Glendale Unified School District

High School

April 16, 2019

Department:	Mathematics
Course Title:	Integrated Math III
Course Code:	3513D/3514D
School(s) Course Offered:	Glendale High School, Clark Magnet High School
UC/CSU Approved (Y/N, Subject):	Y, “c” Mathematics
Course Credits:	Full Year (10)
Recommended Prerequisite:	Integrated Math II A/B
Recommended Textbook:	*Core Connections Integrated III* Judy Kysh, Evra Baldinger, Michael Kassarjian, Karen Wootton, et. al CPM Educational Program Second Edition, Version 5.0

Course Overview:

Integrated Mathematics III aims to apply and extend what students have learned in previous courses by focusing on finding connections between multiple representations of functions, transformations of different function families, finding zeros of polynomials and connecting them to graphs and equations of polynomials, modeling periodic phenomena with trigonometry, and understanding the role of randomness and the normal distribution in making statistical conclusions.

On a daily basis, students in this course use problem-solving strategies, questioning, investigating, analyzing critically, gathering and constructing evidence, and communicating rigorous arguments justifying their thinking. Under teacher guidance, students learn in collaboration with others while sharing information, expertise, and ideas.

The course is well balanced between procedural fluency (algorithms and basic skills), deep conceptual understanding, strategic competence (problem solving), and adaptive reasoning (extension and transference). The lessons in the course meet all of the content standards, including the "plus" standards, of Appendix A of the *Common Core State Standards for Mathematics*. The course imbeds the CCSS Standards for Mathematical Practice as an integral part of the lessons in the course.

Key concepts addressed in this course are:

- Visualize, express, interpret and describe, and graph functions (and their inverses, in many cases). Given a graph, students will be able to represent the function with an equation, and vice-versa, and transform the graph, including the following function families: absolute value, exponential, linear, logarithmic, piecewise-defined, polynomial, quadratic, square root, trigonometric.
- Use of variables and functions to represent relationships given in tables, graphs, situations, and geometric diagrams, and recognize the connections among these multiple representations.
- Application of multiple algebraic representations to model and solve problems presented as real world situations or simulations.
- Solving linear or quadratic equations in one variable, systems of equations in two variables, and linear systems of equations in three or more variables.
- Use of algebra to rewrite complicated algebraic expressions and equations in more useful forms.
- Rewriting rational expressions and arithmetic operations on polynomials.
- The relationship between zeros and factors of polynomials.
- Operations with complex numbers, and solving quadratics with complex solutions.
- Applications of the Law of Sines and Law of Cosines.
- Modeling periodic phenomena with trigonometric functions.
- Calculating the sums of arithmetic and geometric series, including infinite geometric series.
- Concepts of randomness and bias in survey design and interpretation of the results.
- Use of a normal distribution to model outcomes and to make inferences as appropriate.
- Use of computers to simulate and determine complex probabilities.
- Use of margin of error and sample-to-sample variability to evaluate statistical decisions.
- Solving trigonometric equations and proving trigonometric identities.
- Understand logarithms and their inverse relationship with exponentials.
- Use logarithms to solve exponential equations.

Course Content:

Semester A

Unit 1: **<u>Investigations and Functions</u>** *(approximately 14 days)*

STANDARDS
F-IF.4, F-IF.7b, F-BF.1, A-APR.1

A. This unit starts a focus on investigation and justification that continues throughout the course as students formulate and investigate mathematical questions and create logical and convincing arguments to support their findings. Students use a graphing calculator to create multiple representations of a function, and review how to fully describe the graph of a function using precise mathematical language. Students are also introduced to the way a parent graph and parameters define a family of functions. Modeling mathematical problems is a big emphasis from the start.

Progression of Content:
The investigation strategies students have developed throughout the course, especially in this unit, will continue to be used and further elaborated in future units as they study logarithmic, inverse, polynomial, and trigonometric functions.

B. Unit Assignment(s):
Mathematics Practices used in Unit 1:
- Look for and Make Use of Structure as they determine which inputs and outputs are possible for each type of function.
- Construct Viable Arguments and Critique the Reasoning of Others as they determine the order for the function machines.
- Use Appropriate Tools Strategically as they learn features of the graphing calculator.
- Attend to Precision as they graph functions with asymptotes and use proper vocabulary when describing statistical data.
- Look for Regularity in Repeated Reasoning as they investigate a family of functions by changing a parameter.
- Reason Abstractly and Quantitatively as they model the relationship between height and volume of rectangular prisms.

Sample Activities:
Function Exploration - Each group of students will be given a radical function to fully investigate. They will make a complete graph and describe it using the following attributes: shape, line of symmetry, asymptotes, increasing or decreasing, x- and y-intercepts, domain and range, endpoints, maximum or minimum points, continuous or discrete, and whether or not it is a function. After all graphs have been presented to the class, a discussion will follow that will discuss the similarities and differences between the graphs.
Open Box - Modeling a geometric relationship, each group of students will be given six equally sized sheets of grid paper. They will cut the corners from the paper and fold it to make a box

without a lid. After making several boxes, students will use multiple representations (table, diagrams and graph) to determine which box has the greatest volume. Students will then generalize their results by writing an equation to represent the volume with height x. Using technology, students will find the height of the box with the largest possible volume.

Unit 2: **Transformations of Parent Graphs** *(approximately 17 days)*

STANDARDS
F.BF.1, F.BF.3, F-IF.4, F-IF.5, F-IF.6, F-IF.7b, F-IF.7e, A-CED.2, A-SSE.1b, G-GPE.3.1

A. In this unit, students learn how to generate families of functions from parent functions. Students develop a general equation of form f(x) = a(x – h)^2 + k for the family of quadratic functions and learn to graph a parabola quickly by identifying its orientation, vertical stretch (or compression), and vertex. Students then continue to generalize families of functions by applying the same kinds of transformations to other parent functions, describing the role of the locator point (h, k) for each family of functions.

Progression of Content:
The idea of families of functions will be revisited several times throughout this course and the next. Each time students are introduced to a new parent functions (e.g., inverses and logarithmic functions in Unit 5), they will be asked to graph members of its family and write an equation in graphing form for the family. The members of a family of functions are all related to a parent function and to each other by a sequence of transformations. As students gain familiarity with the properties of new functions, they will build their ability to choose the appropriate function to model a particular relationship.

B. Unit Assignment(s):
   Mathematical Practices used in Unit 2:
   - Look for and Make Use of Structure when they graph quadratic functions and rewrite the equations of quadratic functions from standard form to graphing form; make connections between the transformations of parabolas and other parent graphs; apply knowledge of parabolas and other parent functions to identify the locator point (h, k) for different families of functions; explore odd and even functions; and complete the square for equations of parabolas and circles and identify the vertex or center and radius.
   - Model with Mathematics as they write quadratic functions to represent relationships, check the reasonableness of their answers, and make predictions.
   - Look for and Express Regularity in Repeated Reasoning as they explore transformations of graphs that are not functions.

Sample Activities:

Transforming Other Parent Graphs - Having transformed quadratic equations earlier in the chapter, students will now discover the transformations of five other parent graphs. Each group will organize their work into a poster that clearly shows: each parent graph, examples of transformations and each equation in graphing form. As a challenge for the other groups, each poster will also show a graph for which other teams need to write the equations and will give an equation for each of the other teams to graph.

Unit 3: **Solving and Inequalities** *(approximately 14 days)*

STANDARDS
A-APR.4, A-REI.2, A-REI.11, A-SSE.1b, A-SSE.2, A-CED.2, A-CED.3, F-BF.1, F-IF.4

A. In this unit, students are asked to think about or visualize the kinds and number of solutions that an equation, inequality, or system of equations or inequalities might have. Another main focus is the application of equations, inequalities, and systems to solve problems. Students will use graphing as a powerful method for solving equations and systems as well as for visualizing the solutions, then reverse the process, when given solutions and asked to visualize the graphs.

Progression of Content:
Students will return to the focus on solving and solutions in the first section of Unit 11, when they will extend their ideas to solving systems of equations with three variables. At the end of that section, they will return to the idea of applications of systems as they solve a system of three equations with three variables to determine the equation of a parabola, $y = ax^2 + bx + c$, that passes through three known points. One benefit of solving equations by graphing is that students soon face equations that they cannot solve using algebraic methods, so they need to use graphing to determine a solution. Graphing becomes a very powerful mathematical tool that students can use to solve polynomial equations in Unit 8 and trigonometric equations in Unit 9.

B. Unit Assignment(s):
Mathematics Practices used in Unit 3:

- Look for and Make Use of Structure as they choose methods for solving linear and nonlinear equations and inequalities.
- Attend to Precision when determining and verifying solutions graphically and algebraically.
- Model with Mathematics as they use systems of equations to model and analyze situations, including problems with constraints to determine an optimal solution.

Sample Activity:
How Tall is Harold? - After learning to solve systems of equations both algebraically and graphically, students are given a scenario in which foods in a food fight hit Harold in the head. Given information about the flight of the food and Harold's distance from the food, students will

model both graphically and algebraically and from their models, they will determine Harold's height.

Unit 4: **<u>Normal Distributions and Geometric Modeling</u>** *(approximately 14 days)*

STANDARDS
S-IC.1, S-IC.2, S-IC.3, S-IC.4, S-IC.5, S-IC.6, G-GMD.4, G-MG.1, G-MG.3

A. In this unit, students will begin their studies of the fundamentals of designing studies and experiments, as well as their limitations. The importance of random sampling for studies and random assignment for experiments is stressed. Students begin by looking at surveys, and gain experience with two common sources of bias in those surveys. Students will next perform an experiment to help them compare and contrast experiments with observational studies (such as opinion surveys), stressing distinctions between experiments and observational studies. Relative frequency histograms will record the data, and will be modeled with normal distributions. Finally, students focus on geometric modeling, looking at cross-sections and solids of revolution.

Progression of Content:
Designing studies and experiments and using normal density curves as models to calculate probabilities is important for Unit 6. In Unit 6, students begin to explore inferential statistics, using samples to make predictions about populations.

B. Unit Assignment(s):
Mathematics Practices used in Unit 4:
- Construct Viable Arguments and Critique the Reasoning of Others as they write research questions and consider issues of bias, explore convenience sampling and try to incorporate some level of random selection into their own sampling for their survey, and explore the differences between observational studies, surveys, and experiments.
- Model with Mathematics as they explore randomly selected samples and samples selected intentionally, explore the impact of a lurking variables, and compare relative frequency histograms to normal probability density models.
- Use Appropriate Tools Strategically as they create relative frequency histograms and explore statistics with their calculator.
- Reason Abstractly and Quantitatively as they continue to explore normal distributions and predict percentiles.

Sample Activities:
Conclusions from Studies - After learning about survey design, samples, randomness and bias, students will write their own survey questions. After gathering data, they will use histograms, percentile, and a normal probability density function to determine if their data is valid. They will then be given an opportunity to run an experiment, adjust the experiment, and re-evaluate their data.

How Can I Get That Cross-Section? - In order to show students that volumes of solids can be found using cross- sections, they are first asked to slice a cube to get different cross-sections. Students will be given a glob of clay that they must first make into a cube. Using dental floss to slice the cube, students will slice the cube to get 4 different cross-sections: a square, a rectangle that is not a square, a triangle, and a hexagon.

Unit 5: **Inverses and Logarithms** *(approximately 11 days)*

STANDARDS
F-BF.3, F-BF-4, F-BF.4a, F-LE.4, F-LE.4.2, I-IF.7e

A. Reversing is an important theme in the early part of this chapter. The first section introduces the concept of inverse relations. Students learn that reversing, or working backward to undo the action of a function, can create a new function. They explore multiple representations of functions and their inverses, and recognize that many functions have inverses that are not functions. In the second, students determine inverses of parent functions. They learn that the inverse of an exponential function is a logarithm. Reversing is emphasized once again as they learn how to convert exponential equations into logarithmic form, and vice versa. Students investigate the new family of logarithmic functions f(x) = logb(x) for different values of b, test values on their calculators to determine the base the calculators work in, and learn to graph transformations of f(x) = log(x).

Progression of Content:
This chapter adds to students' lists of parent functions, which will continue to expand with the addition of polynomial functions in Unit 8 and trigonometric functions in Unit 9.

B. Unit Assignment(s):
Mathematics Practices used in Unit 5:

- Students will look for and make use of structure and construct viable arguments as they develop and justify strategies for undoing functions and as they investigate different bases for logarithms..
- Students will use appropriate tools strategically and look for and make use of structure when they graph inverses of functions and write their equations.
- Students will look for and make use of structure as they verify inverses using multiple representations, and attend to precision as they restrict the domain of a function to ensure that its inverse is also a function.
- Students will construct viable arguments and critique the reasoning of others and look for and make use of structure as they apply their knowledge of parent graphs and inverses to learn about logarithms.
- Students will look for and make use of structure and express regularity in repeated reasoning as they learn the definition of logarithm and calculate the values of logarithms.

- Students will construct viable arguments and critique the reasoning of others, use appropriate tools strategically, and look for and make use of structure as they investigate logarithms with different bases.

Sample Activities:
Guess My Number Game - Students are asked to guess the number the teacher is thinking of based on the order of operations applied to the number and what the mystery number has ultimately transformed into. Students may or may not write an equation, but you may want to encourage them to do so as it will help when they progress to working with functions and inverses. Making sure that the idea of reversing, or undoing, comes up in the discussion of the "Guess My Number" game. You undo each step, reversing the original Order of Operations.

Graph the Inverse Function - Students will be looking at strategies for creating graphs of inverse functions. The first two graphs have functions that they are capable of finding the equation of their inverse functions but the third function does not lend itself to be solved for the inverse function. Teams could make a mini-table of some coordinates from the graph and then use it to help make a mini-table for the inverse graph. Students will soon discover that the line $y = x$ is the line of symmetry.

Semester B

Unit 6: **<u>Simulating Sampling Variability</u>** *(approximately 12 days)*

STANDARDS
S-IC.1, S-IC.2, S-IC.4, S-IC.5, S-IC.6, S-MD.6+, S-MD.7+

A. This unit introduces students to techniques for computing complex probabilities through simulations. Students also begin working with inferential statistics and statistical hypothesis testing. Students will develop an understanding of counterintuitive probability problems by using area models or tree diagrams. Students learn how to use simulations to estimate complex probabilities. Students then investigate the concept of natural variation in samples and how that variation can be modeled and controlled through sample size. Students will use the concept of sample-to-sample variation from Unit 6.1 to perform informal hypothesis testing using margin of error in Unit 6.2. The section concludes with an introduction to statistical process control. Any system designed to detect rare events may be highly accurate but still have problems with false positives. In Unit 6.3, students will look at several such systems, such as HIV and drug tests, and decide whether the social cost of false positives is greater than the benefits of true positive results.

Progression of Content:
This unit provides an introduction to inferential statistics. Students will make statements about populations based on information obtained from a sample. They will study this topic more extensively if they take a formal course in statistics.

B. Unit Assignment(s):
Mathematics Practices used in Unit 6:

- Model with Mathematics as they simulate the probability of a newborn being a boy or girl; simulate the number of streaks they can expect in a random process; take a random sample of candies to understand margin of error; explore sample-to-sample variability by conducting a hypothesis test; and use simulations to determine if a manufactured part is within typical quality specifications.
- Attend to Precision as they evaluate the mean and margin of error from a data set, evaluate the results from their simulations, simulate the quality control process for a specific company.
- Use Appropriate Tools Strategically as they compare the effects of two treatments in an experiment, evaluate results of simulations, analyze decisions and strategies in situations that are counterintuitive.

Sample Activity:
AIDS in South Africa - Students will showcase their understanding of estimating sample-to-sample variability and conducting a hypothesis test. Given the claim of a drug manufacturing company, students will simulate 100 samples of 125 residents. They will determine the mean, margin of error, and determine if they can support the claim of the drug company. Students are asked if their margin of error is reasonable and then tell what must be done to make the margin of error smaller.

Unit 7: **Logarithms and Triangles** *(approximately 13 days)*

STANDARDS
F-LE.3, F-LE.4, F-LE.4.1, F-LE.4.3, A-SSE.2, F-BF.1, F-IF.7e, G-SRT.9+, G-SRT.10+, G-SRT.11+

A. In this unit, students return to their work with logarithms to develop tools they can use when solving application problems involving exponential equations. In the first half of this unit, students investigate the family y = log(mn) and discover the Power Property of Logarithms, which allows them to solve exponential equations by using logs to undo or rewrite the equation. Students generalize from number patterns to make conjectures about other properties of logarithms and then prove these properties. Furthermore, students develop and share strategies to write the equation of the exponential function with a given asymptote that passes through two given points. Then they use that equation to make predictions.

The remainder of this unit focuses on completing a tool kit for calculating missing parts of non-right triangles. Students identify the types of information needed to determine all of the missing sides and angles of a triangle. Through this exercise, students also identify triangles for which they do not yet have the tools to determine missing parts. Students notice that they *do* have enough tools to calculate the measures and side lengths of right triangles. This leads to the question, *"What if the triangle is not a right triangle?"* Students then develop the Law of Sines and Law of Cosines so that they have a complete set of tools to determine the other missing parts of any triangle (when sufficient information is provided).

The unit concludes with students looking at different application problems using triangles and identifying which tools are most useful in each situation. In addition, students investigates the ambiguous case of triangles: SSA. This lesson is offered for accelerated classes or those that could benefit from a complete view of the relationships between the sides and angles of a triangle. Working through the problems of this lesson before you decide to use the lesson with your students is highly recommended.

Progression of Content:
The work with logarithms in this unit prepares students for future work in a pre-calculus course. Working with Law of Sines and Cosines and reviewing the use of right triangle trigonometry and special right triangles prepares students for working with the trigonometric family of functions in Unit 9.

B. Unit Assignment(s):
Mathematics Practices used in Unit 7:

- Students will look for and make use of structure and express regularity in repeated reasoning while they develop the Power Property of Logarithms, learn other properties of logs and how to rewrite equations with different bases.
- Students will make sense of problems and reason abstractly and quantitatively as they write the equation of an exponential function given two points and an asymptote.
- Students will make sense of problems and persevere in solving them, model with mathematics, and reason abstractly and quantitatively as they explore exponential functions with an asymptote other than $y = 0$ and apply logarithms to solve an exponential equation.
- Students will make sense of problems and persevere in solving them as they figure out what information they need to solve for parts of triangles. They will need to attend to precision as they communicate what they know and do not know.
- Students will look for and express regularity in repeated reasoning as they develop the ratios for the Law of Sines.
- Students will make sense of problems and persevere in solving them using the Law of Cosines.
- Students will attend to precision as they work with triangles involving the SSA relationship. They must also use appropriate tools strategically as they explore the ambiguous case of the Law of Sines.
- Students will reason abstractly and quantitatively as they make sense of problems and persevere in solving them. They will attend to precision as they solve the problems and communicate within their teams, labeling diagrams, attending to units, and calculating their answers accurately.

Sample Activities:
"The Case of the Cooling Corpse" - a problem in which students model the falling temperature of a corpse to establish time of death and solve a murder mystery. In order to solve the mystery, students will need to decide which information is relevant to solving the problem (e.g., body temperature, times listed on sign-in sheets, etc.) . Students will need to write and solve a system of exponential functions, with a horizontal asymptote representing the room temperature, the independent variable as time, and the dependent variable as the temperature of the body.

Solving Triangles - This activity consists of eight independent triangle problems (suggestion is to assign one problem to a group of 3-4 students) that will provide students with a chance to consolidate their understanding of the various tools and strategies they have developed so far to solve triangle problems. Some problems may be solved using the Law of Sines, Law of Cosines, or a combination of both. Students will present their assigned problem to the entire class and their process for solving the assigned problem.

Unit 8: **Polynomials** *(approximately 15 days)*

STANDARDS
A-APR.1, A-APR.2, A-APR.4, A-APR.6, A-APR.3, F-IF.4, F-IF.7c, N-CN.8+, N-CN.9+, A-SSE.2, A-CED.2, F-BF.1

A. In the first section, students will investigate the equation ↔ graph connections for polynomial functions. They will recognize that equations in factored form are much easier to sketch, and they will understand the relationship between the factors and the x-intercepts of the graph. Then, in the second section, they will develop an understanding of imaginary and complex numbers and recognize that polynomial functions can have complex roots. In the third and last section, they will learn to divide polynomials by a known factor to find other factors. This will allow them to determine complex and irrational roots of some cubic and quartic functions.

Progression of Content:
Students will build on their understanding of function families in Unit 9, where they study trigonometric functions and transform the graphs of sine and cosine functions. Students will use their algebra skills when they study rational expressions in Unit 11. Students will also use their algebra skills when they prove formulas for sums of series in Unit 10 and solve trigonometric identities in Unit 12.

B. Unit Assignment(s):
Mathematics Practices used in Unit 8:

- Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others as they discuss factored form and describe the graphs of polynomials and their understanding of the stretch. Students will also look for and make use of structure as they draw graphs of polynomials.

- Students will look for and make use of structure as they continue their polynomial investigation. Students will also construct viable arguments and critique the reasoning of others as they consolidate their results on polynomials.
- Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others as they develop their understanding of a stretch factor. Student will also model with mathematics as they develop an equation for the roller coaster problem first introduced in the first lesson of Unit 8.
- Students will look for and make use of structure as they use polynomial division to determine factors of polynomials. Students will also construct viable arguments and critique the reasoning of others as they develop their understanding of polynomial division.
- Students will look for and make use of structure as they use complex roots to write equations of quadratic functions and express regularity in repeated reasoning as they identify polynomial identities to help them factor. Students will also look for and express regularity in repeated reasoning as they identify patterns in the sums and products of complex roots.
- Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others when they determine all roots of a polynomial with a degree greater than two and when they determine all roots of a polynomial with a degree greater than two.

Sample Activities:
Polynomial Function Investigation - Students are instructed to look for, label, and describe the x-intercepts and "bounces" (double roots, although students will probably not use this term), the y-intercept, the number of turns, and the behavior of graphs for very large and very small x-values. They should share and discuss all of their graphs and their observations within their teams. The discussion should lead to conjectures and the creation of several new equations to try. Students are coming up with methods for determining the x- and y-intercepts, predictions about the numbers of crossings of the x-axis, and ideas for determining a reasonable window or maximum and minimum approximations.

Game of Polydoku - By treating division as a puzzle and using the organizational device of an area model, students can use logical reasoning to reverse the multiplication process and figure out a missing factor. Once they have worked through the process several times, they should be able to set up and solve their own division problems. This method is as efficient as (if not more efficient than) polynomial "long division" and you can use it to develop synthetic division if that is part of your curriculum. This game introduces polynomial division by challenging students to reverse the process of polynomial multiplication.

Unit 9: **<u>Trigonometric Functions</u>** *(approximately 16 days)*

STANDARDS
F-TF.5, F-BF.1, F-IF.4, F-IF.7e, F-TF.2.1, F-TF.5, F-TF.2, F-BF.2.1, F-BF.3

A. In this chapter, students will extend their understanding of trigonometric ratios in right triangles to trigonometric functions. The unit circle is introduced as a representation of trigonometric relationships, and students explore the connections between the unit circle and graphs of trigonometric functions. They look at sine as the height, cosine as the base, and tangent as the slope of the hypotenuse of a right triangle within the unit circle. Students are often confused by the use of variables in trigonometric relationships. When they think of x as the horizontal coordinate in the unit circle, seeing the function y = sin(x) is confusing. For this reason, the notation y = sin(θ) appears through most of the first section. When investigating trigonometric functions whose periods are not 2π the transition is made to y = sin(x). In the second section, students investigate y = sin(x) and y = cos(x) as parent functions and explore their transformations. They develop understanding of the idea of period and its role in the general equation. By the end of the chapter, teams will be able to generate graphs from sinusoidal equations and vice versa.

Progression of Content:
In Unit 12, students will return to the study of trigonometry. They will solve trigonometric equations, learn reciprocal trigonometric functions, develop trigonometric identities, and further explore the connections between sine, cosine, and tangent.

B. Unit Assignment(s):
Mathematics Practices used in Unit 9:

- Students will model with mathematics as they determine how to create an equation to represent a periodic situation.
- Students will model measurement data with mathematics and reason abstractly and quantitatively as they validate their model. They will also look for and make use of structure as they create a sine graph to represent a situation.
- Students will look for and make use of structure and look for and express regularity in repeated reasoning as they make connections between different angles on the unit circle that have the same sine ratio.
- Students will use appropriate tools strategically as they find sine and cosine values. They will also look for and make use of structure and look for and express regularity in repeated reasoning as they make connections between the sine and cosine values of points on the unit circle and their x- and y-values on a coordinate graph.
- Students will use appropriate tools strategically, look for and make use of structure, and look for and express regularity in repeated reasoning as they apply the definition of a radian and learn how to convert between radian and degree measures.
- Students will use appropriate tools strategically, look for and make use of structure, and look for and express regularity in repeated reasoning as they complete key points on a unit circle using radian measures.
- Students will make sense of problems and persevere in solving them as they figure out how to create a graph of the tangent function. They will look for and make use of structure

and express regularity in repeated reasoning as they connect the tangent function to what they already know about the sine and cosine functions.

- Students will use appropriate tools strategically as they investigate transformations of trigonometric functions using technology. They will also look for and make use of structure and express regularity in repeated reasoning as they develop a general equation for the sine and cosine family of functions.
- Students will make sense of problems and persevere in solving them as they model a situation using a sine function. They will use appropriate tools strategically and look for and make use of structure as they identify the period of periodic functions and further investigate the general equation for sine functions.
- Students will use appropriate tools strategically, look for and make use of structure, and express regularity in repeated reasoning as they make graphs of sine functions and write equations from sine graphs.
- Students will model with mathematics and use appropriate tools strategically as they make connections between periodic graphs and their equations. Students will also look for and express regularity in repeated reasoning as they see that sine and cosine functions are horizontally shifted versions of each other.

Sample Activities:

Blood Drip Lab - Students either in their teams or as a class will conduct the lab with simulated blood dripping from an IV bag. This is a preview of transformations of sinusoidal functions, which students will explore in the next section. At that time, they will revisit this lesson's activity and write equations to model the curve that they create. They will investigate what are the different factors that play a role in how each sinusoidal graph differs from each other.

The Screamer Ferris Wheel - Students use a ferris wheel model to find the height in which a rider will need to either climb up or down from the ferris in the event of the ferris wheel halting. So by measuring the height of a car on the ferris wheel at different positions, students will be able to create the sine graph unknowingly.

Unit 10: **Series** *(approximately 19 days)*

STANDARDS

A-SSE.1a, A.SSE.2, A.SSE.4, A-APR.5+, A-APR.4

A. This unit provides an opportunity for students to use what they have learned during earlier units, and then extend their knowledge to new contexts. The development of formulas for sums of geometric and arithmetic series depends on the work with sequences students started in previous courses and further develops their skills with algebraic procedures. The development of the Binomial Theorem involves working with combinations, a topic from previous courses. The lesson on mathematical induction provides an introduction to this form of proof as well as practice with algebraic manipulation.

Progression of Content:
Using a graphical approach to develop a formula for the sum of an arithmetic series previews the use of the area of rectangles to calculate the area under curves in a later course. Summation notation and series will be developed further in precalculus and calculus.

B. Unit Assignment(s):
Mathematics Practices used in Unit 10:

- Students will make sense of problems and persevere in solving them as they determine how to calculate the sum of an arithmetic sequence, and they will look for and express regularity in repeated reasoning as they develop strategies that allow them to do this efficiently.
- Student will make sense of problems as they use geometric representations to represent arithmetic sequences and use them to calculate sums. They will also look for and express regularity in repeated reasoning as they begin to generalize their methods.
- Students will look for and make use of structure as they learn how to combine known series to form new ones. Additionally, they will look for and express regularity in repeated reasoning as they construct formulas for sums that have an unspecified number of terms.
- Student will look for and express regularity in repeated reasoning and attend to precision as they use summation notation to compute the sums of arithmetic sequences algebraically.
- Students will make sense of problems and persevere in solving them, and construct viable arguments and critique the reasoning of others as they explore proof by induction.
- Students will make sense of problems and persevere in solving them as they create a strategy for calculating the sums of geometric series. They will also reason abstractly and quantitatively as they solve problems using either sums of arithmetic series, geometric series, or both.
- Students will make sense of problems, model with mathematics, and look for and make use of structure as they use binomial probability models to represent and analyze situations.
- Students will look for and express regularity in repeated reasoning and look for and make use of structure as they describe patterns in Pascal's triangle and make connections between the patterns in the triangle and binomial expansions.
- Students will look for and express regularity in repeated reasoning to make sense of problems as they discover how the Binomial Theorem is related to the derivation of *e*.

Sample Activities:
Sum of an Arithmetic Series - Before getting to the general equation to find the sum of *k* terms in an arithmetic series, S(k)=t(1)+t(k)k2, students find the sum graphically. The visualization of forming a rectangle by stacking the first term and the last term of the series, and noticing that a rectangle with a base length of *k*/2 and a height equivalent to the first + last term is a way of finding the sum of the terms in the series.

Pascal's Triangle and the Binomial Theorem - During this activity, students connect two topics in mathematics that they may not think are related: counting combinations and equivalent algebraic expressions.

Unit 11: **Rational Expressions and Three-Variable Systems** *(approximately 12 days)*

STANDARDS
A-APR.7+, A-CED.2

A. In the first section of this chapter, students will focus on operations with rational expressions. Students did a function investigation in Unit 1 that previewed the investigation of rational expressions. Students learn a powerful method of simplifying rational expressions that uses properties of the number 1 and the properties of exponents. The last lessons in this chapter build upon and reinforce students' understanding of operations with rational numbers while developing procedures for operations with rational expressions. In the second section, students are introduced to the three-dimensional Cartesian coordinate system and are challenged with the task of graphing an equation with three variables. Students use hands-on graphing techniques and technology to help them visualize graphs of planes. In a previous course, students learned to solve a system of two linear equations in two variables, and they reviewed this topic in Chapter 3 of this text. At the end of Section 11.2, students extend these methods to develop a procedure for solving a system of three linear equations in three variables.

Progression of Content:
The remaining chapter requires students to use their algebra skills to solve trigonometric identities. The three-dimensional visualization in this chapter prepares students for visualizing intersections of planes with cones to form conic sections in future courses. In future courses, students may use matrices to solve systems of equations.

B. Unit Assignment(s):
Mathematics Practices used in Unit 11:

- Students will look for and make use of structure as well as look for and express regularity in repeated reasoning as they use the number 1 to understand rational expressions.
- Students will also look for and make use of structure as well as look for and express regularity in repeated reasoning as they connect multiplication and division of fractions to that of rational expressions.
- Students will look for and make use of structure as they make connections between adding and subtracting fractions and adding and subtracting rational expressions, and as they locate points in three-dimensional space. They will construct viable arguments and critique the reasoning of others as they justify their strategies.
- Students will continue to look for and make use of structure as well as look for and express regularity in repeated reasoning as they work with rational expressions. They will

construct viable arguments and critique the reasoning of others as they investigate the closure of rational expressions under operations.

- Students will look for and make use of structure when they graph on three-dimensional axes.
- Students will look for and make use of structure and use appropriate tools strategically when they solve a system of three equations with three variables algebraically and investigate the different ways three planes can intersect.
- Students will reason abstractly and quantitatively and look for and express regularity in repeated reasoning as they write the equation of a quadratic function by solving a system of three equations with three unknowns.

Sample Activities:
Giant One - Students are encouraged to rewrite rational expressions to create fractions that will reduce to one. The new expressions will be reduced to its simplest form. This strategy to rewrite will allow students to simplify rational expression after doing basic operations on the original expressions.

3-D Coordinate System - Students use cubes to build prisms in their three-dimensional system to help them visualize different points. Students will need cubes to build these prisms as well as isometric dot paper to represent their solutions. Using the dimensions of the prism, students can name the coordinate farthest from the origin on which the opposite corner is already lying on.

Unit 12: **Analytic Trigonometry** *(approximately 15 days)*

STANDARDS
F-IF.7e, F-TF.2.1, F-TF.9+

A. The intent of this chapter is to give students a strong exposure to analytical trigonometry that will prepare them to enter a pre-calculus or calculus course. The chapter centers on solving equations and asks students to think about the circumstances under which an equation is true. Section 12.1 focuses on equations that are sometimes true, giving students the tools to find all relevant solutions. Section 12.2 focuses on identities, that is, equations that are always true. Through the study of identities, students gain the understanding necessary to rewrite equations, which expands the range of equations they are able to solve.

Progression of Content:
By the end of this course, students will have completed trigonometric units which will prepare them for a fourth year in math, whether it be a trigonometric course or Precalculus.

B. Unit Assignment(s):
Mathematics Practices used in Unit 12:

- Students will construct viable arguments and critique the reasoning of others and attend to precision as they investigate trigonometric and other algebraic equations to determine under what conditions they are true, and as they investigate the Angle Sum and Difference Identities.
- Students will use appropriate tools strategically as they represent solutions in multiple ways. They will look for and make use of structure as they represent all of the solutions for trigonometric equations.
- Students will look for and make use of structure and attend to precision as they consider the inverses of trigonometric functions and their domains.
- Students construct viable arguments and critique the reasoning of others as well as attend to precision in their use of mathematical terms and notation and in their creation of graphs of the reciprocal trigonometric functions.
- Students will look for and make use of structure and attend to precision as they use graphs to discover trigonometric identities and apply the trigonometric identities to solve problems.

Sample Activities:

Carbon Copy to find inverses of Trigonometric Functions - Students will graph sine, cosine, and tangent functions on a resource page then use the "Carbon Copy" method to find the inverse graph of these three trigonometric functions. This is a method used in the activity mentioned in Unit 5: Graphing the Inverse Function Activity.The focus of this activity is for students recognize how the domain and range of these functions change. Students already know that the domain and range switch, but this is the first time that they see it in trigonometric functions.

Graphs of Reciprocal Trigonometric Functions - In this activity, students are introduced for the first time to the reciprocal trigonometric functions (secant, cosecant, and cotangent). For example, after graphing the sine function, students use the concept of reciprocals to graph y=1sinx to create the graph of the cosecant function. Using the new graph, student conduct a full investigation of this new function and make observations, including the fact that the x-intercepts of sine graph become vertical asymptotes for cosecant function. A similar process is done with the secant and cotangent functions.

Assignments, Student Engagement, and Assessments

Through the use of technology (including graphing calculators, Desmos, and etools embedded throughout the curriculum), whole class/pair/group discussions, and independent practice, students will deepen their understanding of the mathematical content standards covered in this course. Furthermore, each unit will have multiple investigative activities that will require students to synthesize the information from the current unit as well as previous units and will require them to use their problem solving skills. For each unit, students will be assigned daily classwork and daily homework. Group activities will be incorporated into classwork assignments. Students will often engage with fellow students in the investigative and modeling process. Teachers will encourage all students to participate and explore. Opting out of learning will not be an option. While students are encouraged to construct their own viable arguments, they will also be encouraged to appraise those of others during discussion sessions.

Assessments

The instructional methods and strategies listed below support the delivery of this Integrated Mathematics I course with emphasis on group work, investigative activities, the Standards for Mathematical Practice (SMP) will be applied throughout the curriculum. SMP) will be applied throughout the curriculum. Students' mathematical skills and understanding will be assessed through a range of strategies, such as:

- **Individual Tests** that will allow the teacher to determine a student's ability to solve mathematical problems, level of skill mastery, and conceptual understanding of topics or ideas.
- **Team Tests** that will be used primarily as a formative assessment and an opportunity to assess the SMPs, which include problems designed to inspire in-depth conversations and collaboration around essential mathematics.
- **Participation Quizzes** that will allow the teacher to assess, and therefore support, the quality of the teams' cooperation, independent of mathematical content. In a Participation Quiz, the quality of the teamwork on any given task is documented and assessed directly by the teacher, rather than the mathematical content.
- **Student Presentations** that will allow students to exchange insights, use the language of mathematics, and deepen their understanding at the same time that they allow teachers to assess mathematical communication, justification, and making connections.
- **Class Observations** that will allow the teacher to assess, with varying degrees of formality, the quality of the discussion of mathematics in the interactions created as teams work together. Daily Assignments will vary giving students opportunities to practice what they've learned in class. This instructional approach allows students to look for and express regularity in repeated reasoning by practicing mathematical strategies learned in the classroom.
- **Independent Practice (Homework)** that will allow students the opportunity to enhance their learning and extend their practice. Nightly homework may be an extension of an investigation, more practice with similar scenarios, or practice with basic symbolic skills. Students will use appropriate tools strategically and attend to precision while extending their learning.
- **Math Portfolios** that will give students a chance to "show off" their learning, taking pride in their own perseverance, growth over time, and appropriate use of math tools, techniques and proficiency. Simultaneously, the teacher will be able to assess understanding and make instructional decisions accordingly, without the pressure of a test.
- **Investigative Activities** that will encourage students to ask questions about a complicated situation and apply mathematics in pursuit of a solution. In the process, students will make assumptions and approximations, understanding that revisions might be needed at a later time. Abstract and quantitative reasoning is applied in considering quantities and their relationships during problem solving.