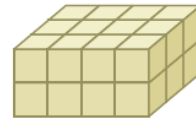# Indicator 31 Class Notes by Mrs. Joshi

## Volume of Rectangular Prism with Fractional Edge Lengths-(6.G.2)
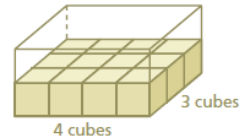
**EXAMPLE 1** **Counting Cubes**

**Key Vocabulary**
rectangular prism,
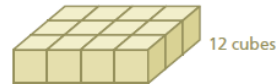  p. 314
volume, p. 314
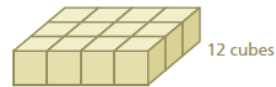cubic units, p. 314

How many cubes do you need to fill the box?

The bottom layer of the box is 4 cubes long and 3 cubes wide. So, you need $4 \times 3$, or 12 cubes to cover the bottom layer.

3 cubes

4 cubes

To fill the box, you need two layers of 12 cubes.

12 cubes

So, you need $2 \times 12 = 24$ cubes to fill the box.

12 cubes

**Remember**

A three-dimensional figure has length, width, and height.

A **rectangular prism** is a three-dimensional figure that has six rectangular sides. The **volume** of a prism is a measure of the amount of space that it occupies. Volume is measured in **cubic units**.

## Key Idea

**Volume of a Rectangular Prism**

**Words** The volume $V$ of a rectangular prism is the product of its length $\ell$, width $w$, and height $h$.

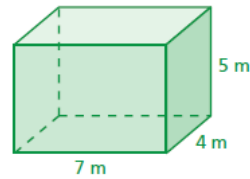**Algebra** $V = \ell w h$

Height, $h$

Width, $w$

Length, $\ell$
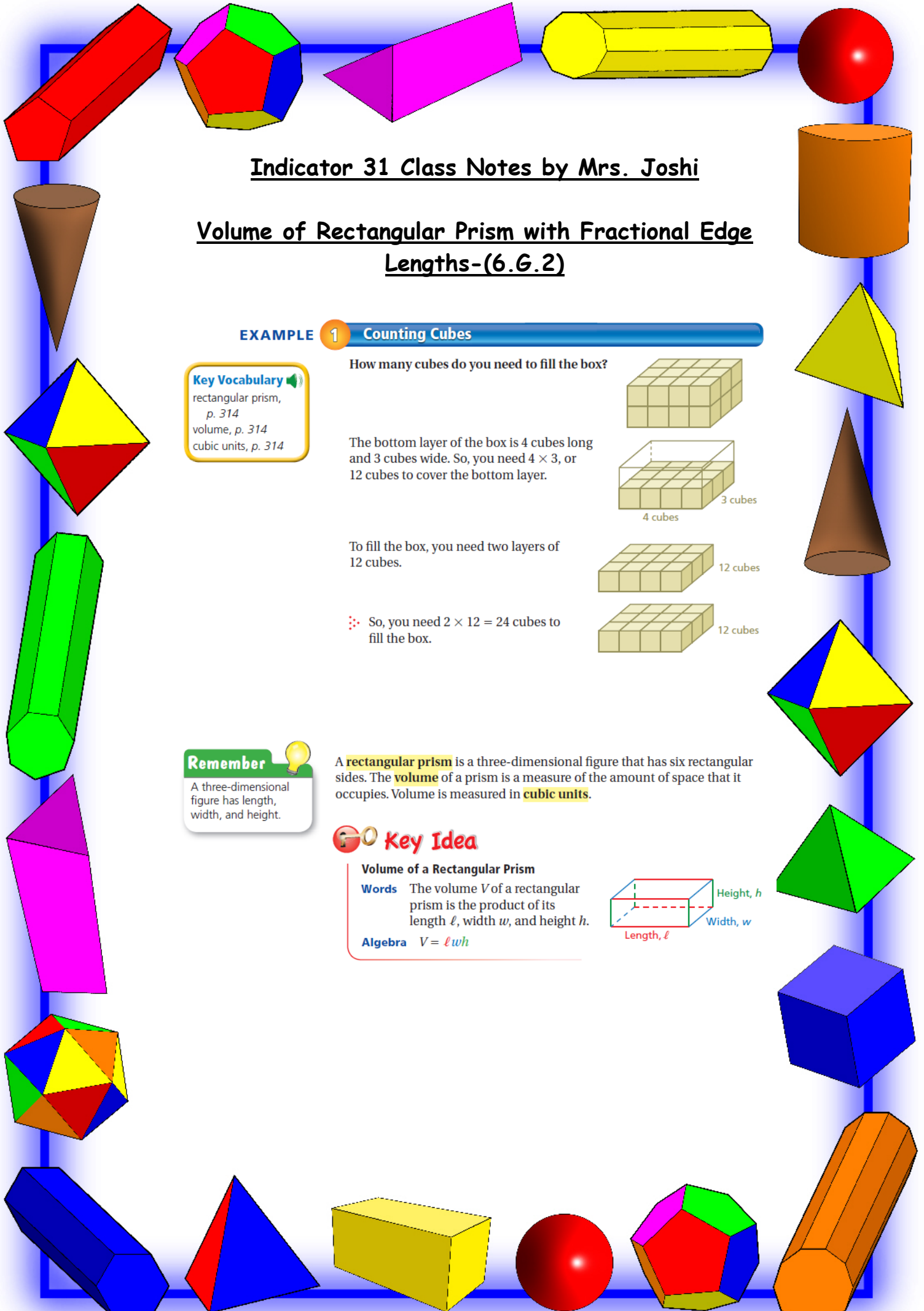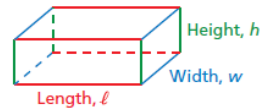
# Indicator 31 Class Notes by Mrs. Joshi

**EXAMPLE 2** **Finding the Volume of a Rectangular Prism**

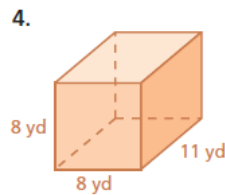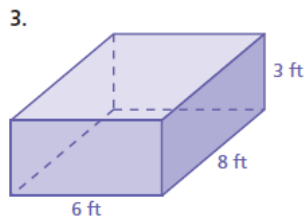Find the volume of the rectangular prism.

$V = \ell w h$      Write formula for volume.

$= 7(4)(5)$      Substitute values.

$= 140$      Multiply.

∴ The volume is 140 cubic meters.

🔴 **On Your Own**

*Now You're Ready*
Exercises 6–8

Find the volume of the rectangular prism.

**3.**

3 ft

8 ft

6 ft

**4.**

8 yd

11 yd

8 yd

**EXAMPLE 3** **Finding a Missing Dimension of a Rectangular Prism**

Write and solve an equation to find the height of the computer tower.

$h$

16 in.

7 in.

Volume = 1792 in.$^3$

$V = \ell w h$      Write formula for volume.

$1792 = 7(16)h$      Substitute 1792 for $V$, 7 for $\ell$, and 16 for $w$.

$1792 = 112h$      Simplify.

$\dfrac{1792}{112} = \dfrac{112h}{112}$      Divide each side by 112.

$16 = h$      Simplify.

∴ The height is 16 inches.
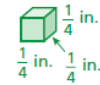
# Indicator 31 Class Notes by Mrs. Joshi

To find the volume of a rectangular prism, multiply the number of cubes needed to fill the prism by the volume of one of the cubes.
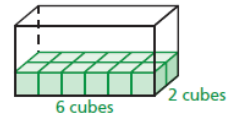
**EXAMPLE  1   Finding the Volume of a Rectangular Prism**
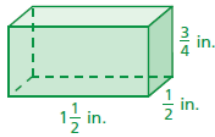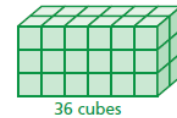
Use cubes to find the volume of the rectangular prism.

Use cubes with an edge length of $\frac{1}{4}$ inch.

**Check**

$V = Bh = \ell w h$

$= \left(1\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)$

$= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)$

$= \frac{9}{16}$ in.³ ✓

The bottom layer is 6 cubes long and 2 cubes wide. So, you need $6 \times 2$, or 12 cubes to cover the bottom layer.

To fill the prism, you need three layers of 12 cubes. So, you need $3 \times 12$, or 36 cubes.

The volume of each cube is $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$ cubic inch.

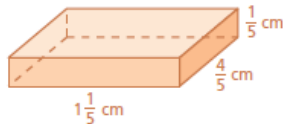So, the volume of the prism is $36 \times \frac{1}{64} = \frac{9}{16}$ cubic inch.
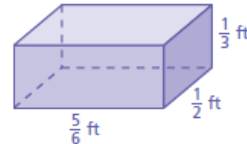
## Practice

Use cubes with the given edge length to find the volume of the rectangular prism. Check your answer using the volume formula.
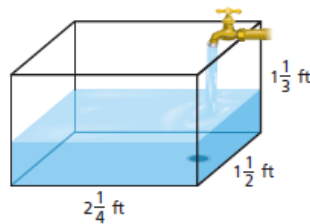
1. Edge length: $\frac{1}{5}$ cm
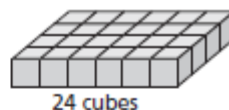
2. Edge length: $\frac{1}{6}$ ft

3. **SINK**  A sink is shaped like a rectangular prism. How much water can the sink hold?

**1.** To fill the prism, you need only one layer of cubes.

The layer is 6 cubes long and 4 cubes wide. So, you need $6 \times 4$, or 24 cubes to fill the prism.

24 cubes

The volume of each cube is $\left(\frac{1}{5}\right)^3 = \frac{1}{125}$ cubic centimeter.
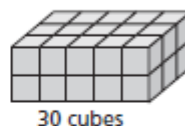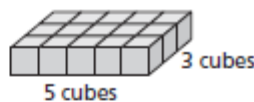
So, the volume of the prism is

$24 \times \frac{1}{125} = \frac{24}{125}$ cubic centimeter.

**Check:** $V = Bh = \ell w h$

$$= \left(1\frac{1}{5}\right)\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)$$

$$= \left(\frac{6}{5}\right)\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)$$

$$= \frac{24}{125} \text{ cm}^3 \checkmark$$

**2.** The bottom layer is 5 cubes long and 3 cubes wide. So, you need $5 \times 3$, or 15 cubes to cover the bottom layer.

To fill the prism, you need two layers of 15 cubes. So, you need $2 \times 15$, or 30 cubes.

5 cubes    3 cubes

30 cubes

The volume of each cube is

$$\left(\frac{1}{6}\right)^3 = \frac{1}{216} \text{ cubic foot.}$$

So, the volume of the prism is $30 \times \frac{1}{216} = \frac{5}{36}$ cubic foot.

**Check:** $V = Bh = \ell w h$

$$= \left(\frac{5}{6}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$$

$$= \frac{5}{36} \text{ ft}^3 \checkmark$$

3. Use cubes with an edge length of $\frac{1}{12}$ foot.

The bottom layer is 27 cubes long and 18 cubes wide. So, you need $27 \times 18$, or 486 cubes to cover the bottom layer.

To fill the prism, you need 16 layers of 486 cubes. So, you need $16 \times 486$, or 7776 cubes.

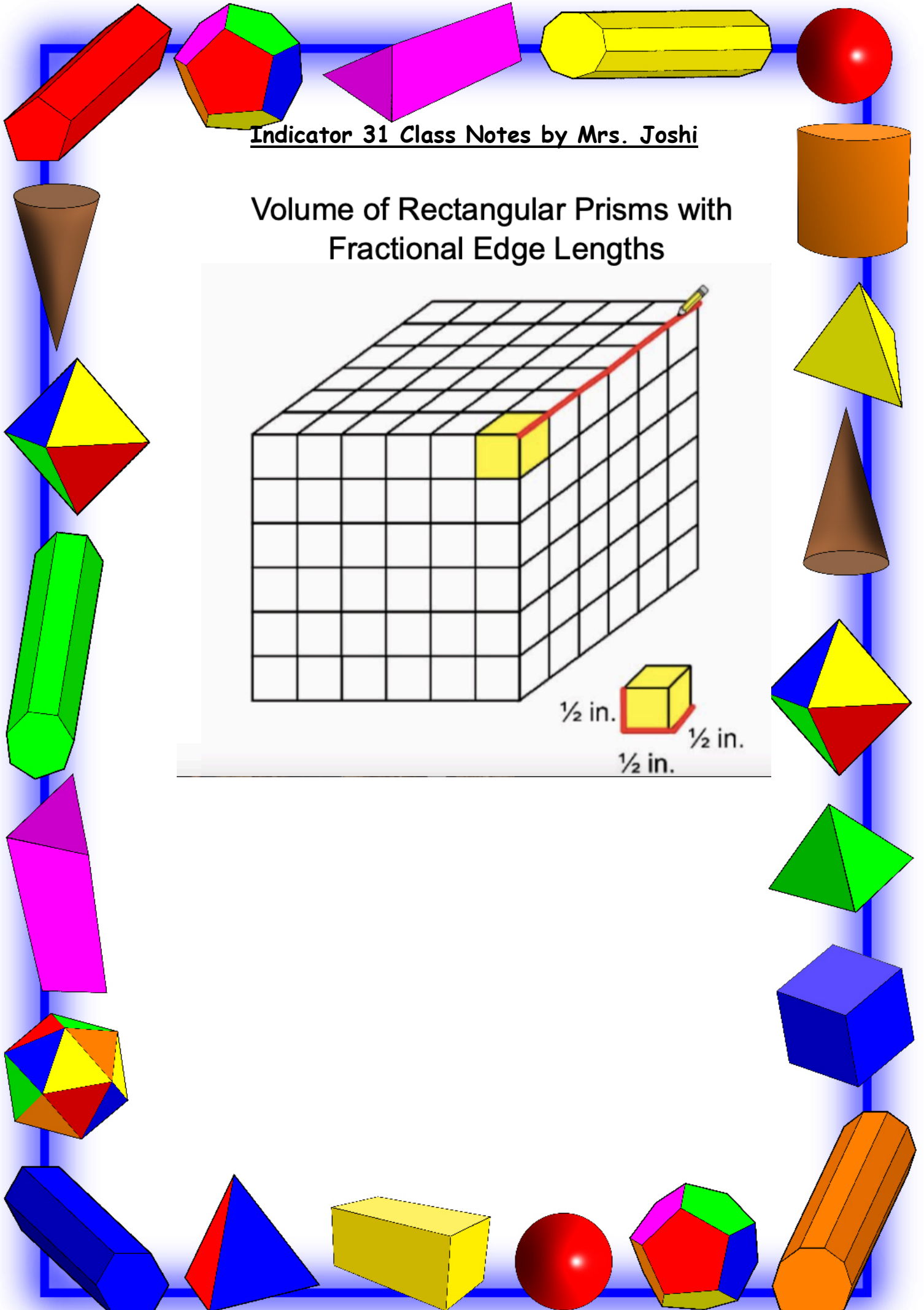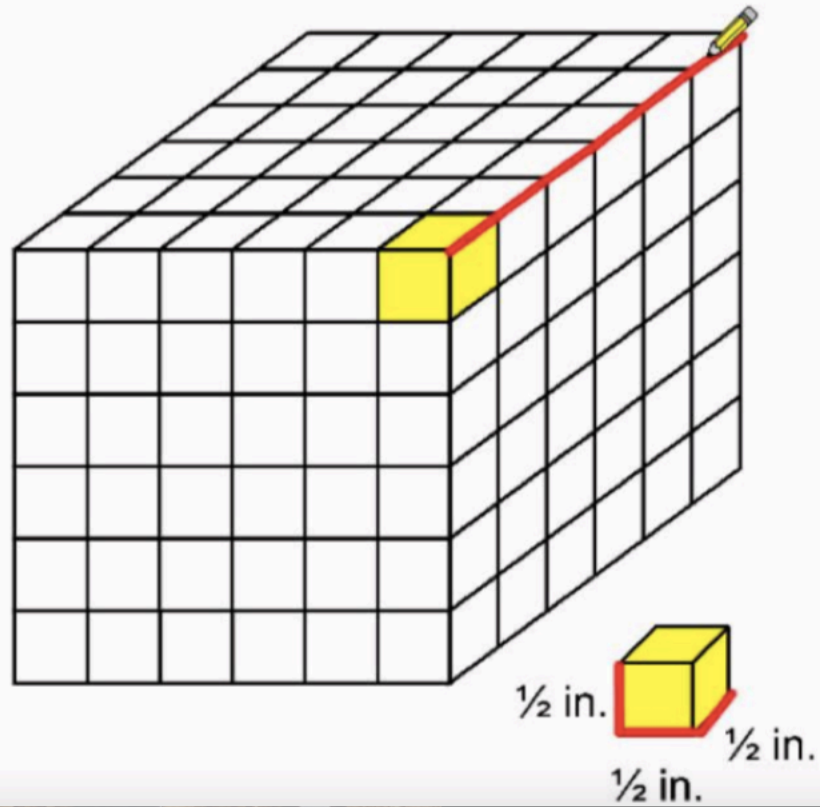The volume of each cube is $\left(\frac{1}{12}\right)^3 = \frac{1}{1728}$ cubic foot.

So, the volume of the prism is

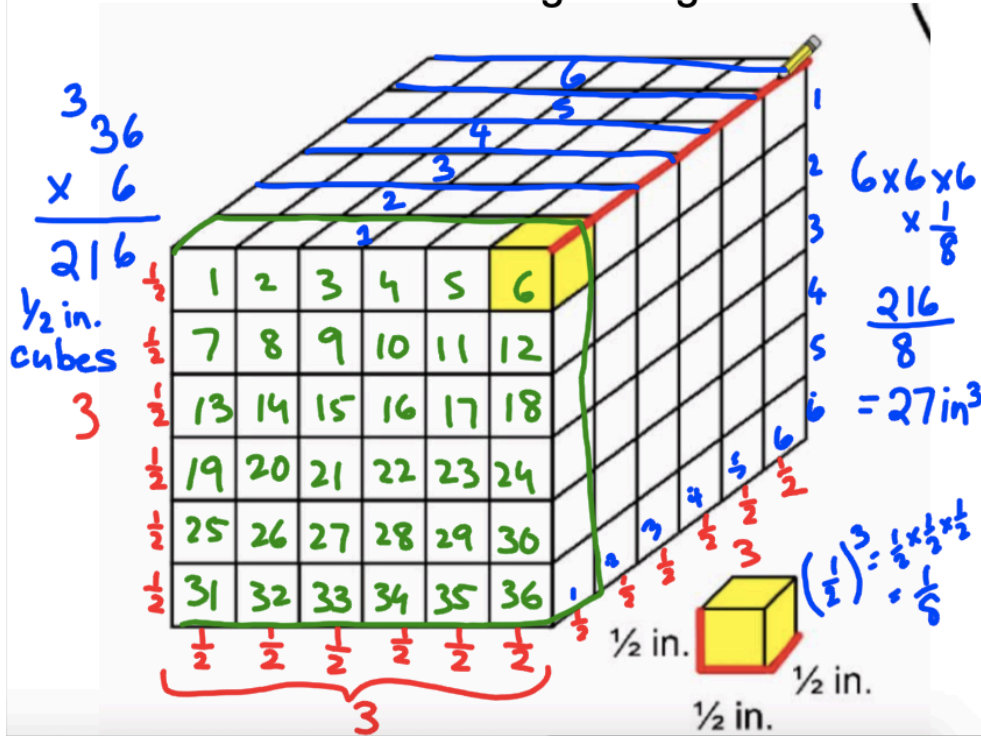$$7776 \times \frac{1}{1728} = \frac{9}{2} \text{ cubic feet.}$$

The sink can hold $\frac{9}{2}$, or $4\frac{1}{2}$ cubic feet of water.

# Volume of Rectangular Prisms with Fractional Edge Lengths

½ in.

½ in.

½ in.

# Volume of Rectangular Prisms with Fractional Edge Lengths



$\begin{array}{r} \overset{3}{3}6 \\ \times\ 6 \\ \hline 216 \end{array}$

½ in.
cubes

3

$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 & 17 & 18 \\ 19 & 20 & 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 & 29 & 30 \\ 31 & 32 & 33 & 34 & 35 & 36 \end{array}$

$6 \times 6 \times 6$
$\times \frac{1}{8}$

$\dfrac{216}{8}$

$= 27 in^3$

$\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
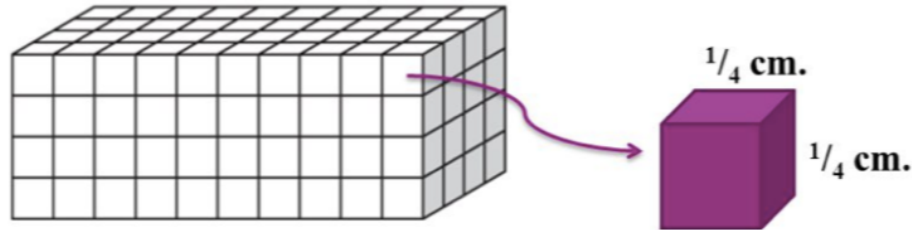
½ in.
½ in.
½ in.

Mrs. Joshi

Volume of each cube $= l\,w\,h$
$= \left(\dfrac{1}{2}\right)^3$
$= \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2}$
$= \dfrac{1}{8} \ in^3$

$V = l\,w\,h$
$= \overset{3}{6} \times \overset{3}{6} \times \overset{3}{6} \times \dfrac{1}{8}$
$V = \boxed{27\ in^3}$

$V = l\,w\,h$
$= 3 \times 3 \times 3$
$= \boxed{27\ in^3}$

**Volume of Rectangular Prisms with Fractional Edge Lengths**

<u>Mrs. Joshi</u>

Volume of each cube = $l \cdot w \cdot h$

$$= \left(\frac{1}{4}\right)^3$$

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{1}{64} \ cm^3$$

$V = l \cdot w \cdot h$      (Simplify before you multiply)

$$\overset{5}{10} \times \overset{}{4} \times \overset{1}{4} \times \frac{1}{\underset{16 \ 4 \ 2}{64}}$$

$$= \frac{5}{2} = 2\frac{1}{2} \ cm^3$$

$V = 2\frac{1}{2} \times 1 \times 1$      $\boxed{\frac{4}{4} = 1}$

$$= \frac{5}{2} \times 1 \times 1$$

$$= 2\frac{1}{2} \ cm^3$$