

AMSCO'S

ALGEBRA 2
and
TRIGONOMETRY

Ann Xavier Gantert



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Dedication

To Jessica Alexander and Uriel Avalos in gratitude for their invaluable work in preparing this text for publication.

Ann Xavier Gantert

The author has been associated with mathematics education in New York State as a teacher and an author throughout the many changes of the past fifty years. She has worked as a consultant to the Mathematics Bureau of the Department of Education in the development and writing of Sequential Mathematics and has been a coauthor of Amsco's *Integrated Mathematics* series, which accompanied that course of study.

Reviewers:

Richard Auclair
Mathematics Teacher
La Salle School
Albany, NY

Steven J. Balasiano
Assistant Principal,
Supervision Mathematics
Canarsie High School
Brooklyn, NY

Debbie Calvino
Mathematics Supervisor,
Grades 7–12
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Montgomery, NY

Domenic D'Orazio
Mathematics Teacher
Midwood High School
Brooklyn, NY

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Baldwin, NY

Ronald Hattar
Mathematics Chairperson
Eastchester High School
Eastchester, NY

Raymond Scacalossi Jr.
Mathematics Coordinator
Manhasset High School
Manhasset, NY

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PREFACE

Algebra 2 and Trigonometry is a new text for a course in intermediate algebra and trigonometry that continues the approach that has made Amsco a leader in presenting mathematics in a modern, integrated manner. Over the last decade, this approach has undergone numerous changes and refinements to keep pace with ever-changing technology.

This textbook is the final book in the three-part series in which Amsco parallels the integrated approach to the teaching of high school mathematics promoted by the National Council of Teachers of Mathematics in its *Principles and Standards for School Mathematics* and mandated by the New York State Board of Regents in the *Mathematics Core Curriculum*. The text presents a range of materials and explanations that are guidelines for achieving a high level of excellence in their understanding of mathematics.

In this book:

- ✓ **The real numbers** are reviewed and the understanding of operations with irrational numbers, particularly radicals, is expanded.
- ✓ **The graphing calculator** continues to be used as a routine tool in the study of mathematics. Its use enables the student to solve problems that require computation that more realistically reflects the real world. The use of the calculator replaces the need for tables in the study of trigonometry and logarithms.
- ✓ **Coordinate geometry** continues to be an integral part of the visualization of algebraic and trigonometric relationships.
- ✓ **Functions** represent a unifying concept throughout. The algebraic functions introduced in *Integrated Algebra 1* are reviewed, and exponential, logarithmic, and trigonometric functions are presented.
- ✓ **Algebraic skills** from *Integrated Algebra 1* are maintained, strengthened, and expanded as both a holistic approach to mathematics and as a bridge to advanced studies.
- ✓ **Statistics** includes the use of the graphing calculator to reexamine range, quartiles, and interquartile range, to introduce measures of dispersion such as variance and standard deviation, and to determine the curve that best represents a set of bivariate data.

- ✓ **Integration** of geometry, algebra, trigonometry, statistics, and other branches of mathematics begun in *Integrated Algebra 1* and *Geometry* is continued and further expanded.
- ✓ **Exercises** are divided into three categories. *Writing About Mathematics* encourages the student to reflect on and justify mathematical conjectures, to discover counterexamples, and to express mathematical ideas in his or her own words. *Developing Skills* provides routine practice exercises that enable the student and teacher to evaluate the student's ability to both manipulate mathematical symbols and understand mathematical relationships. *Applying Skills* provides exercises in which the new ideas of each section, together with previously learned skills, are used to solve problems that reflect real-life situations.
- ✓ **Problem solving**, a primary goal of all learning standards, is emphasized throughout the text. Students are challenged to apply what has been learned to the solution of both routine and non-routine problems.
- ✓ **Enrichment** is stressed both in the text and in the Teacher's Manual where many suggestions are given for teaching strategies and alternative assessment. The Manual provides opportunities for extended tasks and hands-on activities. Reproducible *Enrichment Activities* that challenge students to explore topics in greater depth are provided in each chapter of the Manual.

In this text, the real number system is expanded to include the complex numbers, and algebraic, exponential, logarithmic, and trigonometric functions are investigated. The student is helped to understand the many branches of mathematics, to appreciate the common threads that link these branches, and to recognize their interdependence.

The intent of the author is to make this book of greatest service to the average student through detailed explanations and multiple examples. Each section provides careful step-by-step procedures for solving routine exercises as well as the non-routine applications of the material. Sufficient enrichment material is included to challenge students of all abilities.

Specifically:

- ✓ Concepts are carefully developed using appropriate language and mathematical symbolism. General principles are stated clearly and concisely.
- ✓ Numerous examples serve as models for students with detailed explanations of the mathematical concepts that underlie the solution. Alternative approaches are suggested where appropriate.
- ✓ Varied and carefully graded exercises are given in abundance to develop skills and to encourage the application of those skills. Additional enrichment materials challenge the most capable students.

This text is offered so that teachers may effectively continue to help students to comprehend, master, and enjoy mathematics as they progress in their education.

CONTENTS

Chapter 1

THE INTEGERS

I-1	Whole Numbers, Integers, and the Number Line	2
I-2	Writing and Solving Number Sentences	5
I-3	Adding Polynomials	9
I-4	Solving Absolute Value Equations and Inequalities	13
I-5	Multiplying Polynomials	17
I-6	Factoring Polynomials	22
I-7	Quadratic Equations with Integral Roots	27
I-8	Quadratic Inequalities	30
	Chapter Summary	35
	Vocabulary	36
	Review Exercises	37

Chapter 2

THE RATIONAL NUMBERS

2-1	Rational Numbers	40
2-2	Simplifying Rational Expressions	44
2-3	Multiplying and Dividing Rational Expressions	48
2-4	Adding and Subtracting Rational Expressions	53
2-5	Ratio and Proportion	57
2-6	Complex Rational Expressions	61
2-7	Solving Rational Equations	64
2-8	Solving Rational Inequalities	70
	Chapter Summary	74
	Vocabulary	74
	Review Exercises	75
	Cumulative Review	77

Chapter 3

REAL NUMBERS AND RADICALS

3-1	The Real Numbers and Absolute Value	80
3-2	Roots and Radicals	84
3-3	Simplifying Radicals	88

3-4	Adding and Subtracting Radicals	94
3-5	Multiplying Radicals	98
3-6	Dividing Radicals	102
3-7	Rationalizing a Denominator	104
3-8	Solving Radical Equations	108
	Chapter Summary	113
	Vocabulary	114
	Review Exercises	114
	Cumulative Review	117

Chapter 4

RELATIONS AND FUNCTIONS **119**

4-1	Relations and Functions	120
4-2	Function Notation	127
4-3	Linear Functions and Direct Variation	130
4-4	Absolute Value Functions	136
4-5	Polynomial Functions	140
4-6	The Algebra of Functions	149
4-7	Composition of Functions	155
4-8	Inverse Functions	160
4-9	Circles	167
4-10	Inverse Variation	174
	Chapter Summary	178
	Vocabulary	180
	Review Exercises	180
	Cumulative Review	184

Chapter 5

QUADRATIC FUNCTIONS AND COMPLEX NUMBERS **186**

5-1	Real Roots of a Quadratic Equation	187
5-2	The Quadratic Formula	193
5-3	The Discriminant	198
5-4	The Complex Numbers	203
5-5	Operations with Complex Numbers	209
5-6	Complex Roots of a Quadratic Equation	217
5-7	Sum and Product of the Roots of a Quadratic Equation	219
5-8	Solving Higher Degree Polynomial Equations	224
5-9	Solutions of Systems of Equations and Inequalities	229
	Chapter Summary	239
	Vocabulary	240
	Review Exercises	241
	Cumulative Review	244

*Chapter 6***SEQUENCES AND SERIES 247**

6-1	Sequences	248
6-2	Arithmetic Sequences	252
6-3	Sigma Notation	257
6-4	Arithmetic Series	262
6-5	Geometric Sequences	266
6-6	Geometric Series	270
6-7	Infinite Series	273
	Chapter Summary	279
	Vocabulary	280
	Review Exercises	280
	Cumulative Review	283

*Chapter 7***EXPONENTIAL FUNCTIONS 286**

7-1	Laws of Exponents	287
7-2	Zero and Negative Exponents	289
7-3	Fractional Exponents	293
7-4	Exponential Functions and Their Graphs	298
7-5	Solving Equations Involving Exponents	304
7-6	Solving Exponential Equations	306
7-7	Applications of Exponential Functions	308
	Chapter Summary	314
	Vocabulary	315
	Review Exercises	315
	Cumulative Review	316

*Chapter 8***LOGARITHMIC FUNCTIONS 319**

8-1	Inverse of an Exponential Function	320
8-2	Logarithmic Form of an Exponential Equation	324
8-3	Logarithmic Relationships	327
8-4	Common Logarithms	332
8-5	Natural Logarithms	336
8-6	Exponential Equations	340
8-7	Logarithmic Equations	344
	Chapter Summary	347
	Vocabulary	347
	Review Exercises	348
	Cumulative Review	351

*Chapter 9***TRIGONOMETRIC FUNCTIONS 353**

9-1	Trigonometry of the Right Triangle	354
9-2	Angles and Arcs as Rotations	357
9-3	The Unit Circle, Sine, and Cosine	362
9-4	The Tangent Function	368
9-5	The Reciprocal Trigonometric Functions	374
9-6	Function Values of Special Angles	378
9-7	Function Values from the Calculator	381
9-8	Reference Angles and the Calculator	386
	Chapter Summary	392
	Vocabulary	394
	Review Exercises	394
	Cumulative Review	396

*Chapter 10***MORE TRIGONOMETRIC FUNCTIONS 399**

10-1	Radian Measure	400
10-2	Trigonometric Function Values and Radian Measure	406
10-3	Pythagorean Identities	411
10-4	Domain and Range of Trigonometric Functions	414
10-5	Inverse Trigonometric Functions	419
10-6	Cofunctions	425
	Chapter Summary	428
	Vocabulary	430
	Review Exercises	430
	Cumulative Review	431

*Chapter 11***GRAPHS OF TRIGONOMETRIC FUNCTIONS 434**

11-1	Graph of the Sine Function	435
11-2	Graph of the Cosine Function	442
11-3	Amplitude, Period, and Phase Shift	447
11-4	Writing the Equation of a Sine or Cosine Graph	455
11-5	Graph of the Tangent Function	460
11-6	Graphs of the Reciprocal Functions	463
11-7	Graphs of Inverse Trigonometric Functions	468
11-8	Sketching Trigonometric Graphs	472
	Chapter Summary	475
	Vocabulary	476
	Review Exercises	476
	Cumulative Review	479

*Chapter 12***TRIGONOMETRIC IDENTITIES 482**

12-1	Basic Identities	483
12-2	Proving an Identity	485
12-3	Cosine ($A - B$)	488
12-4	Cosine ($A + B$)	493
12-5	Sine ($A - B$) and Sine ($A + B$)	496
12-6	Tangent ($A - B$) and Tangent ($A + B$)	500
12-7	Functions of $2A$	504
12-8	Functions of $\frac{1}{2}A$	508
	Chapter Summary	513
	Vocabulary	514
	Review Exercises	514
	Cumulative Review	515

*Chapter 13***TRIGONOMETRIC EQUATIONS 518**

13-1	First-Degree Trigonometric Equations	519
13-2	Using Factoring to Solve Trigonometric Equations	526
13-3	Using the Quadratic Formula to Solve Trigonometric Equations	530
13-4	Using Substitution to Solve Trigonometric Equations Involving More Than One Function	534
13-5	Using Substitution to Solve Trigonometric Equations Involving Different Angle Measures	538
	Chapter Summary	542
	Vocabulary	542
	Review Exercises	543
	Cumulative Review	545

*Chapter 14***TRIGONOMETRIC APPLICATIONS 547**

14-1	Similar Triangles	548
14-2	Law of Cosines	552
14-3	Using the Law of Cosines to Find Angle Measure	557
14-4	Area of a Triangle	559
14-5	Law of Sines	564
14-6	The Ambiguous Case	569
14-7	Solving Triangles	575
	Chapter Summary	581
	Vocabulary	582
	Review Exercises	582
	Cumulative Review	585

Chapter 15

STATISTICS

587

15-1	Univariate Statistics	588
15-2	Measures of Central Tendency	596
15-3	Measures of Central Tendency for Grouped Data	605
15-4	Measures of Dispersion	614
15-5	Variance and Standard Deviation	619
15-6	Normal Distribution	628
15-7	Bivariate Statistics	634
15-8	Correlation Coefficient	641
15-9	Non-Linear Regression	647
15-10	Interpolation and Extrapolation	655
	Chapter Summary	662
	Vocabulary	664
	Review Exercises	664
	Cumulative Review	669

Chapter 16

PROBABILITY AND THE BINOMIAL THEOREM

672

16-1	The Counting Principle	673
16-2	Permutations and Combinations	678
16-3	Probability	687
16-4	Probability with Two Outcomes	695
16-5	Binomial Probability and the Normal Curve	701
16-6	The Binomial Theorem	708
	Chapter Summary	711
	Vocabulary	713
	Review Exercises	713
	Cumulative Review	715

INDEX

718