Glendale Unified School District

Senior High

February 17, 2015

Department: Mathematics

Course Title: Linear Algebra 1-2

Course Number: 3160-3161

Grade Level: 10-12

Semester Hours: 10

Recommended Prerequisite: AP Calculus AB

Recommended Textbook: Linear Algebra with Applications (5th Edition) by Otto K. Bretscher

Course Description: Students will be able to understand and apply concepts of Linear Algebra to real world applications as well as understanding the theory. Emphasis will be placed on the purpose and motivation behind concepts discussed through formal instruction and collaboration amongst students. Proofs of theorems will be elaborated on and used to help gain a deeper understanding of topics. Students will be able to use algorithms and other tools to help approximate solutions to systems of equations, which will allow them to make connections to problems dealing with engineering. Lectures will require students to give their input and to question as to how and why certain concepts work the way they do.

I. Standards

A. Specific Content Standard

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v). Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. Solve problems involving velocity and other quantities that can be represented by vectors. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in

the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. Compute the magnitude of a scalar multiple $\boldsymbol{cv}$ using $||\boldsymbol{cv}|| = |c|\boldsymbol{v}$. Compute the direction of $\boldsymbol{cv}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $\boldsymbol{cv}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$). Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. Add, subtract, and multiply matrices of appropriate dimensions. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

2. Students will learn how to apply topics of Linear Algebra to a broad range of sciences such as Physics, Computer Science, Chemistry, Biology, Economics, and of course Mathematics. These consist of concepts dealing with solution of linear systems such as simple matrix applications, working with linear combinations, Linear Transformations (defining what a linear transformation is and applying them to matrices and finding their inverse) and linear spaces as well as properties relating to these concepts (i.e. Subspaces, Image, Kernel, Bases, Linear Independence). Topics such as Matrix and Vector theory, identifying angles between vectors by using definitions of orthogonality and correlation coefficient will also be discussed. The ideas of eigenvalues and eigenvectors will be used to diagonalize matrices along with employing the properties of eigenvalues to deal with orthogonal matrices. Students will be able to find determinants of matrices and apply their properties to reach certain solutions. Some examples include finding determinants of two and three-dimensional matrices, apply the Laplace expansion to find determinants, finding the determinant of the transpose of a matrix, apply determinants to find volumes of solids, and apply Cramer's rule to explicitly find the solution of a system of linear equations. Complex eigenvalues will be explored to discuss stability of

dynamical systems along with their phase portraits. Linear Differential equations will also be covered if time permits. Direction fields will be used to sketch trajectories of matrices, stability of dynamical system along with their phase portraits will be covered, as well as solving Linear differential equations by the eigenfunction approach.

II. A. The format of the assessments will be designed as follows: There will be questions that are strictly computational, problems that will require explanation and reasoning to solutions and proofs. The distribution of questions corresponding to the level of difficulty will roughly follow: 20% of questions will consist of simple computations, 20% of questions will place emphasis on problems that place emphasis on theory through the use of proofs, and 60% of questions will consist of a combination of computational and theoretical type problems that require students to show understanding towards each unit.

III. Topic of Study - Suggested Time Distribution

A. *Linear Equations (2-3 weeks)*
B. *Linear Transformations (3-4 weeks)*
C. *Subspaces of $\Re^n$ and Their Dimensions (3-4 weeks)*
D. *Linear Spaces (3 weeks)*
E. *Orthogonality and Least Squares (4-5 weeks)*
F. *Determinants (3-4 weeks)*
G. *Eigenvalues and Eigenvectors (5-6 weeks)*
H. *Symmetric Matrices and Quadratic Forms (3-4 weeks)*
I. *Linear Differential Equations (3 weeks)*

IV. Recommended Materials

A. *Textbook*