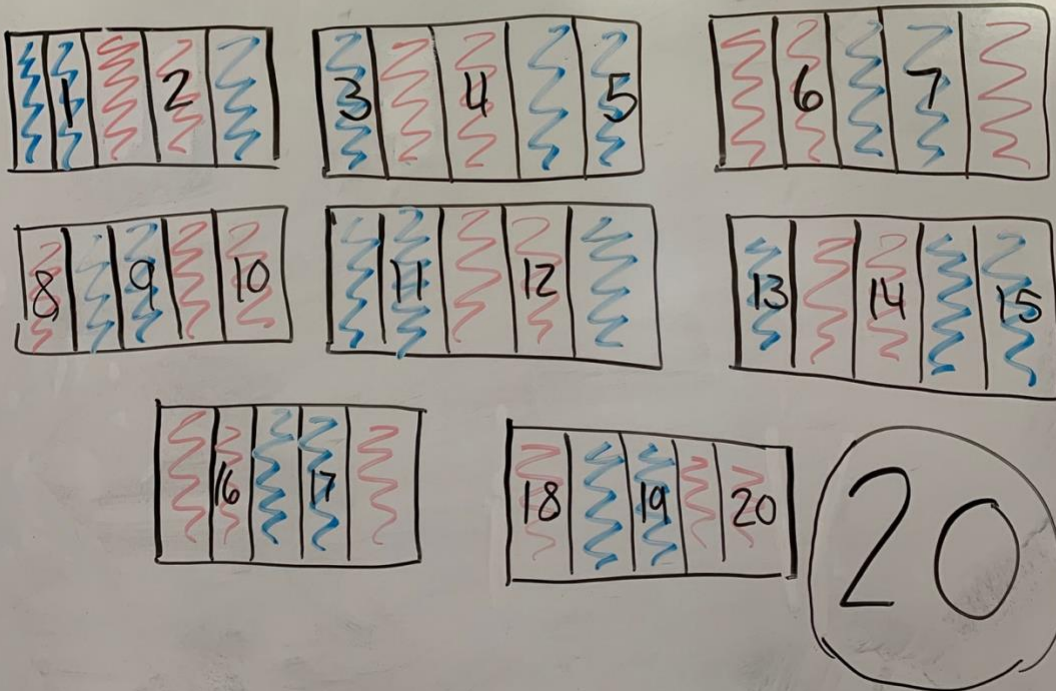
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Visual Models of Dividing Fractions

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Charlotte

2) $8 \div \frac{2}{5} = 20$



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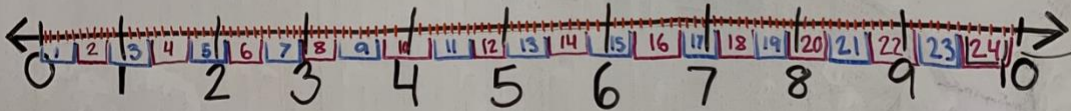
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Indicator 6

Dividing Fractions

Clara Maehlmann

$$10 \div \frac{5}{12} = 24$$



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Visual Models of Dividing Fractions

Mrs. Joshi
Indicator 6

Dividing Fractions

Tabytha

$$10 \div \frac{5}{12} = 24$$



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Visual Models of Dividing Fractions

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$3 \overline{) 10} = \frac{5}{12}$

$\frac{10}{1} \times \frac{12}{5}$
 \downarrow
 $\frac{3}{1} \times \frac{12}{1} = \frac{24}{1} = 24$

Ben

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Visual Models of Dividing Fractions

$7 \div \frac{1}{2}$

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Fraction Model

$7 \div \frac{1}{2} = 14$

Hayden

14

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Visual Models of Dividing Fractions

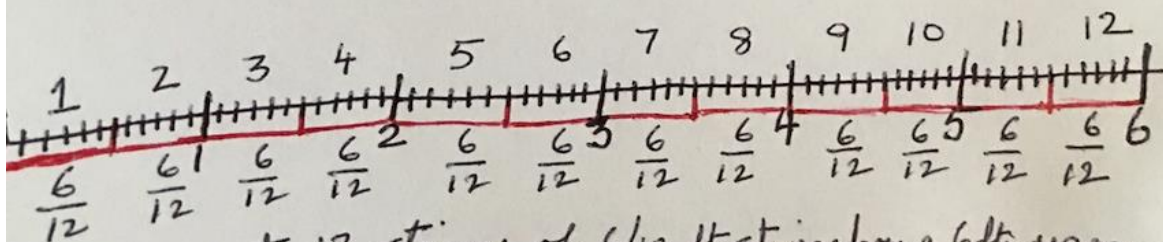
Mrinal Joshi

A string of yarn is 6 feet long. How many $\frac{6}{12}$ feet strings of yarn can I cut from the 6 foot yarn? Draw a visual model to represent the problem.

Answer: $6 \div \frac{6}{12}$

How many groups of $\frac{6}{12}$ are in 6?

Visual Model



I can cut 12 strings of $\frac{6}{12}$ ft. strings from a 6ft. yarn.

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Visual Models of Dividing Fractions



Mrs. Joshi

$12 \div \frac{1}{3}$ Disha



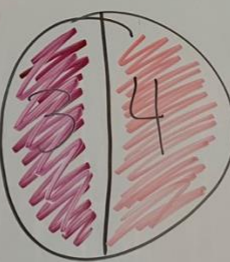
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Visual Models of Dividing Fractions

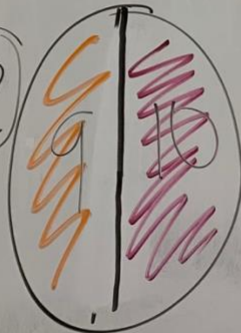
Mrs. Joshi

Om

$$6 \div \frac{1}{2}$$



$$6 \div \frac{1}{2} = 12$$

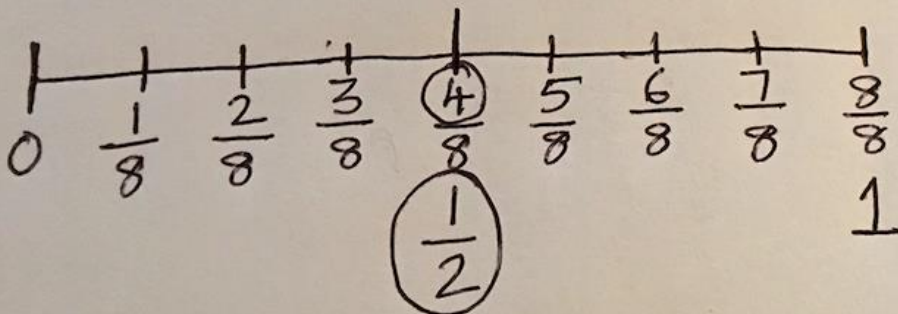


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Visual Models of Dividing Fractions

Mrinal Joshi
Fraction by Fraction Model

$\frac{1}{2} \div \frac{1}{8}$ This basically means:
How many groups of $\frac{1}{8}$ are
in $\frac{1}{2}$?



In $\frac{1}{2}$ we have 4 groups of
 $\frac{1}{8}$, since the answer is $\frac{4}{8}$.

Traditional way: $\frac{1}{2} \times \frac{8}{1} = \textcircled{4}$

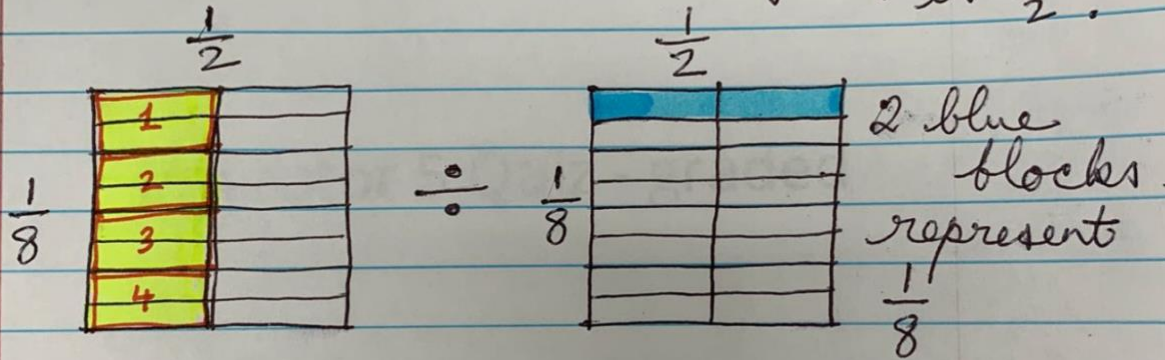
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Visual Models of Dividing Fractions

Mrs. Joshi (Fraction by fraction model)

$$\frac{1}{2} \div \frac{1}{8}$$

How many $\frac{1}{8}$ groups are in $\frac{1}{2}$?



$$\frac{1}{2} \div \frac{1}{8} = 4$$

4 groups of $\frac{1}{8}$ are in $\frac{1}{2}$.

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Visual Models of Dividing Fractions

$\frac{1}{2} \div \frac{1}{8}$

$\frac{1}{2}$

$\frac{1}{8}$

1	2
2	
3	
4	
5	
6	
7	
8	

- Count the squares in $\frac{1}{2}$ as numerator
- Count the circled squares in $\frac{1}{8}$ as denominator

$\frac{8}{2} = 4$

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Visual Models of Dividing Fractions

how many fit?

$\frac{4}{5} \div \frac{2}{3} = 1 \frac{2}{10} = 1 \frac{1}{5}$

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EXAMPLE 4 Evaluating an Algebraic Expression

Evaluate $a \div b$ when $a = \frac{4}{5}$ and $b = 2$.

$$\begin{aligned} a \div b &= \frac{4}{5} \div 2 && \text{Substitute } \frac{4}{5} \text{ for } a \text{ and } 2 \text{ for } b. \\ &= \frac{4}{5} \times \frac{1}{2} && \text{Multiply by the reciprocal of } 2, \text{ which is } \frac{1}{2}. \\ &= \frac{\cancel{4}^2 \times 1}{5 \times \cancel{2}_1} && \text{Multiply fractions. Divide out the common factor } 2. \\ &= \frac{2}{5} && \text{Simplify.} \end{aligned}$$

EXAMPLE 5 Using Order of Operations

Evaluate $\frac{3}{8} + \frac{5}{6} \div 5$.

$$\begin{aligned} \frac{3}{8} + \frac{5}{6} \div 5 &= \frac{3}{8} + \frac{5}{6} \times \frac{1}{5} && \text{Multiply by the reciprocal of } 5, \text{ which is } \frac{1}{5}. \\ &= \frac{3}{8} + \frac{\cancel{5}^1 \times 1}{6 \times \cancel{5}_1} && \text{Multiply } \frac{5}{6} \text{ and } \frac{1}{5}. \text{ Divide out the common factor } 5. \\ &= \frac{3}{8} + \frac{1}{6} && \text{Simplify.} \\ &= \frac{9}{24} + \frac{4}{24} && \text{Rewrite fractions using the LCD } 24. \\ &= \frac{13}{24} && \text{Add.} \end{aligned}$$

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2.6 Lesson



Key Idea

Dividing Mixed Numbers

Write each mixed number as an improper fraction. Then divide as you would with proper fractions.

EXAMPLE 1 Dividing a Mixed Number by a Fraction

Find $4\frac{1}{2} \div \frac{3}{8}$.

$$\begin{aligned} 4\frac{1}{2} \div \frac{3}{8} &= \frac{9}{2} \div \frac{3}{8} \\ &= \frac{9}{2} \times \frac{8}{3} \\ &= \frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}}} \\ &= 12 \end{aligned}$$

∴ So, the quotient is 12.

Estimate $5 \div \frac{1}{2} = 10$

Write $4\frac{1}{2}$ as the improper fraction $\frac{9}{2}$.

Multiply by the reciprocal of $\frac{3}{8}$, which is $\frac{8}{3}$.

Multiply fractions. Divide out common factors.

Simplify.

Reasonable? $12 \approx 10$ ✓

EXAMPLE 2 Dividing Mixed Numbers

Find $3\frac{5}{6} \div 1\frac{2}{3}$.

$$\begin{aligned} 3\frac{5}{6} \div 1\frac{2}{3} &= \frac{23}{6} \div \frac{5}{3} \\ &= \frac{23}{6} \times \frac{3}{5} \\ &= \frac{\overset{1}{\cancel{23}} \times \overset{3}{\cancel{3}}}{\underset{2}{\cancel{6}} \times 5} \\ &= \frac{23}{10}, \text{ or } 2\frac{3}{10} \end{aligned}$$

∴ So, the quotient is $2\frac{3}{10}$.

Estimate $4 \div 2 = 2$

Write each mixed number as an improper fraction.

Multiply by the reciprocal of $\frac{5}{3}$, which is $\frac{3}{5}$.

Multiply fractions. Divide out common factors.

Simplify.

Reasonable? $2\frac{3}{10} \approx 2$ ✓

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EXAMPLE 3 Using Order of Operations

Remember

Be sure to check your answers whenever possible. In Example 3, you can use estimation to check that your answer is reasonable.

$$\begin{aligned}5\frac{1}{4} \div 1\frac{1}{8} - \frac{2}{3} &\approx 5 \div 1 - 1 \\ &= 5 - 1 \\ &= 4 \quad \checkmark\end{aligned}$$

Evaluate $5\frac{1}{4} \div 1\frac{1}{8} - \frac{2}{3}$.

$$\begin{aligned}5\frac{1}{4} \div 1\frac{1}{8} - \frac{2}{3} &= \frac{21}{4} \div \frac{9}{8} - \frac{2}{3} \\ &= \frac{21}{4} \times \frac{8}{9} - \frac{2}{3} \\ &= \frac{\overset{7}{\cancel{21}} \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \times \underset{3}{\cancel{9}}} - \frac{2}{3} \\ &= \frac{14}{3} - \frac{2}{3} \\ &= \frac{12}{3}, \text{ or } 4\end{aligned}$$

Write each mixed number as an improper fraction.

Multiply by the reciprocal of $\frac{9}{8}$, which is $\frac{8}{9}$.

Multiply $\frac{21}{4}$ and $\frac{8}{9}$. Divide out common factors.

Simplify.

Subtract.

EXAMPLE 4 Real-Life Application

One serving of tortilla soup is $1\frac{2}{3}$ cups. A restaurant cook makes 50 cups of soup. Is there enough to serve 35 people? Explain.

Divide 50 by $1\frac{2}{3}$ to find the number of available servings.



$$\begin{aligned}50 \div 1\frac{2}{3} &= \frac{50}{1} \div \frac{5}{3} \\ &= \frac{50}{1} \cdot \frac{3}{5} \\ &= \frac{\overset{10}{\cancel{50}} \cdot 3}{1 \cdot \underset{1}{\cancel{5}}} \\ &= 30\end{aligned}$$

Rewrite each number as an improper fraction.

Multiply by the reciprocal of $\frac{5}{3}$, which is $\frac{3}{5}$.

Multiply fractions. Divide out common factors.

Simplify.

❖ No. Because 30 is less than 35, there is not enough soup to serve 35 people.