AMSCO'S

GEOMETRY

Ann Xavier Gantert

AMSCO SCHOOL PUBLICATIONS, INC.
315 HUDSON STREET, NEW YORK, N.Y. 10013

Dedication

This book is dedicated to Patricia Sullivan whose friendship and support has always been the mainstay of the author's life and work.

Ann Xavier Gantert

The author has been associated with mathematics education in New York State as a teacher and an author throughout the many changes of the past fifty years. She has worked as a consultant to the Mathematics Bureau of the Department of Education in the development and writing of Sequential Mathematics and has been a coauthor of Amsco's *Integrated Mathematics* series, which accompanied that course of study.

Reviewers:

Steven J. Balasiano
Assistant Principal, Supervision Mathematics
Canarsie High School
Brooklyn, NY

Debbie Calvino
Mathematics Supervisor, Grades 7–12
Valley Central High School
Montgomery, NY

Domenic D'Orazio
Mathematics Teacher
Midwood High School
Brooklyn, NY

Ronald Hattar
Mathematics Chairperson
Eastchester High School
Eastchester, NY

Juanita Maltese
Mathematics, Business, and Technology Chairperson
Carle Place Middle School/ High School
Carle Place, NY

Raymond Scacalossi Jr.
Mathematics Coordinator
Manhasset High School
Manhasset, NY

Sal Sutera
Mathematics Teacher
New Utrecht High School
Brooklyn, NY

Text Designer: Nesbitt Graphics, Inc.
Compositor: Compset, Inc.
Cover Design by Meghan J. Shupe
Cover Art by Brand X Pictures (RF)

Please visit our Web site at: ***www.amscopub.com***

When ordering this book, please specify:
R 80 P *or* GEOMETRY, *Paperback* *or* **R 80 H** *or* GEOMETRY, *Hardbound*

ISBN 978-1-56765-596-4 (Paperback edition)
NYC Item 56765-596-3 (Paperback edition)

ISBN 978-1-56765-595-7 (Hardcover edition)
NYC Item 56765-595-6 (Hardcover edition)

Copyright © 2008 by Amsco School Publications, Inc.
No part of this book may be reproduced in any form without written permission from the publisher.
Printed in the United States of America

2 3 4 5 6 7 8 9 10 11 10 09 08

PREFACE

Geometry is a new text for high school geometry that continues the approach that has made Amsco a leader in presenting mathematics in a contemporary, integrated manner. Over the past decades, this approach has undergone many changes and refinements to keep pace with the introduction and expansion of technology in the classroom.

Amsco texts parallel the integrated approach to the teaching of high school mathematics promoted by the National Council of Teachers of Mathematics in its *Curriculum and Evaluation Standards for School Mathematics*. In addition, the content of the book follows the guidelines mandated by the New York State Board of Regents in the *Mathematics Core Curriculum*. This book presents a range of materials and explanations to enable students to achieve a high level of excellence in their understanding of mathematics.

In this book:

- ✔ **Formal logic** is presented as the basis for geometric reasoning. Most of the geometric facts presented in this text are already familiar to the student. The purpose of this text is to help the student to use the principles of logic to understand the interdependence among these geometric and algebraic concepts.
- ✔ **Coordinate geometry** is presented with a postulational approach and used when appropriate to enhance and clarify synthetic proof.
- ✔ **Transformations** are introduced to further expand the students understanding of function and to relate that concept to geometry.
- ✔ **The concurrence theorems** for the altitudes, angle bisectors, medians and perpendicular bisectors of triangles are proved using a variety of approaches.
- ✔ **Solid geometry** is introduced and students are encouraged to expand their understanding of the three-dimensional world, particularly through the study of perpendicular and parallel lines and planes.
- ✔ **Algebraic skills** from *Integrated Algebra 1* are maintained, strengthened, and expanded as a bridge to *Algebra 2 and Trigonometry*.

- ✔ **Writing About Mathematics** encourages students to reflect on and justify mathematical conjectures, to discover counterexamples, and to express mathematical ideas in their own language.
- ✔ **Enrichment** is stressed both in the text and in the Teacher's Manual where many suggestions are given for teaching strategies and alternative assessment. The Manual provides opportunities for *Extended Tasks* and *Hands-On Activities*. Reproducible *Enrichment Activities* that challenge students to explore topics in greater depth are provided in each chapter of the Manual.

While *Integrated Algebra 1* is concerned with an intuitive approach to mathematics, the emphasis in *Geometry* is proof. In this text, geometry is developed as a postulational system of reasoning beginning with definitions, postulates, and the laws of reasoning. A unique blending occurs when students learn to apply the laws of logic to traditional deductive proof in geometry, both direct and indirect. The integration of traditional synthetic geometry, coordinate geometry, and transformational geometry is seen throughout the text and students learn to appreciate the interdependence of those branches of mathematics.

The intent of the author is to make the book of greatest service to the average student using thorough explanations and multiple examples. Each section provides careful step-by-step procedures for solving routine exercises as well as the nonroutine applications of the material. Sufficient enrichment material is included to challenge students of all abilities. Specifically:

- ✔ Concepts are carefully developed using appropriate language and mathematical symbolism. General principles are stated clearly and concisely.
- ✔ Numerous examples are solved as models for students with detailed explanations of the mathematical concepts that underlie the solution. Where appropriate, alternative approaches are suggested.
- ✔ Varied and carefully graded exercises are given in abundance to develop skills and to encourage the application of those skills. Additional enrichment materials challenge the most capable students.

CONTENTS

Chapter 1

ESSENTIALS OF GEOMETRY **1**

1-1	Undefined Terms	2
1-2	The Real Numbers and Their Properties	3
1-3	Definitions, Lines, and Line Segments	7
1-4	Midpoints and Bisectors	11
1-5	Rays and Angles	14
1-6	More Angle Definitions	19
1-7	Triangles	23
	Chapter Summary	29
	Vocabulary	30
	Review Exercises	31

Chapter 2

LOGIC **34**

2-1	Sentences, Statements, and Truth Values	35
2-2	Conjunctions	42
2-3	Disjunctions	48
2-4	Conditionals	53
2-5	Inverses, Converses, and Contrapositives	60
2-6	Biconditionals	69
2-7	The Laws of Logic	74
2-8	Drawing Conclusions	80
	Chapter Summary	85
	Vocabulary	87
	Review Exercises	87
	Cumulative Review	90

Chapter 3
PROVING STATEMENTS IN GEOMETRY 93

3-1 Inductive Reasoning 94
3-2 Definitions as Biconditionals 97
3-3 Deductive Reasoning 100
3-4 Direct and Indirect Proofs 105
3-5 Postulates, Theorems, and Proof 109
3-6 The Substitution Postulate 115
3-7 The Addition and Subtraction Postulates 118
3-8 The Multiplication and Division Postulates 124
Chapter Summary 128
Vocabulary 128
Review Exercises 129
Cumulative Review 131

Chapter 4
CONGRUENCE OF LINE SEGMENTS, ANGLES, AND TRIANGLES 134

4-1 Postulates of Lines, Line Segments, and Angles 135
4-2 Using Postulates and Definitions in Proofs 141
4-3 Proving Theorems About Angles 144
4-4 Congruent Polygons and Corresponding Parts 154
4-5 Proving Triangles Congruent Using Side, Angle, Side 158
4-6 Proving Triangles Congruent Using Angle, Side, Angle 161
4-7 Proving Triangles Congruent Using Side, Side, Side 165
Chapter Summary 167
Vocabulary 169
Review Exercises 169
Cumulative Review 170

Chapter 5
CONGRUENCE BASED ON TRIANGLES 174

5-1 Line Segments Associated with Triangles 175
5-2 Using Congruent Triangles to Prove Line Segments Congruent and Angles Congruent 178
5-3 Isosceles and Equilateral Triangles 181
5-4 Using Two Pairs of Congruent Triangles 186
5-5 Proving Overlapping Triangles Congruent 188
5-6 Perpendicular Bisector of a Line Segment 191
5-7 Basic Constructions 196
Chapter Summary 204
Vocabulary 204

Review Exercises 204
Cumulative Review 206

Chapter 6
TRANSFORMATIONS AND THE COORDINATE PLANE 209

6-1 The Coordinates of a Point in a Plane 210
6-2 Line Reflections 214
6-3 Line Reflections in the Coordinate Plane 222
6-4 Point Reflections in the Coordinate Plane 227
6-5 Translations in the Coordinate Plane 232
6-6 Rotations in the Coordinate Plane 238
6-7 Glide Reflections 243
6-8 Dilations in the Coordinate Plane 247
6-9 Transformations as Functions 250
Chapter Summary 255
Vocabulary 257
Review Exercises 257
Cumulative Review 259

Chapter 7
GEOMETRIC INEQUALITIES 262

7-1 Basic Inequality Postulates 263
7-2 Inequality Postulates Involving Addition and Subtraction 267
7-3 Inequality Postulates Involving Multiplication and Division 270
7-4 An Inequality Involving the Lengths of the Sides of a Triangle 273
7-5 An Inequality Involving an Exterior Angle of a Triangle 276
7-6 Inequalities Involving Sides and Angles of a Triangle 281
Chapter Summary 285
Vocabulary 286
Review Exercises 286
Cumulative Review 288

Chapter 8
SLOPES AND EQUATIONS OF LINES 290

8-1 The Slope of a Line 291
8-2 The Equation of a Line 295
8-3 Midpoint of a Line Segment 300
8-4 The Slopes of Perpendicular Lines 307
8-5 Coordinate Proof 313
8-6 Concurrence of the Altitudes of a Triangle 317
Chapter Summary 322
Vocabulary 323

Review Exercises 323
Cumulative Review 325

Chapter 9
PARALLEL LINES 328

9-1 Proving Lines Parallel 329
9-2 Properties of Parallel Lines 335
9-3 Parallel Lines in the Coordinate Plane 342
9-4 The Sum of the Measures of the Angles of a Triangle 347
9-5 Proving Triangles Congruent by Angle, Angle, Side 352
9-6 The Converse of the Isosceles Triangle Theorem 357
9-7 Proving Right Triangles Congruent by Hypotenuse, Leg 362
9-8 Interior and Exterior Angles of Polygons 367
Chapter Summary 373
Vocabulary 374
Review Exercises 375
Cumulative Review 376

Chapter 10
QUADRILATERALS 379

10-1 The General Quadrilateral 380
10-2 The Parallelogram 380
10-3 Proving That a Quadrilateral Is a Parallelogram 385
10-4 The Rectangle 389
10-5 The Rhombus 393
10-6 The Square 399
10-7 The Trapezoid 402
10-8 Areas of Polygons 409
Chapter Summary 412
Vocabulary 413
Review Exercises 414
Cumulative Review 417

Chapter 11
THE GEOMETRY OF THREE DIMENSIONS 419

11-1 Points, Lines, and Planes 420
11-2 Perpendicular Lines and Planes 423
11-3 Parallel Lines and Planes 433
11-4 Surface Area of a Prism 440
11-5 Volume of a Prism 446
11-6 Pyramids 449
11-7 Cylinders 453
11-7 Cones 456

11-8 Spheres 459
Chapter Summary 464
Vocabulary 467
Review Exercises 468
Cumulative Review 471

Chapter 12
RATIO, PROPORTION, AND SIMILARITY 474

12-1 Ratio and Proportion 475
12-2 Proportions Involving Line Segments 480
12-3 Similar Polygons 486
12-4 Proving Triangles Similar 489
12-5 Dilations 495
12-6 Proportional Relations Among Segments Related to Triangles 502
12-7 Concurrence of the Medians of a Triangle 506
12-8 Proportions in a Right Triangle 510
12-9 Pythagorean Theorem 515
12-10 The Distance Formula 521
Chapter Summary 527
Vocabulary 529
Review Exercises 529
Cumulative Review 532

Chapter 13
GEOMETRY OF THE CIRCLE 535

13-1 Arcs and Angles 536
13-2 Arcs and Chords 543
13-3 Inscribed Angles and Their Measures 552
13-4 Tangents and Secants 558
13-5 Angles Formed by Tangents, Chords, and Secants 567
13-6 Measures of Tangent Segments, Chords, and Secant Segments 575
13-7 Circles in the Coordinate Plane 581
13-8 Tangents and Secants in the Coordinate Plane 588
Chapter Summary 593
Vocabulary 597
Review Exercises 598
Cumulative Review 600

Chapter 14
LOCUS AND CONSTRUCTION 604

14-1 Constructing Parallel Lines 605
14-2 The Meaning of Locus 609

14-3	Five Fundamental Loci	613
14-4	Points at a Fixed Distance in Coordinate Geometry	616
14-5	Equidistant Lines in Coordinate Geometry	619
14-6	Points Equidistant from a Point and a Line	624
	Chapter Summary	630
	Vocabulary	631
	Review Exercises	631
	Cumulative Review	633

INDEX **634**