Glendale Unified School District

High School

April 16, 2019

Department:	Mathematics
Course Title:	Integrated Math IIIB/Precalculus Accelerated
Course Code:	3517D, 3518D
School(s) Course Offered:	Glendale High School
UC/CSU Approved (Y/N, Subject):	Y, "c" Mathematics
Course Credits:	Full Year (10)
Recommended Prerequisite:	Integrated II/IIIA Accelerated or Integrated II + (Summer) Integrated Math IIIA Accelerated
Recommended Textbook:	*Core Connections Integrated III* Judy Kysh, Evra Baldinger, Michael Kassarjian, Karen Wootton, et. al CPM Educational Program Second Edition, Version 5.0 *Precalculus* Josea Eggink, Samantha Falkner, Emily Kaffel, Mark Ray, Jeanne Villeneuve, Karen Wooton, Erin Yao CPM Educational Program Third Edition

Course Overview: Integrated Mathematics IIIB/Precalculus Accelerated is part two of a two-part compacted math series. Following Integrated Mathematics II/IIIA Accelerated, this course provides students with instruction in the second half of the content of the Integrated Math III and all of the course content for Precalculus. This compression is designed as the single point of acceleration at the high school level as recommended by the California Mathematics Framework. This course is aligned to the California Common Core State standards for high school mathematics and supports the Standards for Mathematical Practice. With this course, students will develop a deep conceptual understanding of the mathematical relationships and concepts needed to succeed in higher level math courses.

In addition to covering the second half of Integrated III standards, this course meets all of the standards for a Common Core 4th Year high school math course. Several big ideas are interwoven, including: functions (e.g., inverse, composite, piecewise), trigonometry, modeling, algebraic manipulation, rates of change, and area under a curve. Students engage with an introduction to several to calculus topics, including limits, area under a curve, and rates of change. On a daily basis, students work collaboratively with others as they use problem-solving strategies, complete investigations, gather evidence, critically analyze results, and communicate clear and effective arguments while justifying their thinking.

The course is well balanced among procedural fluency (algorithms and basic skills), deep conceptual understanding, strategic competence (problem solving), and adaptive reasoning (application and extension). The course embeds the CCSS Standards for Mathematical Practice as an integral part of each lesson. With the emergence of new technology, many lessons have moved beyond a traditional handheld device and are written with Desmos eTools as an integral component. The curriculum contains several key labs and hands-on activities to introduce and connect concepts, with an emphasis on modeling.

A focus on algebra is woven throughout the course. Students investigate equivalent expressions and practice setting up word problems right from the start. Students use algebra to manipulate inverse, composite, and piecewise-defined functions as well as investigate characteristics of functions and transformations of functions. Students continue rewriting expressions, solving complicated equations and systems, and use algebra to solve word problems. Algebraic manipulation is practiced throughout the rest of the course as students work with limits, rates of change, trigonometric expressions, complex numbers, series, conic sections, and area under the curve.

Careful consideration was given to the sequencing of the concepts in the course to allow for mastery over time while meeting the content standards of a 4th year course with focus on the 4th year math standards: algebra, functions, trigonometry, complex numbers, conic sections, probability, vectors, and matrices.

In addition to the second half of Integrated III standards, this courses covers the same material as Precalculus Honors, by adding and adds rates of change, limits and area under the curve to the standard Precalculus course.

Course Content:

Semester A

Unit 5: Inverses and Logarithms *(approximately 11 days)*

STANDARDS
F-BF.3, F-BF-4, F-BF.4a, F-LE.4, F-LE.4.2, I-IF.7e

A. Reversing is an important theme in the early part of this chapter. The first section introduces the concept of inverse relations. Students learn that reversing, or working backward to undo the action of a function, can create a new function. They explore multiple representations of functions and their inverses, and recognize that many functions have inverses that are not functions. In the second, students determine inverses of parent functions. They learn that the inverse of an exponential function is a logarithm. Reversing is emphasized once again as they learn how to convert exponential equations into logarithmic form, and vice versa. Students investigate the new family of logarithmic functions $f(x) = \log_b(x)$ for different values of b, test values on their calculators to determine the base the calculators work in, and learn to graph transformations of $f(x) = \log(x)$.

Progression of Content:
This chapter adds to students' lists of parent functions, which will continue to expand with the addition of polynomial functions in Unit 8 and trigonometric functions in Unit 9.

B. Unit Assignment(s):
Mathematics Practices Used in Unit 5:
- Students will look for and make use of structure and construct viable arguments as they develop and justify strategies for undoing functions and as they investigate different bases for logarithms..

- Students will use appropriate tools strategically and look for and make use of structure when they graph inverses of functions and write their equations.
- Students will look for and make use of structure as they verify inverses using multiple representations, and attend to precision as they restrict the domain of a function to ensure that its inverse is also a function.
- Students will construct viable arguments and critique the reasoning of others and look for and make use of structure as they apply their knowledge of parent graphs and inverses to learn about logarithms.
- Students will look for and make use of structure and express regularity in repeated reasoning as they learn the definition of logarithm and calculate the values of logarithms.
- Students will construct viable arguments and critique the reasoning of others, use appropriate tools strategically, and look for and make use of structure as they investigate logarithms with different bases.

Sample Activities:
Guess My Number Game - Students are asked to guess the number the teacher is thinking of based on the order of operations applied to the number and what the mystery number has ultimately transformed into. Students may or may not write an equation, but you may want to encourage them to do so as it will help when they progress to working with functions and inverses. Making sure that the idea of reversing, or undoing, comes up in the discussion of the "Guess My Number" game. You undo each step, reversing the original Order of Operations.

Graph the Inverse Function - Students will be looking at strategies for creating graphs of inverse functions. The first two graphs have functions that they are capable of finding the equation of their inverse functions but the third function does not lend itself to be solved for the inverse function. Teams could make a mini-table of some coordinates from the graph and then use it to help make a mini-table for the inverse graph. Students will soon discover that the line $y = x$ is the line of symmetry.

Unit 7: Logarithms and Triangles *(approximately 13 days)*

STANDARDS
F-LE.3, F-LE.4, F-LE.4.1, F-LE.4.3, A-SSE.2, F-BF.1, F-IF.7e, G-SRT.9+, G-SRT.10+, G-SRT.11+

A. In this unit, students return to their work with logarithms to develop tools they can use when solving application problems involving exponential equations. In the first half of this unit, students investigate the family $y = \log(m^n)$ and discover the Power Property of Logarithms, which allows them to solve exponential equations by using logs to undo or rewrite the equation. Students generalize from number patterns to make conjectures about other properties of logarithms and then prove these properties. Furthermore, students develop and share strategies to write the equation of the exponential function with a given asymptote that passes through two given points. Then they use that equation to make predictions.

The remainder of this unit focuses on completing a tool kit for calculating missing parts of non-right triangles. Students identify the types of information needed to determine all of the missing sides and angles of a triangle. Through this exercise, students also identify triangles for which they do not yet have the tools to determine missing parts. Students notice that they *do* have enough tools to calculate the measures and side lengths of right triangles. This leads to the question, *"What if the triangle is not a right triangle?"* Students then develop the Law of Sines and Law of Cosines so that they have a complete set of tools to determine the other missing parts of any triangle (when sufficient information is provided).

The unit concludes with students looking at different application problems using triangles and identifying which tools are most useful in each situation. In addition, students investigates the ambiguous case of triangles: SSA. This lesson is offered for accelerated classes or those that could benefit from a complete view of the relationships between the sides and angles of a triangle. Working through the problems of this lesson before you decide to use the lesson with your students is highly recommended.

Progression of Content:
The work with logarithms in this unit prepares students for future work in a pre- calculus course. Working with Law of Sines and Cosines and reviewing the use of right triangle trigonometry and special right triangles prepares students for working with the trigonometric family of functions in Unit 9.

B. Unit Assignment(s):

Mathematics Practices Used in Unit 7:

- Students will look for and make use of structure and express regularity in repeated reasoning while they develop the Power Property of Logarithms, learn other properties of logs and how to rewrite equations with different bases.
- Students will make sense of problems and reason abstractly and quantitatively as they write the equation of an exponential function given two points and an asymptote.
- Students will make sense of problems and persevere in solving them, model with mathematics, and reason abstractly and quantitatively as they explore exponential functions with an asymptote other than y = 0 and apply logarithms to solve an exponential equation.
- Students will make sense of problems and persevere in solving them as they figure out what information they need to solve for parts of triangles. They will need to attend to precision as they communicate what they know and do not know.
- Students will look for and express regularity in repeated reasoning as they develop the ratios for the Law of Sines.

- Students will make sense of problems and persevere in solving them using the Law of Cosines.
- Students will attend to precision as they work with triangles involving the SSA relationship. They must also use appropriate tools strategically as they explore the ambiguous case of the Law of Sines.
- Students will reason abstractly and quantitatively as they make sense of problems and persevere in solving them. They will attend to precision as they solve the problems and communicate within their teams, labeling diagrams, attending to units, and calculating their answers accurately.

Sample Activities:
"The Case of the Cooling Corpse" - a problem in which students model the falling temperature of a corpse to establish time of death and solve a murder mystery. In order to solve the mystery, students will need to decide which information is relevant to solving the problem (e.g., body temperature, times listed on sign-in sheets, etc.) . Students will need to write and solve a system of exponential functions, with a horizontal asymptote representing the room temperature, the independent variable as time, and the dependent variable as the temperature of the body.

Solving Triangles - This activity consists of eight independent triangle problems (suggestion is to assign one problem to a group of 3-4 students) that will provide students with a chance to consolidate their understanding of the various tools and strategies they have developed so far to solve triangle problems. Some problems may be solved using the Law of Sines, Law of Cosines, or a combination of both. Students will present their assigned problem to the entire class and their process for solving the assigned problem.

Unit 8: Polynomials *(approximately 15 days)*

STANDARDS
A-APR.1, A-APR.2, A-APR.4, A-APR.6, A-APR.3, F-IF.4, F-IF.7c, N-CN.8+, N-CN.9+, A-SSE.2, A-CED.2, F-BF.1

A. In the first section, students will investigate the equation ↔ graph connections for polynomial functions. They will recognize that equations in factored form are much easier to sketch, and they will understand the relationship between the factors and the x-intercepts of the graph. Then, in the second section, they will develop an understanding of imaginary and complex numbers and recognize that polynomial functions can have complex roots. In the third and last section, they will learn to divide polynomials by a known factor to find other factors. This will allow them to determine complex and irrational roots of some cubic and quartic functions.

Progression of Content:
Students will build on their understanding of function families in Unit 9, where they study trigonometric functions and transform the graphs of sine and cosine functions. Students will use

their algebra skills when they study rational expressions in Unit 11. Students will also use their algebra skills when they prove formulas for sums of series in Unit 10 and solve trigonometric identities in Unit 12.

B. Unit Assignment(s):

Mathematics Practices Used in Unit 8:

- Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others as they discuss factored form and describe the graphs of polynomials and their understanding of the stretch. Students will also look for and make use of structure as they draw graphs of polynomials.
- Students will look for and make use of structure as they continue their polynomial investigation. Students will also construct viable arguments and critique the reasoning of others as they consolidate their results on polynomials.
- Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others as they develop their understanding of a stretch factor. Student will also model with mathematics as they develop an equation for the roller coaster problem first introduced in the first lesson of Unit 8.
- Students will look for and make use of structure as they use polynomial division to determine factors of polynomials. Students will also construct viable arguments and critique the reasoning of others as they develop their understanding of polynomial division.
- Students will look for and make use of structure as they use complex roots to write equations of quadratic functions and express regularity in repeated reasoning as they identify polynomial identities to help them factor. Students will also look for and express regularity in repeated reasoning as they identify patterns in the sums and products of complex roots.
- Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others when they determine all roots of a polynomial with a degree greater than two and when they determine all roots of a polynomial with a degree greater than two.

Sample Activities:

Polynomial Function Investigation - Students are instructed to look for, label, and describe the x-intercepts and "bounces" (double roots, although students will probably not use this term), the y-intercept, the number of turns, and the behavior of graphs for very large and very small x-values. They should share and discuss all of their graphs and their observations within their teams. The discussion should lead to conjectures and the creation of several new equations to try. Students are coming up with methods for determining the x- and y-intercepts, predictions about the numbers of crossings of the x-axis, and ideas for determining a reasonable window or maximum and minimum approximations.

Game of Polydoku - By treating division as a puzzle and using the organizational device of an area model, students can use logical reasoning to reverse the multiplication process and figure out a missing factor. Once they have worked through the process several times, they should be able to set up and solve their own division problems. This method is as efficient as (if not more efficient than) polynomial "long division" and you can use it to develop synthetic division if that is part of your curriculum. This game introduces polynomial division by challenging students to reverse the process of polynomial multiplication.

Unit 11: Rational Expressions and Three-Variable Systems *(approximately 6 days)*

STANDARDS
A-APR.7+

A. In this chapter, students will focus on operations with rational expressions. Students did a function investigation in Unit 1 that previewed the investigation of rational expressions. Students learn a powerful method of simplifying rational expressions that uses properties of the number 1 and the properties of exponents. In a previous course, students learned to solve a system of two linear equations in two variables, and they reviewed this topic in Chapter 3 of this text.

Progression of Content:
The remaining chapter requires students to use their algebra skills to solve trigonometric identities. In future courses, students may use matrices to solve systems of equations.

B. Unit Assignment(s):
Mathematics Practices used in Unit 11:

- Students will look for and make use of structure as well as look for and express regularity in repeated reasoning as they use the number 1 to understand rational expressions.
- Students will also look for and make use of structure as well as look for and express regularity in repeated reasoning as they connect multiplication and division of fractions to that of rational expressions.
- Students will look for and make use of structure as they make connections between adding and subtracting fractions and adding and subtracting rational expressions, and as they locate points in three-dimensional space. They will construct viable arguments and critique the reasoning of others as they justify their strategies.
- Students will continue to look for and make use of structure as well as look for and express regularity in repeated reasoning as they work with rational expressions. They will construct viable arguments and critique the reasoning of others as they investigate the closure of rational expressions under operations.
- Students will reason abstractly and quantitatively and look for and express regularity in repeated reasoning as they write the equation of a quadratic function by solving a system of three equations with three unknowns.

Sample Activity:
Giant One - Students are encouraged to rewrite rational expressions to create fractions that will reduce to one. The new expressions will be reduced to its simplest form. This strategy to rewrite will allow students to simplify rational expression after doing basic operations on the original expressions.

Unit 1: Preparing for Your Journey *(approximately 15 days)*

STANDARDS
F.if.4, F.IF.5, F.BF.1, A.CED.1, A.CED.2, F.IF.5, A.SSE.2, A.APR.6, F.BF.1c, F.BF.4, F.IF.7b

A. This unit is designed to accomplish several objectives:
- Introduce students to some of the main concepts that are part of this course.
- Introduce students to the modeling cycle.
- Have students work with functions, including inverse, composite, and piecewise-defined functions.
- Introduce students to radians and the unit circle.

The opening of this unit establishes a starting point for many of the concepts in this course. Interpreting functions and their graphs will be used extensively in the modeling process throughout the course. Solving word problems as well as algebraic manipulation are skills that are necessary throughout the course.

Progression of Content:
Students are introduced to radians and the unit circle early in the course to facilitate work with trigonometric functions in later units. Students review the inverse functions from a previous course. Understanding domain restrictions will be necessary when students learn about the graphs of the inverse trigonometric functions contained in Unit 2 and Unit 8. After reviewing inverse functions, students will investigate composite functions and use them to algebraically determine if two given functions are inverses of each other. Lastly, students define a radian and use radians to measure angles in the unit circle. This is the start of using the unit circle, which trigonometry will be applied to throughout the course. The last lesson in this section examines angular motion vs. linear motion.

B. Unit Assignment(s):
Mathematical Practices Used in Unit 1:
- Students will model with mathematics, attend to precision, look for and make use of structure as they collect data from the Spring Problem
- Students will make sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others, look for and make use of structure as they work to find the inverse of functions graphically, on a table, and a rule

Sample Activities:

The Spring Problem - Students engage with a notice and wonder through this activity as they conduct a lab using a spring with a weight attached at the end. In order to gather data from this activity, students video record the motion of the spring and weight in order to make a table and graph the results. Finally, students use the their model to make various predictions of the position of the wight at different times.

The Inverse of Function - Students will review how to write an inverse function by "undoing" and learn how to algebraically verify that functions are inverses. Students will see the inverse of a function graphically, on a table, and through its rule.

Unit 2: Functions and Trigonometry *(approximately 14 days)*

STANDARDS
F.IF.4, F.IF.7, F.BF.3, F.TF.3, F.TF.4, F.IF.7e, F.TF.2, A.REI.10, F.TF.6

A. This unit is designed to accomplish several objectives:

- Describe graphs as increasing/decreasing, concave up/concave down, and state the location(s) of maxima and minima.
- Identify graphs of functions as even, odd, or neither. Or, given the equation of a function, algebraically show that the function is even, odd, or neither.
- Develop fluency with angles and coordinates in the unit circle.
- Generate the parent graphs for sine and cosine and use them in a variety of transformations.
- Generate the parent graphs for inverse sine, inverse cosine, tangent, and inverse tangent.
- Solve basic trigonometric equations over specified domains.

In the first section of this unit, students will review attributes used to describe functions. Concavity will be introduced and students will use algebra to show that functions are even, odd, or neither. Then transformations of functions will be investigated. Again, much of this will be review from a previous course, but will now include horizontal stretches.

In Unit 1, students were introduced to radians and the unit circle. They have practiced a number of problems to reinforce key angle measurements and have worked with special triangles to develop ratios for 45º-45º-90º and 30º-60º-90º triangles. Now they will use these ideas to generate the key coordinates in the unit circle and develop the graphs of sine and cosine.
The second section is devoted to the development of sine and cosine from the standpoint of the unit circle. Students first locate the coordinates for the special angles they used in Unit 1. They then see that the sine ratio corresponds to the y-coordinate and the cosine ratio corresponds to the x-coordinate. Using these values, students develop the Pythagorean Identity. From here, students will graph $y = \sin(q)$ and $y = \cos(q)$ using the unit circle. The section continues with transformations of the graphs of sine and cosine.

In the last section, students begin by solving trigonometric equations. Here students will see that these equations can have multiple, and even an infinite number of, solutions. They will use graphs and the unit circle to generate the solutions to these equations. Students will then use $y = \sin^{-1}(x)$ and $y = \cos^{-1}(x)$ to solve for angles which are not the special angles in the unit circle. Finally, students generate the graphs of inverse sine, inverse cosine, tangent, and inverse tangent.

Progression of Content:
In this unit, students work with transformations of sine and cosine and applied these transformations to a few applications. Then in Unit 8, students extend their work with trigonometric functions to more complex situations; model situations with sinusoidal functions that have both a horizontal shift and a period other than 2π; graph and apply the reciprocal trigonometric functions; develop other trigonometric tools for simplifying expressions using formulas involving sums of angles.

B. Unit Assignment(s):

Mathematical Practices used in Unit 2:

- Look for and make use of structure, reason abstractly and quantitatively.
- Use appropriate tools strategically, look for and make use of structure, look for and express regularity in repeated reasoning.
- Construct viable arguments and critique the reasoning of others, look for and make use of structure, look for and express regularity in repeated reasoning.
- Use appropriate tools strategically, attend to precision.
- Make sense of problems and persevere in solving them, use appropriate tools strategically, attend to precision, look for and make use of structure.

Sample Activities:
The Unit Circle - Students determine the coordinates of the intersection of the terminal side of a special angle with the unit circle. Students will cut out two special right triangles from a resource page. Sides of the special triangles will be labeled using the exact values on both sides of the paper. This will ensure that the labels are shown when the triangles are flipped over. By placing each of the special right triangles on the x-axis, students will be able to find the coordinates of each of the special angles and repeat the process in all four quadrants to complete the unit circle.

Solving Trigonometric Equations - To solve an equation such as sin x =12, students are directed to draw a unit circle and also graph y = 12, which is a horizontal line. The intersections of the unit circle and the horizontal line will show students the solutions as well as number of solutions. By treating this problem as a system of equations, students are made aware of the connections the solutions are graphically and algebraically. In the same problem, students are asked how many more solutions there could be if we were allowed to continue to revolve around the circle. Eventually the next time students come across this type of problem, they will know to convert the unit circle into a sinusoidal graph and will see the horizontal line having multiple intersections with the sinusoidal graph to help them find all solutions.

Semester B

Unit 3: Algebra and Area Under the Curve *(approximately 10 days)*

STANDARDS
A.APR.6, A.APR.7, A.SSE.2, A.REI.7, A.REI.11, F.BF.1, A.CED.1

A. In Unit 3, students begin by practicing and strengthening their algebraic manipulation skills as they continue to write equivalent rational expressions and complex fractions. Students learn how substitution can help solve equations and systems of equations. Students then finish up this unit by solving a series of word problems using their algebra skills.

Another focus of this unit is approximating the area under a curve using summation notation. This concept is one of the major themes of calculus. The goal is to understand what area under a curve can represent and how to approximate it using rectangles.

Progression of Content:
Area under a curve is one of the main ideas in calculus. In this unit, students will be taught background information that will provide a deeper understanding of integration in calculus. The goal in this section is for students to understand the meaning of area under a curve.

B. Unit Assignment(s):
Mathematical Practices used in Unit 3:
- Students will make sense of problems and persevere in solving them, look for and make use of structure, construct viable arguments and critique the reasoning of others as they add, subtract, multiply, and divide rational expressions.
- Students will look for and make use of structure, look for and express regularity in repeated reasoning, reason abstractly and quantitatively as they calculate sums by expanding sigma notation as well as write finite arithmetic series in sigma notation.

Sample Activities
Growth Mindset (Jo Boaler) - In the first lesson of this unit, teachers are encouraged and given the tools to discuss the concepts of the importance of everyone, especially students, learning from their mistakes through error analysis and dendrite growth.

Unit 4: Polynomial and Rational Functions *(approximately 10 days)*

STANDARDS
F.IF.7c, A.APR.3, F.IF.4, N.CN.8, N.CN.9, A.APR.6, F.IF.7d, A.CED.1, A.CED.2

A. In this unit, students will apply their knowledge of families of functions to include polynomial and rational functions. Students will investigate the equation ↔ graph connection for these two families of functions, and learn to describe the end behavior of a function. In the first section, students will learn to graph polynomial functions from the factored forms or their equations.

Students will then work backwards, using graphs to write equations in factored form. Finally, students will learn how to identify all of the roots of a polynomial. Second section begins with a new method for rewriting equations of rational functions is introduced in this section. Students will practice graphing transformations of rational functions. Then students will investigate rational functions that have slant asymptotes and holes.

Finally, students will extend your knowledge of rational functions to graph reciprocal functions. In the last section of the chapter, students will apply what students have learned to solve polynomials and rational equations and inequalities. Finally, students will apply their knowledge of polynomial and rational functions to model and analyze everyday situations.

Progression of Content:
In calculus, polynomials are used to approximate complicated functions because they "behave nicely" and are easy to work with.

B. Unit Assignment(s):
Mathematical Practices used in Unit 4:

- Students will look for and make use of structure, construct viable arguments and critique the reasoning of others as they graph polynomial functions from equations given in factored form.
- Students will look for and make use of structure as they rewrite rational expressions to transform functions in the form $g(x) = \frac{ax+b}{x-c}$ into transformations of $y = \frac{1}{x-h} + k$.
- Students will look for and make use of structure, construct viable arguments and critique the reasoning of others as they solve polynomial and rational inequalities.

Sample Activities
Polynomial Function Investigation - Students will investigate the graphs of polynomial functions in factored form. Use different numbers of factors with different values of the parameters will show how it will change the graph. Sample of an equation in which students need to change the parameters would is as follows: n(x)=x-a(x-b)(x-c) in which a, b, and c will be different values. Students will have a chance to compare their graphs with their teammates to see the behavior of different types of equations without having to do all of them themselves.

Unit 5: Exponentials and Logarithms *(approximately 10 days)*

STANDARDS
A.CED.2, F.IF.7e, F.LE.2, F.BF.5, F.LE.4

A. The start of this unit focuses on exponential functions. Students will apply what they already know about exponential functions to everyday situations. Students will then realize that sometimes two different transformations give the same result, mainly that every exponential function with a horizontal shift is equivalent to a vertical stretch. This shows graphically, as well as algebraically, that every function of the form $y = a(b^{x-h}) + k$ is equivalent to $y = A(b^x) + k$. Furthermore, students will learn about the number e.

In the second half of this unit, students will review, use, and prove the properties of logarithms, each of which corresponds to a property of exponents. Students will also continue solving equations and simplifying expressions that involve logarithms and exponents. This unit is culminated through graphing the family of logarithmic functions and solving application problems.

Progression of Content:
This unit on exponential and logarithmic functions culminates students' engagement in this 4th year course.

B. Unit Assignment(s):
Mathematical Practices used in Unit 5:

- Students will make sense of problems and persevere in solving them, reason abstractly and quantitatively, model with mathematics, look for and make use of structure as they use exponential functions to model everyday situations
- Students will look for and make use of structure as they understand that for exponential functions, a horizontal shift can be equivalently written as a vertical stretch.
- Students will look for and make use of structure, use appropriate tools strategically, look for and express regularity in repeated reasoning as they practice converting between exponential and logarithmic equations, and investigate the basic properties of logarithms, including natural logarithms.

Sample Activities
The number *e* - In this activity, students explore what occurs when1 cm2of bacteria reproduces at 100% rate every hour, every minute, every second, and finally every millisecond for one day. A follow up question to this activity is, "What happens to the bacteria if it reproduces *continuously*?" Students will recognize that the base of their exponential model is getting closer and closer to the number *e* and the continuous compound formula A(t)=Pert.

Proving the Logarithmic Properties Card Sort:
In this activity, students prove the Product Property of Logarithms using a card sort. Each team obtains one set of cards. Students arrange the cards in the center of their team's workspace so that everyone can participate. Students then arrange the cards in a logical manner and provide justification for each part of their proof. Justifications can be written on sticky notes or blank cards so that they are easily moved as well. The teacher verifies that each team has the correct proof before allowing them to move on. Students prove the Quotient and Power Properties in a similar format.

Unit 6: Triangles and Vectors *(approximately 9 days)*

STANDARDS
G.SRT.9, G.SRT.10, G.SRT.11, N.VM.1, N.VM.2, N.VM.4, N.VM.5, N.VM.3

A. Two useful tools for solving situational problems are triangles and vectors. Many everyday situations involve triangles, so students will need to be able to solve any triangle with minimal given information. In first section, students will develop and use the Law of Sines and the Law of Cosines to solve non-right triangles. Students will learn to solve triangles when the given information does not create one unique triangle. In the second section, students will learn how to use vectors to describe motion; complete vector operations both graphically and algebraically; apply your knowledge of vectors to solve everyday problems.

Progression of Content:
Students are reintroduced to Law of Sines and Law of Cosines and their applications. But this will most likely be the last time students see a formal lesson on these topics. As for the lessons on vector, since this is the first introduction to the topic, students who pursue higher math and science course will encounter them again.

B. Unit Assignment(s):
Mathematical Practices used in Unit 6:
- Students will look for and make use of structure as they prove, understand, and apply the Law of Sines and Law of Cosines. They will also derive the formula $A = 1/2ab\sin(C)$ for the area of a triangle.
- Students will look for and make use of structure, attend to precision, use appropriate tools strategically as they are introduced to vectors and vector notation. They will determine magnitude, direction, and/or components of a vector.
- Students will make sense of problems and persevere in solving them, attend to precision, reason abstractly and quantitatively, model with mathematics, use appropriate tools strategically as they use vectors in real world applications.

Sample Activity:
Vector Line Dance - The Vector Line Dance has students perform a series of moves in the form of a line dance. The idea is that all of the students are doing the same movements but from different starting positions. The starting position does not matter when working with vectors. This activity is "self correcting" in that students should notice if they make a mistake in their direction or distance, since students should move in unison (similar to line dancing). Once students have completed the activity, the class can discuss the idea of everyone making the same motion (steps and direction), but from different positions. This develops the notion that a vector has a direction and length (magnitude).

Unit 7: Limits and Rates *(approximately 12 days)*

STANDARDS
Preparation for Calculus (Addition for Precalculus Honors)

A. In mathematics, the concept of a limit can be used to describe the behavior of a function as the independent variable approaches a particular value, or as it becomes arbitrarily large. In the first half of this unit, students will explore how functions behave as x approaches a particular value or goes to infinity. Students will also look at limits from several perspectives including geometry, graphs, tables, and algebra. Students will then learn about one-sided limits and evaluate limits of many functions including rational and piecewise-defined functions. Students will use limits to define continuity.

In the second half of this unit, students will investigate rates of change as they occur in everyday situations and through multiple representations. Often, the most interesting thing about the values in a situation is not the values themselves, but how those values are changing. Is your car speeding up or slowing down? Is the room getting hotter or colder? Furthermore, students will look at the slopes of secant and tangent lines. At the end of this unit, students will use what they have learned about limits to define instantaneous rate of change.

Progression of Content:
Students develop intuitive notions of limits during this unit and refine these ideas in Unit 13. The main focus in this unit is for students to begin to develop the definition of a derivative. The final goal, in Unit 13, is for students to understand what a derivative actually is and what it tells them about a function and its graph.

B. Unit Assignment(s):
Mathematical Practices used in Unit 7:

- Students will use appropriate tools strategically as they evaluate limits geometrically, graphically, on a table, and understand the necessary conditions for a limit to exist.

- Students will make sense of problems and persevere in solving them as they work with limits of functions such asf(x)=sin(x)x.
- Students will look fo and make use of structure as they calculate average rates of change by calculating the slope of the secant line between two data points.
- Students will look for and make use of structure, look for and express regularity in repeated reasoning as they use the limit as $h \to 0$ for the average rate of change to calculate the instantaneous rate of change.

Sample Activity:
Folding Angles - This activity is intended to provide students a tangible example of limits during an engaging activity. In this activity, every student in a group chooses a different acute angle, marks it on their receipt tape, then bisects the supplementary obtuse angle adjacent to their initial angle. By repeating the process, students discover that repeated folds in a given pattern lead to angles ever closer to 60º.

Unit 8: Extending Periodic Functions *(approximately 12 days)*

STANDARDS
F.BF.3, F.TF.7, A.CED.2, F.TF.9, F.TF.10

A. In Unit 2, students worked with transformations of sine and cosine and applied these transformations to a few applications. This unit extends their work with trigonometric functions to more complex situations. Students will model situations with sinusoidal functions that have both a horizontal shift and a period other than 2π. Students will graph and apply the reciprocal trigonometric functions. You will also develop other trigonometric tools for simplifying expressions using formulas involving sums of angles.

Progression of Content:
In Calculus, students will be using trigonometric functions to find derivatives and find integrals.

B. Unit Assignment(s):
Mathematical Practices used in Unit 8:
- Students will look for and make use of structure as they combine a horizontal stretch and shift of the same trigonometric function and set up a modeling problem.
- Students will model with mathematics as they generate trigonometric models for real-world applications.
- Students will look for and make use of structure, look for and express regularity in repeated reasoning, use appropriate tools strategically as they graph $y = \csc(x)$, $y = \sec(x)$, and $y = \cot(x)$; prove trigonometric identities.
- Students will use appropriate tools strategically as they use geometry to visualize the trigonometric functions.

Sample Activity:
Graphing Reciprocal Trigonometric Functions - Students will use the graphs of sine, cosine, and tangent to graph their respective reciprocal functions cosecant, secant, and cotangent. Students are encouraged to use the symmetry of the graph to save themselves time. For example, once they have values for sine and cosecant between $x = 0$ and $x = \pi$, they can use the same values (but negative) between $x = \pi$ and $x = 2\pi$. Students will see that the x-intercepts of $y = \sin(x)$ become the locations of the asymptotes for its reciprocal function. These asymptotes should help students identify the domain and asymptotes for the reciprocal functions.

Unit 9: Matrices *(approximately 9 days)*

STANDARDS
N.VM.6, N.VM.7, N.VM.8, N.VM.9, N.VM.10, N.VM 11, N.VM.12, A.REI.8, A.REI.9

A. In this unit, students learn what a matrix is and how matrices, along with a graphing calculator, can be useful tools for organizing data and solving problems. Students then use matrices to solve complicated systems of equations.

By the second half of the unit, students understand the definition of a linear transformations and relate linear transformations to matrices. Students then investigate compositions of transformations and see how transformations affect geometric figures.

Progression of Content:
This unit is an introduction to matrices, the topic of a Linear Algebra course (usually taken after Calculus) in college. Matrices are also included in college Business Math courses.

B. Unit Assignment(s):
Mathematical Practices used in Unit 9:

- Students will look for and make use of structure, make sense of problems and persevere in solving them as they add, subtract, and start to multiply matrices.
- Students will look for and make use of structure, look for and express regularity in repeated reasoning, make sense of problems and persevere in solving them, use appropriate tools strategically as they multiply a vector (regarded as a matrix with one row/column) by a matrix of suitable dimensions to produce another vector, and use matrix multiplication to solve problems.
- Students will reason abstractly and quantitatively, look for and make use of structure as they perform linear transformations using matrices.

Sample Activity:
The Toy Factory - Students learn to use matrices to represent linear situations that involve several variables needed to make two types of toys, cars and trucks requiring different amount of wheels, seats, and different costs to manufacture each type of toy. The goal of the activity is for students to determine whether or not a particular request from a buyer can be fulfilled.

Unit 10: Conics and Parametric Functions *(approximately 11 days)*

STANDARDS
G.GPE.3.1, F.IF.10

A. In this unit, students will analyze shapes that result from slicing a cone with a plane. These shapes are called conic section. In the first section, students will look at circles, ellipses, hyperbolas, and parabolas. They will generate the conics and derive their equations using the formal definitions. Students will recognize conic sections from their equations and complete the square to rewrite the equations in graphing form. Second section focuses on parametrically-defined functions. Students are introduced to the concept of defining x and y in terms of the parameter t. They will see how parametrically-defined functions can be used to model situations involving motion. The final lesson in this section has students solve problems using projectile motion. This is much more powerful than the past work students have completed with parabolas.

Progression of Content:
In calculus, a chapter is devoted to developing calculus tools for other forms of equations: polar, parametric, and vector.

B. Unit Assignment(s):
Mathematical Practices used in Unit 10:
- Students will look for and make use of structure, reason abstractly and quantitatively, use appropriate tools strategically, attend to precision as they derive the equation of a circle and practice completing the square, derive the equation of a hyperbola and ellipse, rewrite parametric equations representing conic sections in rectangular form, apply their knowledge of parametric equations to everyday situations.

Sample Activities:
Where is the Center - Students are reintroduced to algebra tiles which they have used in previous course to rewrite quadratic equation into perfect square forms to solve them. Now they will use algebra tiles to complete the square on both x and y variables which will help in rewrite conic section equations into graphing form. The use of the manipulatives allows students, especially those unfamiliar with completing the square or that lack the experience with algebra tiles to get a full understanding as to why it is called completing the square. Once equations of a circle in this problem are rewritten in this form, students can easily identify the center of the circle.

Flick a Coin - This demonstrates how vertical motion is not affected by horizontal motion. To do this, one coin is flicked while the other coin is dropped. The teacher will need to do a demonstration for the whole class. Fold the 3" × 5" index card in half lengthwise, then fold each half (again lengthwise) in the opposite direction. Squeeze the two middle parts together with your thumb and forefinger creating a T when viewed at the end. Turn the folded index card so that the T is upside down. Lay the upside-down T on a table and place a coin on either side of the vertical center. The coins should be laid flat, resting against the center of the T, and near the end. Pinch and hold the side opposite of the coins. Slide the card so that the coins and half of the card extend beyond the edge of a flat surface. Now, imagine that this is a pinball machine flipper and quickly flick your wrist. One coin will be projected forward while the other falls straight down. Listen carefully for the sounds as they hit the floor. One sound means they hit at approximately the same time. Theoretically, the two coins should hit the ground at the same time, even though the first coin travels a longer path. This is true because the horizontal and vertical motions are independent.

Unit 11: Polar Functions and Complex Numbers *(approximately 11 days)*

STANDARDS
F.IF.11, N.CN.3, N.CN.4, N.CN.5, N.CN.6

A. In this unit, students transition from graphing rectangular coordinates (x, y), to polar coordinates, which use a distance and an angle. Students will then apply their work with polar coordinates to the world of complex numbers. They will graph complex numbers and learn to rewrite them in polar form. Furthermore, students will also perform operations with complex numbers in polar form, including multiplying, dividing, and computing powers and roots. This unit culminates with students learning to plot points and graph equations using a radius and an angle, and make conversions between polar and rectangular equations

Progression of Content:
It is not until the second year of Calculus that students revisit polar functions through integration and differentiation.

B. Unit Assignment(s):
Mathematical Practices used in Unit 11:
- Students will look for and make use of structure, look for and express regularity in repeated reasoning as they graph polar functions, explore various polar functions, graph complex numbers.
- Students will look for and make use of structure as they convert between polar and rectangular forms.
- Students will look for and make use of structure, use appropriate tools strategically as they represent operations with complex numbers geometrically, and use conjugates to determine moduli and quotients of complex numbers.

Sample Activity:
Polar Coordinates Battleship - Directions/rules for the students are as follows:
- Mark your ships on your polar grid. Each ship must be connected to another ship, either along a ray or around a circle.

- Take turns firing out shots. Each target should be stated in four ways: positive radius and positive angle (+, +), positive radius and negative angle (+, −), negative radius and positive angle (−, +), and negative radius and negative angle (−, −). Both players should record the shots on the charts for further reference.
- If you are the opponent, verify that all of the targets stated are the same location. If not, the player forfeits their turn.
- If the shots miss a ship, the opponent declares "miss" and both players place an open circle in the appropriate location. If the shots hit a ship, the opponent declares "hit" and both players mark an X in the appropriate location.
- You must state when a ship is sunk completely.
- The goal is to sink all of your opponent's ships.

Unit 12: Series and Statistics *(approximately 12 days)*

STANDARDS
A.SSE.2, A.SSE.4, A.APR.5, S.MD.1, S.MD.2, S.MD.3, S.MD.4, S.MD.5

A. In this unit students will begin by calculating the sums of series. Students will derive formulas for and evaluate the sums of arithmetic and geometric series. Students will use limits to evaluate sums of infinite geometric series. At the end of this section students' skills will be applied to analyze some common situations. Second section begins by looking for patterns in the expansion of $(x + y)^n$ and relates the patterns to Pascal's triangle. This leads the use of the Binomial Theorem for the expansion of binomials of the form $(a + b)^n$, where n is a positive integer. Students then apply the patterns from the first lesson of this section to compute probabilities for binomial experiments. In the last section, students will understand what a discrete random variable is as well as the difference between frequency and relative frequency. They will create relative frequency tables for discrete random variables and graph probability distributions as relative frequency histograms. The section continues with calculating the mean and expected value of a discrete random variable. Once students understand these values and how to calculate them, they use these values to make decisions for everyday situations.

Progression of Content:
In Calculus, students will learn tests for determining convergence or divergence of infinite series. And students will learn basic tools if they do decide to move onto a statistical course.

B. Unit Assignment(s):
Mathematical Practices used in Unit 12:
- Students will look for and make use of structure, look for and express regularity in repeated reasoning, reason abstractly and quantitatively, use appropriate tools strategically as they develop a formula for the sum of an arithmetic series.

- Students will use appropriate tools strategically, look for and make use of structure, reason abstractly and quantitatively as they use limits to determine the sum for infinite geometric series.
- Students will make sense of problems and persevere in solving them, use appropriate tools strategically as they apply their knowledge of sums of geometric series to solve problems involving everyday situations and develop the concept of a random variable for discrete random variables. They will graph probability distributions associated with random variables. They will calculate and interpret the mean and expected value of a discrete random variable.

Sample Activity:
Quebare - This problem is launched by showing short video for the "game" they are about to play. For the purpose of this problem, Quebare's moves have been restricted to down left and down right. Students are instructed to discuss strategies for determining the exact number of paths to each block. Using isometric dot paper, plastic sleeves, and dry erase pens. Students can then use the resources to test their strategies. Students may or may not notice at this point that they have seen this pattern before. If they recognize it as Pascal's triangle, allow them to share this with the class. Otherwise, let them continue to work the patterns; the paragraph at the end of the lesson addresses the name and significance of the pattern.

Unit 13: Precalculus Finale *(approximately 13 days)*

STANDARDS
Preparation for Calculus (Addition for Precalculus Honors)

A. Students learned about limits in Unit 7 by using a graphical approach. In this unit, students will use dominant terms to evaluate limits at infinity and algebraic techniques to evaluate limits at a point. Students will also learn to recognize when a technique is appropriate to use.

   Furthermore, in this unit, students will understand what area under a curve represents and how to approximate it using trapezoids, and by writing and using an area under a curve program with a graphing calculator.

   This unit is culminated as students build conceptual understanding between the slope of a function and the area under the curve of a function's derivative, two concepts developed throughout this course. You will also learn how to write slope functions for power functions.

Progression of Content:
By the end of this unit, and essentially this course, students will be ready for Calculus.

B. Unit Assignment(s):

Mathematical Practices used in Unit 13:

- Students will look for and express regularity in repeated reasoning, reason abstractly and quantitatively as they begin a formal understanding of limits. Students will understand what a dominant term is and use the idea of dominant terms to evaluate limits at infinity.
- Students will reason abstractly and quantitatively, attend to precision as they investigate the number e as a limit in the indeterminate form $(1)^\infty$ and learn how the number e is important to mathematics.
- Students will attend to precision, construct viable arguments and critique the reasoning of others as they approximate area under a curve using trapezoids and compare the results obtained using trapezoids to results obtained using left endpoint and right endpoint rectangles. Students will also realize that a trapezoidal approximation is the average of left endpoint and right endpoint rectangle approximations.
- Students will make sense of problems and persevere in solving them as they sketch velocity graphs and position graphs and develop connections between the two types of graphs.

Sample Activity:

A Race to Infinity - Through this activity, students will be evaluating eight functions, whose end behavior can be described as follows: x, y. But, the important part is to determine which of these functions would approach infinity faster when comparing exponential, power, and logarithmic functions with b>1. This activity sets the premise for "dominant" functions when students then evaluate algebraic rational expressions containing any of these functions either in the numerator or on the denominator. The end goal of this activity is for students to realize that: (1) exponential functions with larger bases will dominate the exponential family, (2) of the power functions, the function with the highest power will dominate. Since radical expressions can be rewritten using an exponent, they are in the power function family, and (3) that logarithms with the smallest base ($b > 1$) would dominate.

Comprehensive Final Exam Details

1. Students will be tested on their knowledge of the following topics:
   - 3-Variable Systems
   - Prerequisites from Algebra and Geometry
   - Trigonometric Functions
   - Analytic Trigonometry
   - Laws of Sines, Cosines
   - Polar and Vectors
   - Complex Numbers
   - Exponential and Logarithmic Functions

- Topics in Analytic Geometry, including Conics
- Functions and Models
- Limits and Derivatives
- Differentiation Rules
- Applications of Differentiation

2. The Final Exams - much like unit/chapter exams - are detail-oriented and require students to provide detailed, step-by-step justification for their responses to each of the questions. This way, they get trained and prepared for their future AP Math courses/exams.