Glendale Unified School District

High School

April 16, 2019

Department:	Mathematics
Course Title:	Integrated Math IIIB/Precalculus Accelerated
Course Code:	3517D1/3518D1
School(s) Course Offered:	Clark Magnet High School
UC/CSU Approved (Y/N, Subject):	Y, "c" Mathematics
Course Credits:	Full Year (10)
Recommended Prerequisite:	Integrated II/IIIA Accelerated or Integrated II + (Summer) Integrated Math IIIA Accelerated
Recommended Textbook:	*Core Connections Integrated III* Judy Kysh, Evra Baldinger, Michael Kassarjian, Karen Wootton, et. al CPM Educational Program Second Edition, Version 5.0 *Precalculus: Mathematics for Calculus* James Stewart, Lothar Redlin, Saleem Watson Cengage Learning Seventh Edition

Course Overview:

Integrated Mathematics IIIB/Precalculus Accelerated is part two of a two-part compacted math series. Following Integrated Mathematics II/IIIA Accelerated, this course provides students with instruction in the second half of the content of the Integrated Math III and all of the course content for Precalculus. This compression is designed as the single point of acceleration at the high school level as recommended by the California

Mathematics Framework. This course is aligned to the California Common Core State standards for high school mathematics and supports the Standards for Mathematical Practice. With this course, students will develop a deep conceptual understanding of the mathematical relationships and concepts needed to succeed in higher level math courses.

In addition to covering the second half of Integrated III standards, this course meets all of the standards for a Common Core 4th Year high school math course. Several big ideas are interwoven, including: functions (e.g., inverse, composite, piecewise), trigonometry, modeling, algebraic manipulation, rates of change, and area under a curve. Students engage with an introduction to several to calculus topics, including limits, area under a curve, and rates of change. On a daily basis, students work collaboratively with others as they use problem-solving strategies, complete investigations, gather evidence, critically analyze results, and communicate clear and effective arguments while justifying their thinking.

In addition to the second half of Integrated III standards, this courses covers the same material as Precalculus Honors, by adding and adds rates of change, limits and area under the curve to the standard Precalculus course.

Course Content:

Semester A

Unit 5: Inverses and Logarithms *(approximately 11 days)*

STANDARDS
F-BF.3, F-BF-4, F-BF.4a, F-LE.4, F-LE.4.2, I-IF.7e

A. Reversing is an important theme in the early part of this chapter. The first section introduces the concept of inverse relations. Students learn that reversing, or working backward to undo the action of a function, can create a new function. They explore multiple representations of functions and their inverses, and recognize that many functions have inverses that are not functions. In the second, students determine inverses of parent functions. They learn that the inverse of an exponential function is a logarithm. Reversing is emphasized once again as they learn how to convert exponential equations into logarithmic form, and vice versa. Students investigate the new family of logarithmic functions $f(x) = \log_b(x)$ for different values of b, test values on their calculators to determine the base the calculators work in, and learn to graph transformations of $f(x) = \log(x)$.

Progression of Content:
This chapter adds to students' lists of parent functions, which will continue to expand with the addition of polynomial functions in Unit 8 and trigonometric functions in Unit 9.

B. Unit Assignment(s):

Mathematics Practices Used in Unit 5:

- Students will look for and make use of structure and construct viable arguments as they develop and justify strategies for undoing functions and as they investigate different bases for logarithms..
- Students will use appropriate tools strategically and look for and make use of structure when they graph inverses of functions and write their equations.
- Students will look for and make use of structure as they verify inverses using multiple representations, and attend to precision as they restrict the domain of a function to ensure that its inverse is also a function.
- Students will construct viable arguments and critique the reasoning of others and look for and make use of structure as they apply their knowledge of parent graphs and inverses to learn about logarithms.
- Students will look for and make use of structure and express regularity in repeated reasoning as they learn the definition of logarithm and calculate the values of logarithms.
- Students will construct viable arguments and critique the reasoning of others, use appropriate tools strategically, and look for and make use of structure as they investigate logarithms with different bases.

Sample Activities:
Guess My Number Game - Students are asked to guess the number the teacher is thinking of based on the order of operations applied to the number and what the mystery number has ultimately transformed into. Students may or may not write an equation, but you may want to encourage them to do so as it will help when they progress to working with functions and inverses. Making sure that the idea of reversing, or undoing, comes up in the discussion of the "Guess My Number" game. You undo each step, reversing the original Order of Operations.

Graph the Inverse Function - Students will be looking at strategies for creating graphs of inverse functions. The first two graphs have functions that they are capable of finding the equation of their inverse functions but the third function does not lend itself to be solved for the inverse function. Teams could make a mini-table of some coordinates from the graph and then use it to help make a mini-table for the inverse graph. Students will soon discover that the line $y = x$ is the line of symmetry.

Unit 7: Logarithms and Triangles *(approximately 13 days)*

STANDARDS
F-LE.3, F-LE.4, F-LE.4.1, F-LE.4.3, A-SSE.2, F-BF.1, F-IF.7e, G-SRT.9+, G-SRT.10+, G-SRT.11+

A. In this unit, students return to their work with logarithms to develop tools they can use when solving application problems involving exponential equations. In the first half of this unit, students investigate the family $y = \log(m^n)$ and discover the Power Property of Logarithms, which allows them to solve exponential equations by using logs to undo or rewrite the equation. Students generalize from number patterns to make conjectures about other properties of logarithms and then prove these properties. Furthermore, students develop and share strategies to write the equation of the exponential function with a given asymptote that passes through two given points. Then they use that equation to make predictions.

The remainder of this unit focuses on completing a tool kit for calculating missing parts of non-right triangles. Students identify the types of information needed to determine all of the missing sides and angles of a triangle. Through this exercise, students also identify triangles for which they do not yet have the tools to determine missing parts. Students notice that they *do* have enough tools to calculate the measures and side lengths of right triangles. This leads to the question, "*What if the triangle is not a right triangle?*" Students then develop the Law of Sines and Law of Cosines so that they have a complete set of tools to determine the other missing parts of any triangle (when sufficient information is provided).

The unit concludes with students looking at different application problems using triangles and identifying which tools are most useful in each situation. In addition, students investigates the ambiguous case of triangles: SSA. This lesson is offered for accelerated classes or those that could benefit from a complete view of the relationships between the sides and angles of a triangle. Working through the problems of this lesson before you decide to use the lesson with your students is highly recommended.

Progression of Content:
The work with logarithms in this unit prepares students for future work in a pre-calculus course. Working with Law of Sines and Cosines and reviewing the use of right triangle trigonometry and special right triangles prepares students for working with the trigonometric family of functions in Unit 9.

B. Unit Assignment(s):
Mathematics Practices Used in Unit 7:

- Students will look for and make use of structure and express regularity in repeated reasoning while they develop the Power Property of Logarithms, learn other properties of logs and how to rewrite equations with different bases.

- Students will make sense of problems and reason abstractly and quantitatively as they write the equation of an exponential function given two points and an asymptote.
- Students will make sense of problems and persevere in solving them, model with mathematics, and reason abstractly and quantitatively as they explore exponential functions with an asymptote other than $y = 0$ and apply logarithms to solve an exponential equation.
- Students will make sense of problems and persevere in solving them as they figure out what information they need to solve for parts of triangles. They will need to attend to precision as they communicate what they know and do not know.
- Students will look for and express regularity in repeated reasoning as they develop the ratios for the Law of Sines.
- Students will make sense of problems and persevere in solving them using the Law of Cosines.
- Students will attend to precision as they work with triangles involving the SSA relationship. They must also use appropriate tools strategically as they explore the ambiguous case of the Law of Sines.
- Students will reason abstractly and quantitatively as they make sense of problems and persevere in solving them. They will attend to precision as they solve the problems and communicate within their teams, labeling diagrams, attending to units, and calculating their answers accurately.

Sample Activities:
"The Case of the Cooling Corpse" - a problem in which students model the falling temperature of a corpse to establish time of death and solve a murder mystery. In order to solve the mystery, students will need to decide which information is relevant to solving the problem (e.g., body temperature, times listed on sign-in sheets, etc.) . Students will need to write and solve a system of exponential functions, with a horizontal asymptote representing the room temperature, the independent variable as time, and the dependent variable as the temperature of the body.

Solving Triangles - This activity consists of eight independent triangle problems (suggestion is to assign one problem to a group of 3-4 students) that will provide students with a chance to consolidate their understanding of the various tools and strategies they have developed so far to solve triangle problems. Some problems may be solved using the Law of Sines, Law of Cosines, or a combination of both. Students will present their assigned problem to the entire class and their process for solving the assigned problem.

Unit 8: Polynomials *(approximately 15 days)*

STANDARDS
A-APR.1, A-APR.2, A-APR.4, A-APR.6, A-APR.3, F-IF.4, F-IF.7c, N-CN.8+, N-CN.9+, A-SSE.2, A-CED.2, F-BF.1

A. In the first section, students will investigate the equation ↔ graph connections for polynomial functions. They will recognize that equations in factored form are much easier to sketch, and they will understand the relationship between the factors and the x-intercepts of the graph. Then, in the second section, they will develop an understanding of imaginary and complex numbers and recognize that polynomial functions can have complex roots. In the third and last section, they will learn to divide polynomials by a known factor to find other factors. This will allow them to determine complex and irrational roots of some cubic and quartic functions.

Progression of Content:
Students will build on their understanding of function families in Unit 9, where they study trigonometric functions and transform the graphs of sine and cosine functions. Students will use their algebra skills when they study rational expressions in Unit 11. Students will also use their algebra skills when they prove formulas for sums of series in Unit 10 and solve trigonometric identities in Unit 12.

B. Unit Assignment(s):
Mathematics Practices used in Unit 8:

- Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others as they discuss factored form and describe the graphs of polynomials and their understanding of the stretch. Students will also look for and make use of structure as they draw graphs of polynomials.
- Students will look for and make use of structure as they continue their polynomial investigation. Students will also construct viable arguments and critique the reasoning of others as they consolidate their results on polynomials.
- Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others as they develop their understanding of a stretch factor. Student will also model with mathematics as they develop an equation for the roller coaster problem first introduced in the first lesson of Unit 8.
- Students will look for and make use of structure as they use polynomial division to determine factors of polynomials. Students will also construct viable arguments and critique the reasoning of others as they develop their understanding of polynomial division.
- Students will look for and make use of structure as they use complex roots to write equations of quadratic functions and express regularity in repeated reasoning as they identify polynomial identities to help them factor. Students will also look for and express regularity in repeated reasoning as they identify patterns in the sums and products of complex roots.
- Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others when they determine all

roots of a polynomial with a degree greater than two and when they determine all roots of a polynomial with a degree greater than two.

Sample Activities:
Polynomial Function Investigation - Students are instructed to look for, label, and describe the *x*-intercepts and "bounces" (double roots, although students will probably not use this term), the *y*-intercept, the number of turns, and the behavior of graphs for very large and very small *x*-values. They should share and discuss all of their graphs and their observations within their teams. The discussion should lead to conjectures and the creation of several new equations to try. Students are coming up with methods for determining the *x*- and *y*-intercepts, predictions about the numbers of crossings of the *x*-axis, and ideas for determining a reasonable window or maximum and minimum approximations.

Game of Polydoku - By treating division as a puzzle and using the organizational device of an area model, students can use logical reasoning to reverse the multiplication process and figure out a missing factor. Once they have worked through the process several times, they should be able to set up and solve their own division problems. This method is as efficient as (if not more efficient than) polynomial "long division" and you can use it to develop synthetic division if that is part of your curriculum. This game introduces polynomial division by challenging students to reverse the process of polynomial multiplication.

Unit 11: Rational Expressions and Three-Variable Systems *(approximately 6 days)*

STANDARDS
A-APR.7+

A. In this chapter, students will focus on operations with rational expressions. Students did a function investigation in Unit 1 that previewed the investigation of rational expressions. Students learn a powerful method of simplifying rational expressions that uses properties of the number 1 and the properties of exponents. In a previous course, students learned to solve a system of two linear equations in two variables, and they reviewed this topic in Chapter 3 of this text.

Progression of Content:
The remaining chapter requires students to use their algebra skills to solve trigonometric identities. In future courses, students may use matrices to solve systems of equations.

B. Unit Assignment(s):
Mathematics Practices Used in Unit 11:

- Students will look for and make use of structure as well as look for and express regularity in repeated reasoning as they use the number 1 to understand rational expressions.

- Students will also look for and make use of structure as well as look for and express regularity in repeated reasoning as they connect multiplication and division of fractions to that of rational expressions.
- Students will look for and make use of structure as they make connections between adding and subtracting fractions and adding and subtracting rational expressions, and as they locate points in three-dimensional space. They will construct viable arguments and critique the reasoning of others as they justify their strategies.
- Students will continue to look for and make use of structure as well as look for and express regularity in repeated reasoning as they work with rational expressions. They will construct viable arguments and critique the reasoning of others as they investigate the closure of rational expressions under operations.
- Students will reason abstractly and quantitatively and look for and express regularity in repeated reasoning as they write the equation of a quadratic function by solving a system of three equations with three unknowns.

Sample Activity:
Giant One - Students are encouraged to rewrite rational expressions to create fractions that will reduce to one. The new expressions will be reduced to its simplest form. This strategy to rewrite will allow students to simplify rational expression after doing basic operations on the original expressions.

Chapter 1: Fundamentals *(approximately 15 days)*

STANDARDS
N.Q.1, N.Q.2, N.Q.3, N.CN.7, A.SSE.3a, A.CED.1, A.CED.2, A.CED.3, A.CED.4, A.REI.2, A.REI.3, A.REI.4, A.RED.4a, A.REI.4b, F.IF.1, F.IF.2, F.IF.4, F.IF.5, F.IF.7, F.IF.7b, F.IF.8, G.GPE.1, G.GPE.5, G.GPE.6, G.GPE.7

A. In the first chapter, students will review the real numbers, equations and the coordinate plane. Students will get a fresh look at these review concepts by applying them to real world problems.

Major Topics:
Real numbers
Exponential, radical and rational expressions
Complex numbers
Inequalities
Lines, circles

B. Unit Assignment:
After completing section 1.12, students will use their understanding of proportionality to determine how a frog's size relates to its sensitivity to pollutants in the environment. They will explore how animals, of the same body type, will have a skin area and volume

that are proportional to the length of their body. The students will be able to calculate proportionality according to body length, weight, surface skin area, and volume.

Mathematical Practices Used in Ch. 1:

- MP.1: Students will explain to themselves the meaning of a problem and look for entry points to its solution. They analyze givens, constraints, relationships and goals for modeling questions regarding inequalities and proportions. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.
- MP.2: Students make sense of quantities and their relationships in problem situations. They are able to abstract a given situation and represent the relationship between two variables symbolically, as in direct and indirect variation.
- MP.3: Students learn how to use stated assumptions, definitions and previously established results in constructing arguments and equations to represent real world situations. They will utilize the distance*rate=time equation and proportional relationships to analyze situations that take into account the context of each question.

Chapter 2: Functions *(approximately 10 days)*

STANDARDS
N.Q.1, N.Q.2, N.Q.3, A.SSE.3b, A.CED.2, F.IF.2, F.IF.4, F.IF.5, F.IF.6, F.IF.7, F.IF.7a, F.IF.7b, F.IF.8, F.IF.8a, F.BF.1, F.BF.1a, F.BF.1b, F.BF.3, F.BF.4, F.BF.4a, F.BF.4b, F.BF.4c, F.BF.4d, F.LE.2, S.ID.6a, S.ID.6b, S.ID.7

A. In this chapter students will study functions and their graphs, as well as many real-world applications of the functions.

Major Topics:
Graphs of Functions
Average Rate of Change of a Function
Transformations of Functions
Combining Functions
One-to-one Functions and their Inverses

B. Unit Assignment:
After completing Section 2.3, students will give a verbal summary of a situation described by a graph. In this assignment students describe, or tell the story that corresponds to a, given graph as well as make graphs that corresponds to a real-world "story".

Mathematical Practices Used in Ch. 2:

- MP.4: Students will model linear relationships in real world situations and use average rate of change to interpret and quantify the relationship between two variables.
- MP.6. Students will label axes and specify key points to clearly and accurately represent the different type of basic functions such as the linear function, reciprocal function and greatest integer function. Students will also attend to precision for piece-wise functions, ensuring that endpoints are clearly marked and representative of the corresponding domains.
- MP.7: Students will be able to interpret average rate of change within the context of a question and express the significance of the value to the problem. The students will be able to step back and overview transformations of functions as they apply to all types of parent functions and can interpret the structure of functions as a simple transformation on the parent function.

Chapter 3: Polynomials and Rational Functions *(approximately 9 days)*

STANDARDS
N.Q.1, N.Q.2, N.Q.3, A.SSE.3b, A.SSE.3c, F.IF.2, F.IF.4, F.IF.5, F.IF.6, F.IF.7, F.IF.7c, F.IF.7d, F.IF.8b, F.BF.1a

A. In this chapter students will study polynomial and rational functions and their graphs, as well as many real-world applications of the functions.

Major Topics:
Quadratic Functions and Models
Polynomial Functions and Their Graphs
Real Zeros of Polynomials
Complex Zeros and the Fundamental Theorem of Algebra
Rational Functions
Polynomial and Rational Inequalities

B. Unit Assignment
After completing Section 3.2, students will experience the process of collecting data and then analyzing the data using linear regression. In this assignment students will construct bridges out of paper and use pennies as weights to determine how strong each bridge is. Students will model the data with linear and power functions to determine which model best fits the data. The model will allow students to predict the strength of a large bridge before it is built.

Mathematical Practices Used in Ch. 3:

- MP.2: Students will be able to reason abstractly by understanding the general importance of a power function to the graph of a polynomial. The students will also be able to contextualize the significance of the end behavior and zeros of the function.

- MP.4: Students can model real world situations with fluctuations by using polynomial functions. They can use the properties of polynomial functions to help contextualize the problem.
- MP.7: Students will be able to discern a pattern in the factored form of a polynomial and its relationship to the x-intercepts of a graph. They will also be able to recognize the significance of the power function of the expanded form of a polynomial and the end behavior of the graph.

Chapter 4: Exponential and Logarithmic Functions *(approximately 9 days)*

STANDARDS
N.Q.1, N.Q.3, A.SSE.3c, F.IF.2, F.IF.4, F.IF.5, F.IF.6, F.IF.7, F.IF.7e, F.IF.8b, F.BF.5, F.LE.4

A. In this chapter students will study exponential functions where the independent variable is in the exponent. Exponential functions are used in modeling many real-world situations. The inverse function of exponential functions are called logarithmic functions. Once an exponential model has been obtained, students will use the model to predict things such as the size of a population or the growth of an investment.

Major Topics:
Exponential Functions
The Natural Exponential Function
Logarithmic Functions
Laws of Logarithms
Exponential and Logarithmic Equations
Modeling with Exponential Functions
Logarithmic Scales

B. Unit Assignment:
After completing Section 4.3, students will use logarithms or orders of magnitude to compare sizes of objects. It is often difficult to compare objects that vary enormously in size. In this assignment students learn how logarithms can be used to define the concept of "order of magnitude" which provides a simple way of comparison of these objects.

Mathematical Practices Used in Ch. 4:
- MP.2: Students will be able to quantitatively understand the structure of a logarithmic and exponential function separately as well as the inverse relationship of the functions. They will also be able to understand how the key features of a graph are represented in the algebraic expressions.
- MP.4: Students will be able to model a data set by using a linear, quadratic, exponential or logarithmic expression. Students will be able to determine which function best fits the data by identifying key points as well as end behavior.

- MP.5: Students will be able to determine when it is appropriate to use a graphing utility to fit an exponential or logarithmic curve to model a data set.

Semester B

Chapter 5: Trigonometric Functions: Unit Circle Approach *(approximately 8 days)*

STANDARDS
N.Q.2, N.Q.3, F.IF.8, F.LE.2, F.TF.1, F.TF.2, F.TF.3, F.TF.8, G.SRT.6, G.SRT.7, S.ID.6a

A. In this chapter students will study one way of viewing trigonometric functions, as functions of real numbers. Students will see in this chapter how the trigonometric functions are used to model periodic motion.

Major Topics:
The Unit Circle
Trigonometric Functions of Real Numbers
Trigonometric Graphs
Inverse Trigonometric Functions and Their Graphs
Modeling Harmonic Motion

B. Unit Assignment:
After completing Section 5.3 students will consider an application to biology. In this assignment students will use sine functions to model the population of a predator and its prey. An isolated island is inhabited by two species of mammals: lynx and hares. The lynx are *predators* who feed on the hares, their *prey*. Students will find functions of the form y=asink(t-b)=c that model the lynx and hare populations from a given graph.

Mathematical Practices Used in Ch. 5:

- MP.2: Students will be able to reason abstractly by understanding how the even-odd properties of a trigonometric function is related to the direction of rotation of the angle measure and the quadrant in which the terminal side is drawn.
- MP.5: Students will be able to determine when it is appropriate to use a graphing utility to model periodic phenomena with a sinusoidal function. Students will be able to fit a trigonometric function by identifying the key features of the graph.
- MP.6: Students will be able to clearly define the meaning of the variables in a trigonometric equation by identifying which is an angle and what specific ratio is defined. Students will also be able to accurately label axes according to the expression being graphed, whether the unit circle or a trigonometric function.

Chapter 6: Trigonometric Functions: Right Triangle Approach *(approximately 9 days)*

STANDARDS
F.TF.3, F.TF.4, G.SRT.6, G.C.5

A. In this chapter students will study another way of viewing trigonometric functions, as functions of angles. Students will see in this chapter the relationships between angles and distances.

Major Topics:
Angle Measures
Trigonometry of Right Triangles
Trigonometric Functions of Angles
Inverse of Trigonometric Functions and Right Triangles
The Law of Sines
The Law of Cosines

B. Unit Assignment:
After completing Section 6.2 students will investigate how areas and volumes of similar figures change as the size of the figure increases (or decreases). In this assignment students will find power functions that relate these quantities and use those functions to explore the possibility of the existence of a real-life giant ape. Students will begin by finding some properties of similar figures.

Mathematical Practices Used in Ch. 6:

- MP.1: Students will be able to analyze the given information for an oblique triangle and will persevere in determining the best law (either law of cosines or sines) to solve the triangle. Students will also be able to determine when the ambiguous case for the law of sines applies and will solve for both possible triangles, when necessary.
- MP.3: Students will be able to determine when, for an ambiguous triangle, the second case does or does not work and can create a viable argument as to why.
- MP.4: Students will be able to model real world situations with right and oblique triangles and will be able to use the laws of cosine and sine and the properties of right triangles to determine distances and angles.

Chapter 7: Analytic Trigonometry *(approximately 8 days)*

STANDARDS
N.Q.1, N.Q.2, F.IF.7, F.IF.7e, F.TF.4, F.TF.5, F.TF.7, F.TF.8

A. In this chapter students will study algebraic properties of trigonometric functions. Students will simplify and factor expressions and solve equations that involve trigonometric functions.

Major Topics:
Trigonometric Identities
Addition and Subtraction Formulas
Double-Angle, Half-Angle and Product-Sum Formulas
Basic Trigonometric Equations

B. Unit Assignment:
After completing Section 7.3 students will use trigonometry to find the best location from which to view a painting or a movie. In this assignment, students will use graphing devices and the Law of Cosines to find the best viewing angle possible.

Mathematical Practices Used in Ch. 7:

- MP.1: Students will look for entry points to a trigonometric identity proof and will persevere in looking for relationships to equate the two expressions. Students will analyze the given expressions to determine a solution pathway instead of jumping into a solution attempt.
- MP.2: Students will be able to reason through a trigonometric equation by being able to reason abstractly and seeing a quadratic form that requires factorization. Students will also be able to reason quantitatively by identifying the relationships between the ratios of sides with the corresponding angle measures.
- MP.3: Students will be able to build a logical progression of statements to provide a proof for a trigonometric identity by utilizing the quotient identities, Pythagorean identities, as well as the sum, difference, double- and half-angle formulas.

Chapter 8: Polar Coordinates and Parametric Equations *(approximately 7 days)*

STANDARDS
F.IF.7, F.IF.7e, F.TF.4, F.TF.5, F.TF.8

A. In this chapter students will study a different way of locating points in the plane. Instead of using rectangular coordinates to specify a location, students will give distance and direction from a fixed reference point.

Major Topics:
Polar Coordinates
Graphs of Polar Equations
Polar Form of Complex Numbers: De Moivre's Theorem
Plane Curves and Parametric Equations

B. Unit Assignment:
After completing the chapter, students will use parametric equations and trigonometry to model the path of a projectile in motion. Students will be able to utilize coordinate geometry to realize how changing the angle of projection affects the domain and range of motion.

Mathematical Practices Used in Ch. 8:

- MP.1: Students will be able to visualize the graphing of a complex number as it relates to right triangle trigonometry and how this relationship allows for the conversion between rectangular and polar systems.
- MP.2: Students will be able to relate the roots of a complex number as points equally spaced about a circle and will be able to relate the periodicity of a sinusoidal function to help identify the polar form of the roots.
- MP.3: Students will be able to analyze the relationship between rectangular coordinates and the path of a point moving on a plane to eliminate a parameter and convert between rectangular and parametric equations.

Chapter 9: Vectors in Two and Three Dimensions *(approximately 9 days)*

STANDARDS
N.Q.1, N.Q.2, N.Q.3, F.TF.6, F.TF.7, G.SRT.8, G.SRT.9

A. In this chapter students will be able to represent quantities, such as force, with vectors that are defined by magnitude and direction. Students will also study how several forces acting on an object will affect the movement of that object.

Major Topics:
Vectors in Two Dimensions
The Dot Product
Three-Dimensional Coordinate Geometry
Vectors in Three Dimensions
The Cross Product
Equations of Lines and Planes

B. Unit Assignment:
After completing the chapter, students will use vector fields to model real-world scenarios, like the gravitational force on the earth or wind on the surface of the earth. Students will be able to then analyze the vector fields to identify the center and direction of motion.

Mathematical Practices Used in Ch. 9:

- MP.4: Students will be able to graph vectors in the cartesian system and use the dot product to find the angle between two vectors as well as the direction angles of a vector.

- MP.5: Students will use proper graphing utilities to help them visualize and analyze three-dimensional vectors and cross product vectors.
- MP.6: Students will attend to precision by properly drawing and labeling axes to graph in three dimensions.

Chapter 10: Systems of Equations and Inequalities *(approximately 15 days)*

STANDARDS
N.Q.1-3, N.CN.4, N.CN.5, N.VM.1-5, A.REI.5, N.REI.6, G.SRT.10, G.SRT.11

A. In this chapter, students will use more than two variables to create and solve a system of equations and inequalities. Students will be able to adapt the elimination and substitution methods of solving systems with two variables to now solve systems with three or more variables as well as learning new methods of solving, such as utilizing matrices and Cramer's Rule.

Major Topics:
Systems of Linear Equations in Two Variables
Systems of Linear Equations in Several Variables
Matrices and Systems of Linear Equations
The Algebra of Matrices
Inverses of Matrices and Matrix Equations
Determinants and Cramer's Rule
Partial Fractions
Systems of Nonlinear Equations
Systems of Inequalities

B. Unit Assignment:
After section 10.4, students will use matrix multiplication to project the population proportions of the young, juvenile and adult proportions of a population of animals. Students will use these predictions to analyze the health and growth of the population for the coming seasons in order to understand the long-term prospects of the population.

Mathematical Practices Used in Ch. 10:
- MP.2: Students will analyze a linear system of equations and inequalities and understand that there are multiple methods of solving a system – i.e. substitution, elimination, augmented matrices, Cramer's rule, etc.
- MP.4: Students will understand how to create a matrix to represent data and how matrix operations can be used to analyze the data.
- MP.5: Students will understand how a matrix can be used to represent equations and data and how to algebraically manipulate matrices to solve linear systems.

Chapter 11: Conic Sections *(approximately 9 days)*

STANDARDS
N.VM.7-12, A.REI.7-9, A.REI.12

A. In this chapter, students will understand how conic sections are different cross-sections of a cone. Students will be able to identify the equations of different conics as well as their geometric properties and their graphs.

Major Topics:
Parabolas
Ellipses
Hyperbolas
Shifted Conics
Rotation of Axes
Polar Equations of Conics

B. Unit Assignment:
At the end of the chapter, students will study the use of conics in architecture and how the geometric properties of each conic can create architectural features, such as a whispering gallery. Students will also learn practical methods by which conics could be constructed with by using practical construction tools.

Mathematical Practices Used in Ch. 11:

- MP.1: Students will be able to transform the general form of a conic into the standard form of the respective conic to determine key features.
- MP.2: Students will be able to identify the type of conic from the general form.
- MP.4: Students will be able to model real world situations with the appropriate conic type and use the key features to solve problems.

Chapter 12: Sequences and Series *(approximately 9 days)*

STANDARDS
G.GPE.2, G.GPE.3, G.GMD.4

A. In this chapter, students will study different types of sequences, such as geometric, arithmetic and the Fibonacci sequences and their applications in the real world. Students will be able to recognize, formulate and apply sequences to real world situations as well as being able to evaluate the sum of a series.

Major Topics:
Sequences and Summation Notation
Arithmetic Sequences

Geometric Sequences
Mathematics of Finance
Mathematical Induction
The Binomial Theorem

B. Unit Assignment:
At the end of this chapter, students will be able to apply their knowledge of recursive and explicit formulas of a sequence to analyze and evaluate practical problems, such as a monthly savings account as well as the accumulation of pollutants in the environment. They will be able to predict future values of account balances as well as the percentage of pollutants in the environment after *n* number of years.

Mathematical Practices Used in Ch. 12:

- MP.5: Students will be able to identify the strengths of using the binomial theorem vs. Pascal's triangle but still understand the complementary relationship between the two.
- MP.7: Students will discern a pattern in a sequence or series and identify whether it is arithmetic or geometric.
- MP8: Students will be able to formulate an explicit and/or recursive formula to represent various sequences and series.

Chapter 13: Limits - A Preview of Calculus *(approximately 8 days)*

STANDARDS
N.CN.8, N.CN.9, A-APR.2, A.APR.3

A. In this chapter, students will understand the definition of one-sided and the overall limit and its applications in calculus, such as finding the instantaneous rate of change as well as the area of a bounded region. Students will understand the laws and the definition of alimit of an average rate of change as the instantaneous speed.

Major Topics:
Finding Limits Numerically and Graphically
Finding Limits Algebraically
Tangent Lines and Derivatives
Limits at Infinity; Limits of Sequences
Areas

B. Unit Assignment:
By the end of the chapter, students will apply their knowledge of finding the area under a curve by finding the total distance of a car traveling for a set amount of time at a designated speed, in miles per hour.

Mathematical Practices Used in Ch. 13:

- MP.2: Students will be able to determine when the limit of a function exists both algebraically and graphically.
- MP.3: Students will be able to justify why the slope of a tangent line must equal 0 at any local maximum or minimum value and its significance.
- MP:6: Students will be able to clearly communicate the conditions for which the limit of a function does and does not exist.

Chapter 14: Probability and Statistics *(approximately 13 days)*

STANDARDS
A.SSE.4, A.APR.5, A.APR.6, A.APR.7, F.IF.3, F.BF.1a, F.BF.2, F.LE.2

A. In this chapter, students will study events that are governed by randomness and have a predictability to them. Students will understand that not all situations can be precisely determined or calculated, but can be analyzed and predicted using probability rules. Students will be able to collect, organize and analyze data using statistical methods, such as the 5-number summary.

Major Topics:
Counting
Probability
Binomial Probability
Expected Value
Descriptive Statistics (Numerical)
Descriptive Statistics (Graphical)
Introduction to Statistical Thinking
Introduction to Inferential Statistics

B. Unit Assignment:
After section 14.6, students will understand how statistics can be used to mislead and misinform. Students will then be able to analyze studies and newspaper articles that quote statistics and identify sources of potential bias.

Mathematical Practices Used in Ch. 14:

- MP.2: Students will be able to use quantitative reasoning to find the probabilities for events, including conditional probability.
- MP.5: Students will be able to use Venn diagrams appropriately to represent the probabilities for the union and intersection of two events as well as for conditional probability. Students will also be able to use a stem and leaf plot to represent data and to find a 5-number summary.

- MP.6 Students will accurately be able to calculate a 5-number summary in order to properly create a visual summary of the statistics, such as a normalized distribution or a box and whisker plot.

Comprehensive Final Exam Details

1. Students will be tested on their knowledge of the following topics:
    - 3-Variable Systems
    - Prerequisites from Algebra and Geometry
    - Trigonometric Functions
    - Analytic Trigonometry
    - Laws of Sines, Cosines
    - Polar and Vectors
    - Complex Numbers
    - Exponential and Logarithmic Functions
    - Topics in Analytic Geometry, including Conics
    - Functions and Models
    - Limits and Derivatives
    - Differentiation Rules
    - Applications of Differentiation
2. The Final Exams - much like unit/chapter exams - are detail-oriented and require students to provide detailed, step-by-step justification for their responses to each of the questions. This way, they get trained and prepared for their future AP Math courses/exams.