## Glendale Unified School District

# High School

## April 16, 2019

Department: Mathematics

Course Title: Integrated Math IIIB/Precalculus Accelerated

Course Code: 3517DH/3518DH

School(s)

Course Offered: Hoover High School, Crescenta Valley High School, Verdugo Academy

UC/CSU Approved

(Y/N, Subject): Y, mathematics "c"

Course Credits: Full Year (10)

Recommended

Prerequisite: Integrated II/IIIA Accelerated

Recommended

Textbook: California Integrated Mathematics 3

Timothy D. Kanold, Edward B. Burger, Juli K. Dixon,

Matthew R. Larson, Steven J. Leinwand

Houghton Mifflin Harcourt

2015

Precalculus: Mathematics for Calculus

James Stewart, Lothar Redlin, Saleem Watson

Cengage Learning Seventh Edition

Course Overview:

Integrated Mathematics IIIB/Precalculus Accelerated is part two of a two-part compacted math series. Following Integrated Mathematics II/IIIA Accelerated, this course provides students with instruction in the second half of the content of the Integrated Math III and all of the course content for Precalculus. This compression is designed as the single point of acceleration at the high school level as recommended by the California

Mathematics Framework. This course is aligned to the California Common Core State standards for high school mathematics and supports the Standards for Mathematical Practice. With this course, students will develop a deep conceptual understanding of the mathematical relationships and concepts needed to succeed in higher level math courses.

In addition to covering the second half of Integrated III standards, this course meets all of the standards for a Common Core 4th Year high school math course. Several big ideas are interwoven, including: functions (e.g., inverse, composite, piecewise), trigonometry, modeling, algebraic manipulation, rates of change, and area under a curve. Students engage with an introduction to several to calculus topics, including limits, area under a curve, and rates of change. On a daily basis, students work collaboratively with others as they use problem-solving strategies, complete investigations, gather evidence, critically analyze results, and communicate clear and effective arguments while justifying their thinking.

In addition to the second half of Integrated III standards, this courses covers the same material as Precalculus Honors, by adding and adds rates of change, limits and area under the curve to the standard Precalculus course.

Course Content:

Semester A

## **Unit 6 Trigonometric Functions**

(approximately 15 days)

**STANDARDS** 

G-STR.8, G-SRT.10, F-TF.1, F-TF.2, F-TF.5, F-TF.8, F-IF.7e

A. This unit builds on the introductory trigonometric function learning from CC Integrated Mathematics II. Students will solve real life problems using trigonometric functions and Pythagorean identities. They will also work with transformations of graphs of trigonometric functions.

Major Topics:

Defining trigonometric functions with the unit circle
Laws of sines and cosines
Evaluating trig functions
Angle rotation and radian measure
Transformations of the parent graphs of trigonometric functions

## B. Unit Assignment(s):

At the end of this unit, students will complete a Math in Careers task by using models to represent the motion of a paddle wheel. Critical skills include graphing a trigonometric function, describing what its parameters mean for the real world situation, and using the function to make a prediction.

Students will graph a function for a riverboat paddle wheel with a given diameter and revolution rate that hangs a specific distance below the water line during an identified time interval. Students will describe the significance of the intercepts, the maximum and minimum values for the situation. They will also predict how revolutions it will take for a point on the wheel to have traveled one mile.

## Mathematical Practices Used in Unit:

- MP.2 Students will be able to reason abstractly in order to understand the relationship between the unit circle and the graphs of the trigonometric functions.
- MP.5 Students will be able to determine when it is appropriate to model periodic phenomena with a sinusoidal function. Students will use their knowledge of the key features of a trigonometric function to help fit the function to the model.
- MP.8 Students will look for and express regularity in repeated reasoning when organizing
  their work to find the angle measures of a triangle. Students will reason whether the
  measures given produce one triangle, two triangles or no triangle by using the Law of
  Sines.

# **Unit 7: Statistics and Decision Making**

(approximately 12 days)

#### **STANDARDS**

S-IC.1, S-IC.2, S-IC.3, S-IC.4, S-IC.5, S-ID.4, S-MD.6, S-CP.4

A. In this unit students will learn about statistics and using them to make sound decisions. They will gather and display data and find specific data points.

Major Topics:

Gathering and displaying data
Shape, center, and spread
Data distributions
Confidence intervals and margins of error
Using probability in making and analyzing decisions

## B. Unit Assignment(s):

After completing this unit, the students will perform a statistical hypothesis test. Critical skills include writing a null hypothesis, creating visual representations of data, and proving or disproving the null hypothesis.

Students will work as a pharmaceutical scientist would by testing whether a certain medication for raising glucose levels is more effective at higher doses. They will be given pre-generated test results for ten patients, five high level doses and five normal level doses. They will state their null hypothesis for the experiment. Then students will compare the results using box plots. Students will then explain why or why not they have enough evidence to reject the hypothesis.

## Mathematical Practices Used in Unit:

- MP.3 Students will be able to analyze whether a study is a survey experiment or an observational study and will be able to critique the results accordingly.
- MP.2 Students will be able to calculate confidence intervals and margins of error and be able to understand that statistics can help predict outcomes.
- MP.5 Students will be able to use a graphing utility to normalize data.

# **Unit 8: Equations of Circles and Parabolas**

(approximately 11 days)

STANDARDS G-C.2, G-C.3, G-C.5, G-GMD.1, G-GPE.1

A. In this unit, students will identify whether a quadratic equation in general form represents a circle by completing the square to get the equation in standard form.

# Major Topics:

Changing general form of quadratic equations into standard form Identify whether the quadratic equations produces a circle or a parabola

# B. Unit Assignment(s):

After completing this unit, students will complete a Module Performance Task to determine if a helicopter with a range of 290 nautical miles is in a close enough range to rescue a sailboat off the coast of California. Critical skills includes plotting, on a coordinate plane, the location of LAX and SFO airports along with the location of the sailboat given the latitude and longitude coordinates. Students need to determine which airport should send the rescue helicopter through graphing the equation of the circle circles. Students will justify their reasons and show the evidence in their work for how they came up with their answer.

## Mathematical Practices Used in Unit:

- MP.3 Students will construct viable arguments and justify their conclusions when identifying whether a quadratic equation in general form is a circle or a parabola.
- MP.8 Students will use repeated reasoning and algebraic steps when completing the square to change a general form quadratic equation into standard form.

## **Chapter 2: Functions**

(approximately 10 days)

## **STANDARDS**

N.Q.1, N.Q.2, N.Q.3, A.SSE.3b, A.CED.2, F.IF.2, F.IF.4, F.IF.5, F.IF.6, F.IF.7, F.IF.7a, F.IF.7b, F.IF.8, F.IF.8a, F.BF.1, F.BF.1a, F.BF.1b, F.BF.3, F.BF.4, F.BF.4a, F.BF.4b, F.BF.4c, F.BF.4d, F.LE.2, S.ID.6a, S.ID.6b, S.ID.7

A. In this chapter students will study functions and their graphs, as well as many real-world applications of the functions.

Major Topics:

Graphs of Functions
Average Rate of Change of a Function
Transformations of Functions
Combining Functions
One-to-one Functions and their Inverses

## B. Unit Assignment:

After completing Section 2.3, students will give a verbal summary of a situation described by a graph. In this assignment students describe, or tell the story that corresponds to a, given graph as well as make graphs that corresponds to a real-world "story".

## Mathematical Practices Used in Ch. 2:

- MP.4: Students will model linear relationships in real world situations and use average rate of change to interpret and quantify the relationship between two variables.
- MP.6. Students will label axes and specify key points to clearly and accurately represent
  the different type of basic functions such as the linear function, reciprocal function and
  greatest integer function. Students will also attend to precision for piece-wise functions,
  ensuring that endpoints are clearly marked and representative of the corresponding
  domains.
- MP.7: Students will be able to interpret average rate of change within the context of a
  question and express the significance of the value to the problem. The students will be
  able to step back and overview transformations of functions as they apply to all types of
  parent functions and can interpret the structure of functions as a simple transformation
  on the parent function.

## **Chapter 3: Polynomials and Rational Functions**

(approximately 9 days)

#### **STANDARDS**

N.Q.1, N.Q.2, N.Q.3, A.SSE.3b, A.SSE.3c, F.IF.2, F.IF.4, F.IF.5, F.IF.6, F.IF.7, F.IF.7c, F.IF.7d, F.IF.8b, F.BF.1a

A. In this chapter students will study polynomial and rational functions and their graphs, as well as many real-world applications of the functions.

Major Topics:

Quadratic Functions and Models
Polynomial Functions and Their Graphs
Real Zeros of Polynomials
Complex Zeros and the Fundamental Theorem of Algebra
Rational Functions
Polynomial and Rational Inequalities

## B. Unit Assignment:

After completing Section 3.2, students will experience the process of collecting data and then analyzing the data using linear regression. In this assignment students will construct bridges out of paper and use pennies as weights to determine how strong each bridge is. Students will model the data with linear and power functions to determine which model best fits the data. The model will allow students to predict the strength of a large bridge before it is built.

#### Mathematical Practices Used in Ch. 3:

- MP.2: Students will be able to reason abstractly by understanding the general importance of a power function to the graph of a polynomial. The students will also be able to contextualize the significance of the end behavior and zeros of the function.
- MP.4: Students can model real world situations with fluctuations by using polynomial functions. They can use the properties of polynomial functions to help contextualize the problem.
- MP.7: Students will be able to discern a pattern in the factored form of a polynomial and its relationship to the x-intercepts of a graph. They will also be able to recognize the significance of the power function of the expanded form of a polynomial and the end behavior of the graph.

Semester B

## **Chapter 4: Exponential and Logarithmic Functions**

(approximately 9 days)

**STANDARDS** 

N.Q.1, N.Q.3, A.SSE.3c, F.IF.2, F.IF.4, F.IF.5, F.IF.6, F.IF.7, F.IF.7e, F.IF.8b, F.BF.5, F.LE.4

A. In this chapter students will study exponential functions where the independent variable is in the exponent. Exponential functions are used in modeling many real-world situations. The inverse function of exponential functions are called logarithmic functions. Once an exponential model has been obtained, students will use the model to predict things such as the size of a population or the growth of an investment.

Major Topics:

Exponential Functions
The Natural Exponential Function
Logarithmic Functions
Laws of Logarithms
Exponential and Logarithmic Equations
Modeling with Exponential Functions
Logarithmic Scales

## B. Unit Assignment:

After completing Section 4.3, students will use logarithms or orders of magnitude to compare sizes of objects. It is often difficult to compare objects that vary enormously in size. In this assignment students learn how logarithms can be used to define the concept of "order of magnitude" which provides a simple way of comparison of these objects.

#### Mathematical Practices Used in Ch. 4:

- MP.2: Students will be able to quantitatively understand the structure of a logarithmic
  and exponential function separately as well as the inverse relationship of the functions.
  They will also be able to understand how the key features of a graph are represented in
  the algebraic expressions.
- MP.4: Students will be able to model a data set by using a linear, quadratic, exponential or logarithmic expression. Students will be able to determine which function best fits the data by identifying key points as well as end behavior.
- MP.5: Students will be able to determine when it is appropriate to use a graphing utility to fit an exponential or logarithmic curve to model a data set.

## Chapter 5: Trigonometric Functions: Unit Circle Approach

(approximately 8 days)

## **STANDARDS**

N.Q.2, N.Q.3, F.IF.8, F.LE.2, F.TF.1, F.TF.2, F.TF.3, F.TF.8, G.SRT.6, G.SRT.7, S.ID.6a

A. In this chapter students will study one way of viewing trigonometric functions, as functions of real numbers. Students will see in this chapter how the trigonometric functions are used to model periodic motion.

Major Topics:

The Unit Circle
Trigonometric Functions of Real Numbers
Trigonometric Graphs
Inverse Trigonometric Functions and Their Graphs
Modeling Harmonic Motion

## B. Unit Assignment:

After completing Section 5.3 students will consider an application to biology. In this assignment students will use sine functions to model the population of a predator and its prey. An isolated island is inhabited by two species of mammals: lynx and hares. The lynx are *predators* who feed on the hares, their *prey*. Students will find functions of the form y=asink(t-b)=c that model the lynx and hare populations from a given graph.

## Mathematical Practices Used in Ch. 5:

- MP.2: Students will be able to reason abstractly by understanding how the even-odd properties of a trigonometric function is related to the direction of rotation of the angle measure and the quadrant in which the terminal side is drawn.
- MP.5: Students will be able to determine when it is appropriate to use a graphing utility to model periodic phenomena with a sinusoidal function. Students will be able to fit a trigonometric function by identifying the key features of the graph.
- MP.6: Students will be able to clearly define the meaning of the variables in a trigonometric equation by identifying which is an angle and what specific ratio is defined. Students will also be able to accurately label axes according to the expression being graphed, whether the unit circle or a trigonometric function.

Semester B

**Chapter 6: Trigonometric Functions: Right Triangle Approach** (approximately 9 days)

STANDARDS

F.TF.3, F.TF.4, G.SRT.6, G.C.5

A. In this chapter students will study another way of viewing trigonometric functions, as functions of angles. Students will see in this chapter the relationships between angles and distances.

Major Topics:

**Angle Measures** 

Trigonometry of Right Triangles

Trigonometric Functions of Angles

Inverse of Trigonometric Functions and Right Triangles

The Law of Sines

The Law of Cosines

#### B. Unit Assignment:

After completing Section 6.2 students will investigate how areas and volumes of similar figures change as the size of the figure increases (or decreases). In this assignment students will find power functions that relate these quantities and use those functions to

explore the possibility of the existence of a real-life giant ape. Students will begin by finding some properties of similar figures.

## Mathematical Practices Used in Ch. 6:

- MP.1: Students will be able to analyze the given information for an oblique triangle and will persevere in determining the best law (either law of cosines or sines) to solve the triangle. Students will also be able to determine when the ambiguous case for the law of sines applies and will solve for both possible triangles, when necessary.
- MP.3: Students will be able to determine when, for an ambiguous triangle, the second case does or does not work and can create a viable argument as to why.
- MP.4: Students will be able to model real world situations with right and oblique triangles and will be able to use the laws of cosine and sine and the properties of right triangles to determine distances and angles.

# Chapter 7: Analytic Trigonometry

(approximately 8 days)

**STANDARDS** 

N.Q.1, N.Q.2, F.IF.7, F.IF.7e, F.TF.4, F.TF.5, F.TF.7, F.TF.8

A. In this chapter students will study algebraic properties of trigonometric functions. Students will simplify and factor expressions and solve equations that involve trigonometric functions.

Major Topics:

Trigonometric Identities Addition and Subtraction Formulas Double-Angle, Half-Angle and Product-Sum Formulas Basic Trigonometric Equations

## B. Unit Assignment:

After completing Section 7.3 students will use trigonometry to find the best location from which to view a painting or a movie. In this assignment, students will use graphing devices and the Law of Cosines to find the best viewing angle possible.

#### Mathematical Practices Used in Ch. 7:

- MP.1: Students will look for entry points to a trigonometric identity proof and will persevere in looking for relationships to equate the two expressions. Students will analyze the given expressions to determine a solution pathway instead of jumping into a solution attempt.
- MP.2: Students will be able to reason through a trigonometric equation by being able to reason abstractly and seeing a quadratic form that requires factorization. Students will also be able to reason quantitatively by identifying the relationships between the ratios of sides with the corresponding angle measures.

MP.3: Students will be able to build a logical progression of statements to provide a proof
for a trigonometric identity by utilizing the quotient identities, Pythagorean identities, as
well as the sum, difference, double- and half-angle formulas.

## **Chapter 8: Polar Coordinates and Parametric Equations**

(approximately 7 days)

**STANDARDS** 

F.IF.7, F.IF.7e, F.TF.4, F.TF.5, F.TF.8

A. In this chapter students will study a different way of locating points in the plane. Instead of using rectangular coordinates to specify a location, students will give distance and direction from a fixed reference point.

Major Topics:

**Polar Coordinates** 

**Graphs of Polar Equations** 

Polar Form of Complex Numbers: De Moivre's Theorem

Plane Curves and Parametric Equations

# B. Unit Assignment:

After completing the chapter, students will use parametric equations and trigonometry to model the path of a projectile in motion. Students will be able to utilize coordinate geometry to realize how changing the angle of projection affects the domain and range of motion.

#### Mathematical Practices Used in Ch. 8:

- MP.1: Students will be able to visualize the graphing of a complex number as it relates to right triangle trigonometry and how this relationship allows for the conversion between rectangular and polar systems.
- MP.2: Students will be able to relate the roots of a complex number as points equally spaced about a circle and will be able to relate the periodicity of a sinusoidal function to help identify the polar form of the roots.
- MP.3: Students will be able to analyze the relationship between rectangular coordinates and the path of a point moving on a plane to eliminate a parameter and convert between rectangular and parametric equations.

## **Chapter 9: Vectors in Two and Three Dimensions**

(approximately 9 days)

**STANDARDS** 

N.Q.1, N.Q.2, N.Q.3, F.TF.6, F.TF.7, G.SRT.8, G.SRT.9

A. In this chapter students will be able to represent quantities, such as force, with vectors that are defined by magnitude and direction. Students will also study how several forces acting on an object will affect the movement of that object.

Major Topics:

Vectors in Two Dimensions
The Dot Product
Three-Dimensional Coordinate Geometry
Vectors in Three Dimensions
The Cross Product
Equations of Lines and Planes

## B. Unit Assignment:

After completing the chapter, students will use vector fields to model real-world scenarios, like the gravitational force on the earth or wind on the surface of the earth. Students will be able to then analyze the vector fields to identify the center and direction of motion.

## Mathematical Practices Used in Ch. 9:

- MP.4: Students will be able to graph vectors in the cartesian system and use the dot product to find the angle between two vectors as well as the direction angles of a vector.
- MP.5: Students will use proper graphing utilities to help them visualize and analyze three-dimensional vectors and cross product vectors.
- MP.6: Students will attend to precision by properly drawing and labeling axes to graph in three dimensions.

## Chapter 10: Systems of Equations and Inequalities

(approximately 15 days)

## **STANDARDS**

N.Q.1-3, N.CN.4, N.CN.5, N.VM.1-5, A.REI.5, N.REI.6, G.SRT.10, G.SRT.11

A. In this chapter, students will use more than two variables to create and solve a system of equations and inequalities. Students will be able to adapt the elimination and substitution methods of solving systems with two variables to now solve systems with three or more variables as well as learning new methods of solving, such as utilizing matrices and Cramer's Rule.

#### *Major Topics:*

Systems of Linear Equations in Two Variables Systems of Linear Equations in Several Variables Matrices and Systems of Linear Equations The Algebra of Matrices Inverses of Matrices and Matrix Equations

Determinants and Cramer's Rule Partial Fractions Systems of Nonlinear Equations Systems of Inequalities

## B. Unit Assignment:

After section 10.4, students will use matrix multiplication to project the population proportions of the young, juvenile and adult proportions of a population of animals. Students will use these predictions to analyze the health and growth of the population for the coming seasons in order to understand the long-term prospects of the population.

#### Mathematical Practices Used in Ch. 10:

- MP.2: Students will analyze a linear system of equations and inequalities and understand that there are multiple methods of solving a system i.e. substitution, elimination, augmented matrices, Cramer's rule, etc.
- MP.4: Students will understand how to create a matrix to represent data and how matrix operations can be used to analyze the data.
- MP.5: Students will understand how a matrix can be used to represent equations and data and how to algebraically manipulate matrices to solve linear systems.

## **Chapter 11: Conic Sections**

(approximately 9 days)

STANDARDS N.VM.7-12, A.REI.7-9, A.REI.12

A. In this chapter, students will understand how conic sections are different cross-sections of a cone. Students will be able to identify the equations of different conics as well as their geometric properties and their graphs.

Major Topics:
Parabolas
Ellipses
Hyperbolas
Shifted Conics
Rotation of Axes
Polar Equations of Conics

## B. Unit Assignment:

At the end of the chapter, students will study the use of conics in architecture and how the geometric properties of each conic can create architectural features, such as a whispering gallery. Students will also learn practical methods by which conics could be constructed with by using practical construction tools.

#### Mathematical Practices Used in Ch. 11:

- MP.1: Students will be able to transform the general form of a conic into the standard form of the respective conic to determine key features.
- MP.2: Students will be able to identify the type of conic from the general form.
- MP.4: Students will be able to model real world situations with the appropriate conic type and use the key features to solve problems.

## **Chapter 12: Sequences and Series**

(approximately 9 days)

STANDARDS G.GPE.2, G.GPE.3, G.GMD.4

A. In this chapter, students will study different types of sequences, such as geometric, arithmetic and the Fibonacci sequences and their applications in the real world. Students will be able to recognize, formulate and apply sequences to real world situations as well as being able to evaluate the sum of a series.

## *Major Topics:*

Sequences and Summation Notation Arithmetic Sequences Geometric Sequences Mathematics of Finance Mathematical Induction The Binomial Theorem

## B. Unit Assignment:

At the end of this chapter, students will be able to apply their knowledge of recursive and explicit formulas of a sequence to analyze and evaluate practical problems, such as a monthly savings account as well as the accumulation of pollutants in the environment. They will be able to predict future values of account balances as well as the percentage of pollutants in the environment after n number of years.

## Mathematical Practices Used in Ch. 12:

- MP.5: Students will be able to identify the strengths of using the binomial theorem vs. Pascal's triangle but still understand the complementary relationship between the two.
- MP.7: Students will discern a pattern in a sequence or series and identify whether it is arithmetic or geometric.
- MP8: Students will be able to formulate an explicit and/or recursive formula to represent various sequences and series.

# **Chapter 13: Limits - A Preview of Calculus**

(approximately 8 days)

STANDARDS

N.CN.8, N.CN.9, A-APR.2, A.APR.3

A. In this chapter, students will understand the definition of one-sided and the overall limit and its applications in calculus, such as finding the instantaneous rate of change as well as the area of a bounded region. Students will understand the laws and the definition of alimit of an average rate of change as the instantaneous speed.

Major Topics:

Finding Limits Numerically and Graphically Finding Limits Algebraically Tangent Lines and Derivatives Limits at Infinity; Limits of Sequences Areas

B. Unit Assignment:

By the end of the chapter, students will apply their knowledge of finding the area under a curve by finding the total distance of a car traveling for a set amount of time at a designated speed, in miles per hour.

Mathematical Practices Used in Ch. 13:

- MP.2: Students will be able to determine when the limit of a function exists both algebraically and graphically.
- MP.3: Students will be able to justify why the slope of a tangent line must equal 0 at any local maximum or minimum value and its significance.
- MP:6: Students will be able to clearly communicate the conditions for which the limit of a function does and does not exist.

## **Chapter 14: Probability and Statistics**

(approximately 13 days)

**STANDARDS** 

A.SSE.4, A.APR.5, A.APR.6, A.APR.7, F.IF.3, F.BF.1a, F.BF.2, F.LE.2

A. In this chapter, students will study events that are governed by randomness and have a predictability to them. Students will understand that not all situations can be precisely determined or calculated, but can be analyzed and predicted using probability rules. Students will be able to collect, organize and analyze data using statistical methods, such as the 5-number summary.

Major Topics:

Counting

Probability

Binomial Probability

**Expected Value** 

Descriptive Statistics (Numerical)

Descriptive Statistics (Graphical)

Introduction to Statistical Thinking

Introduction to Inferential Statistics

# B. Unit Assignment:

After section 14.6, students will understand how statistics can be used to mislead and misinform. Students will then be able to analyze studies and newspaper articles that quote statistics and identify sources of potential bias.

#### Mathematical Practices Used in Ch. 14:

- MP.2: Students will be able to use quantitative reasoning to find the probabilities for events, including conditional probability.
- MP.5: Students will be able to use Venn diagrams appropriately to represent the probabilities for the union and intersection of two events as well as for conditional probability. Students will also be able to use a stem and leaf plot to represent data and to find a 5-number summary.
- MP.6 Students will accurately be able to calculate a 5-number summary in order to properly create a visual summary of the statistics, such as a normalized distribution or a box and whisker plot.

## Comprehensive Final Exam Details

- 1. Students will be tested on their knowledge of the following topics:
  - o Equations of Circles and Parabolas
  - Prerequisites from Algebra and Geometry
  - Trigonometric Functions
  - Analytic Trigonometry
  - Laws of Sines, Cosines
  - Polar and Vectors
  - Complex Numbers
  - Exponential and Logarithmic Functions
  - Topics in Analytic Geometry, including Conics
  - Functions and Models
  - Limits and Derivatives
  - Differentiation Rules
  - Applications of Differentiation

2. The Final Exams - much like unit/chapter exams - are detail-oriented and require students to provide detailed, step-by-step justification for their responses to each of the questions. This way, they get trained and prepared for their future AP Math courses/exams.