Glendale Unified School District

School Level
High School

Date
*(Meeting date will be typed in **after** Board Approval)*

Department: Mathematics

Course Title: Integrated Mathematics II/IIIA Accelerated A/B

Course Code: 3511DH/3512DH
School(s) Course Offered: Crescenta Valley High School, Hoover High School

UC/CSU Approved: Yes, mathematics "c"

Course Credits: 10, Full Year

Recommended Prerequisite: Integrated Math I A/B

Recommended Textbook:
California Integrated Mathematics 2
Timothy D. Kanold, Edward B. Burger, Juli K. Dixon, Matthew R. Larson, Steven J. Leinwand
Houghton Mifflin Harcourt
2015

California Integrated Mathematics 3
Timothy D. Kanold, Edward B. Burger, Juli K. Dixon, Matthew R. Larson, Steven J. Leinwand
Houghton Mifflin Harcourt
2015

Course Description
Course overview: Integrated Mathematics II/IIIA Accelerated is the first course of a two course accelerated sequence that includes Integrated Mathematics II, Integrated Mathematics III, and Pre-Calculus. Integrated Mathematics II/IIA will cover all of the material from Integrated Mathematics II and will also include material from the first half of Integrated Mathematics III. This course satisfies the California Common Core State Standards Integrated Pathway model and follows a suggested option for accelerating three years of content into two years' time as prescribed within the Common Core State Standards Appendix A. It will strengthen and build on students' previous knowledge from Integrated Mathematics I standards. This course will focus on polynomial expressions and functions, quadratic functions and equations, probability,

two- and three-dimensional geometry, similarity, rational functions and equations, radical functions and equations, and connecting algebra and geometry through reasoning and proof as well as real life applications.

Course content:
Semester A

Unit 1 Characteristics of Functions *(approximately 11 instructional days)*
A. *Content Standards:* F-IF.4, F-IF.5, F-IF 7b, A-REI 3.1
In this unit, students will learn about analyzing functions, including domain, range and end behavior. Students will explore the inverses of functions. Students will graph, write, and solve functions including absolute value functions, equations, and inequalities.
Major Topics:

- Domain, range, and end behavior
- Function inverses
- Graphing and solving with absolute value

B. Unit Assignment(s):
After completing this unit, students will complete a Math in Careers task by creating and graphing an algebraic model for the revenue of a community theater at differing ticket prices. Critical skills include representing real-world situations algebraically, determining domain and range, and interpreting graphs. Students will write an expression for the costs of a season ticket after n price increases and another expression for the number of season ticket holders after n price increases. Students will create a revenue function from survey information. They will then determine the constraint on the value of n and state a reasonable domain for the revenue functions. They will also represent the revenue function as a labeled graph. Finally, students will write a brief paragraph describing what actions the theater owner should take to maximize revenue.

<u>Mathematical Practices used in Unit 1:</u>
MP.6 - Students learn to describe intervals using inequalities, set notation, and interval notation. They also learn how to use mathematical notation to describe end behavior of a function. Furthermore, students find the solutions to absolute value equations both by graphing them, with and without technology, and through algebra. Students learn that a disjunction is often used to express the solutions to absolute value equations, and they use properties of algebra to accurately and efficiently find the solutions to various types of absolute value equations.
MP.7 - Students learn how the attributes of functions are represented graphically. Students learn to make connections between functions and the situation they represent.
MP.4 - Students learn the meaning of the parameters a, b, h, and k in an absolute value function, and use those parameters to graph and draw conclusions about absolute value functions.

Unit 2 Polynomial Operations *(approximately 15 instructional days)*

A. Content Standards: N-RN.1, N-RN.2, A-APR.1, A-SSE 1a

Students will learn about expressions with rational exponents. They will perform operations on polynomial expressions. Students will also explore special products of binomials.

Major Topics:

- Simplifying expressions with rational exponents
- Simplifying expressions with radicals
- Adding and subtracting polynomials
- Multiplying polynomials

B. Unit Assignment(s):

Students will complete a performance task by writing and performing operations on several functions based on summer camp enrollment and expenses. Critical skills include modeling real-world situations and polynomial addition, subtraction, and multiplication. Students will write function rules for the number of campers (girls, boys, and total) as a function of time. Additional functions will be written for cost per child and the revenue generated by the total enrollment, expenses, and profit. Students will explain the meaning of a constant function. In addition, students will determine the annual rate of change in the revenue.

Mathematical Practices used in Unit 2:

MP.8 - Students will recognize that all problems with radicals and rational exponents can be expressed in equivalent forms, b^m/n=<n root>b^m .

MP.8 - Students learn that a polynomial written in standard form has exponents of a variable decreasing from left to right.

MP.3 - Students will recognize that the lowest degree a polynomial can have is 0.

Unit 3 Quadratic Functions *(approximately 11 instructional days)*

A. Content Standards: F-BF.3, F-IF.4, F-IF.7a, A-APR.3, A-REI.4

In this unit, students will learn about graphing quadratic functions. They will interpret vertex and standard form of quadratic functions. Students will explore the connection between intercepts and zeros. They will solve quadratic equations using the Zero Product Property.

Major Topics:

- Transforming quadratic functions
- Connecting intercepts, zeros and linear factors
- Applying the Zero Product Property to solve equations

B. Unit Assignment(s):

After completing this unit, students will complete a Math in Careers task by graphing and writing an equation for a function that fits a data set comparing gas mileage to speed. Critical skills include modeling real-world situations and fitting a function to a set of data. Students will see how a transportation engineer uses math on the job. Students will identify independent and dependent variables and the units associated with each variable. They will sketch a parabola that best fits their plotted points. Student will explain why a parabola is a reasonable curve to fit

to the data. They will write an equation for the function and identify the vertex, intercepts, and one other point. Finally, students will write a paragraph describing how they can use their model to find a car's speed and why there may be more than one answer.

Mathematical Practices used in Unit 3:
MP.2 - Students learn to recognize the parent function of a quadratic equation and then to analyze the relationship between parameter a and the graph of the quadratic function. Students determine what kinds of real-world problems can be solved by identifying the zeros of a quadratic function.
MP.4 - Students show that graphs of parabolas can model real-world situations, such as height over time when throwing a ball.
MP.5 - Students will use appropriate tools strategically by using graphing calculators or Desmos to graph parent functions and their different a values.
MP.8 - Students look for and express regularity in repeated reasoning by noting that parabolas are symmetrical through the vertical line through their vertex and how that can be identified in a table of values (single least or greatest values).

Unit 4 Quadratic Equations and Models *(approximately 20 instructional days)*

A. Content Standards: A-SSE.2, A-REI.4a, A-REI.4b, A-REI.7, A-SSE.3a, S-ID.6b, F-IF.7e, A.CED.2, F-LE.1b

In this unit, students expand their understanding and skills when dealing with quadratic functions. Using multiple methods to solving quadratic equations is the focus of the unit. Special attention is given to choosing which method best fits the situation and purpose.
Major Topics:

- Solving quadratics by factoring
- Solving quadratics by taking square roots
- Solving quadratics using the quadratic formula
- Solving non-linear systems
- Comparing linear, exponential, and quadratic models

B. Unit Assignment(s):

Students will complete a performance task by modeling the heights of two divers with equations for projectile motion. Critical skills include modeling real-world situations and interpreting the graphs of quadratic functions. Students will write functions representing two divers jumping from different heights and speeds modeled by h(t)=-5t2+vt+h0. They then graph both functions on the same coordinate grid, labeling each function. They determine the domain and range of each function in terms of the situation and explain why they made those choices. Students compare and discuss the maximum heights and elapsed time before each diver hits the water.
Mathematical Practices used in Unit 4:
MP.2 - Students will relate completing the square with using the quadratic formula and conclude that the same solutions are found. Students will explain the meaning of the

components of a graph of a real-world situation, correlating them to the real-world parts of the situation, for example the position of two cars in a race, when the pace car leaves ahead of the race car.
MP.3 - Students will explain why it makes sense for the graph of f(x) = ab^x to stretch when a>1, and for the graph to shrink when 0 < a < 1, compared to the graph of f(x) = b^x. For quadratic regressions, students will interpret and explain the value of R^2.
MP.4 - Students will model factoring polynomials with algebra tiles. Students will use equations of parabolas to model upward velocity of an object thrown in the air.
MP.5 - Students will choose the appropriate tool for solving a system of equations (algebraically or graphing calculator, factoring or quadratic formula). Students will use a graphing calculator or a graphing program to find line of best fit. Students use paper and pencil to find the first differences and second differences in order to determine whether a quadratic function can fit given data. Students also use graphing calculators/program to find a quadratic function that fits a data set, and use that function to solve real-world problems.
MP.8 - Students will independently notice the sign of the x-term in a perfect square trinomial, then apply the correct form of the expression, (a + b)^2 or (a - b)^2, to factor.

Unit 5 Extending Quadratic Equations *(approximately 18 instructional days)*
A. Content Standards: N-CN.1, N-CN.2, N-CN.7, G-GPE.3.1, A-REI.7, F-IF.7b, F-IF.7c, F-BF.4a
Students, during this unit, will learn about quadratic equations with complex number solutions. They will continue to practice solving quadratic equations. Students will explore quadratic equations as they pertain to circles and parabolas by deriving the standard form of their equations. They will solve linear- quadratic systems of equations.

Major Topics:

- Complex numbers
- Complex solutions to quadratic equations
- Equation of a circle
- Completing the square
- Linear-quadratic systems

B. Unit Assignment(s):
At the end of this unit, students will complete a Math in Careers task by graphing and interpreting a quadratic function that models the profitability of a toy. Critical skills include completing the square, graphing quadratic functions, and interpreting graphs. Students will create and graph a quadratic function given information about the profitability of specific product. They will interpret the information that both the equation and graph indicate. In a group of 3 to 4, students will provide a presentation (Powerpoint or Prezi) that they would use to convince the board of directors to either produce or not produce this toy.

Mathematical Practices used in Unit 5:
MP.2 - Students will reason abstractly and quantitatively as they compare how the completing the square process is used to write a quadratic function in vertex form to how it is used to write a circle in standard form. Students will identify real-world situations that can be modeled by the intersection of a line and a circle or parabola. Students use multiple representations (table, inequality, and graph) when working with translations of cube root functions.
MP.3 - Students will construct viable arguments and critique the reasoning of others as they discuss how the imaginary solutions of equations in the form of ax^2=c are related and what their sum might be. Students will explain the relationship of the product of 2 + 3i and 2 - 3i and the product of 3 + 2i and 3 - 2i. Students will compare and describe the graphs of multiple transformations of circles. Students discuss and justify the restrictions in the domain in real-world problems.
MP.4 - Students will model with mathematics when they use algebra tiles to practice factoring quadratic expressions. Students will use patty paper or transparencies to prove that an odd function has rotational symmetry about the origin. Students will also use patty paper or transparencies to model parent functions and their translations before writing the function rule for each transformation.
MP.5 - Students will use graphing calculators or online graphing programs, such as Desmos, to find a maximum or minimum value of a quadratic function and to check solutions to problems that were solved algebraically. Similarly, students will use a graphing calculator to observe that a circle is not a function, therefore equations for the top and bottom half of the circle must be entered.
MP.6 - Students will use precise language to accurately identify the differences between polynomial functions of even and odd degree, including symmetry, number of turning points, and end behavior at each end of the domain and range.
MP.7 - Students look for the structure of equations that form parabolas and their vertices and directrices and make use of the structure of the equations to graph parabolas. Students will compare that graphs of parent functions with the related functions, discovering that adding two even functions produces an even function, while adding an even function and an odd function does not.

Semester B

Unit 6 Geometric Proof *(approximately 13 instructional days)*

A. *Content Standards:* G.CO.9, G-CO.10, G-CO.11, G-C.3, G-SRT.B.5

This unit will expand students understanding of reasoning and proof. Students will learn about parallel and perpendicular lines, transversals, and angle relationships. They explore using constructions for bisectors. The students will apply the properties of parallelograms.

Major Topics:

- Slopes and equations of parallel and perpendicular lines
- Isosceles and equilateral triangles
- Conditions for rectangles, rhombuses, and squares
- Theorems of parallelograms
- Special segments of triangles

B. Unit Assignment(s):

Students will complete a performance task by analyzing the shape of a park. Critical skills include modeling real-world situations and applying knowledge of angle measurements in triangle and polygons. Students are given a map of an area that includes a park bounded on all sides by roads. Several different angles are formed by the intersection of the surrounding streets and some are included on the map. Students are to find the measurements of the remaining angles and explain in detail using geometric arguments how they determined their answers.

Mathematical Practices used in Unit 6

MP.2 - Students reason abstractly and quantitatively to apply construction tools to a map to determine actual distances between locations.

MP.3 - Students use proofs to prove theorems of lines, angles, triangles and quadrilaterals. Students discuss and argue the benefits of a variety of construction tools for a given figure, including tracing paper, compass and straightedge, geometry software, protractor and reflective devices, and check the accuracy of their constructions.

MP.5 - Students use compasses, straightedges, and technology to practice and review constructions from Integrated I (parallel lines, perpendicular bisectors, angle bisectors, congruent triangles, etc.) Students will also use a variety of tools to create polygons with given conditions.

MP.6 - In student-to-student discussions, precise vocabulary will be used to describe the angles formed by two coplanar lines intersected by a transversal.

Unit 7 Similarity and Right Triangles *(approximately 17 instructional days)*

A. *Content Standards:* G-SRT.1a, G-SRT.2, G-SRT.3, G-SRT.4, G-SRT.5, G-SRT.6, G-SRT.8, G-GPE.6, F-TF.8

Students will learn about similarity and dilations. They will solve problems and work with proofs involving similar triangles. Students will solve problems using trigonometric ratios.

Major Topics:

- Similarity of circles
- Corresponding parts of similar figures
- Dividing segments in a given ratio
- Using sine, cosine, and tangent
- Using laws of sines and cosines to solve right triangles

B. Unit Assignment(s):

Upon completion of this unit, students will complete a Math in Careers task by using similarity to make a calculation for a special effects engineer. Critical skills include modeling real-world

situations and using similar figures to find missing measurements. Students will assist the engineer as he/she is creating a digital scene for a movie that involves a totem pole and a six-foot-tall man standing next to it. Students will create an image with given information and then use the image to find the length of the shadow that the engineer needs to create for the totem pole.

Mathematical Practices used in Unit 7:

MP.3 - Students will construct viable arguments and critique the reasoning of others by explaining in groups or with a partner whether or not two triangles are similar. If they are similar, the students will determine the scale factor.

MP.4 - Students will use drawings as models to discuss how similar triangles can be used to find the heights of building or trees that are difficult to measure directly.

MP.5 - Students will use tools of construction (compass and straightedge, protractor, geometry software, graph paper, etc.) to explore properties of dilation in lines and polygons.

MP.2 - Students reason abstractly and quantitatively as they derive the formula for the area of a triangle by recognizing the relationships that occur within the triangle as the altitude is constructed.

Unit 8 Understanding Probability *(approximately 12 instructional days)*

A. *Content Standards:* S-CP.1, S-CP.2, S-CP.4, S-CP.8, S-CP.9, S-MD.6

Students will expand their learning about probability and set theory. They will explore the topics of permutations and combinations. Students will use probability in making and analyzing decisions.

Major Topics:

- Probability-set theory, permutations, and combinations
- Mutually exclusive and overlapping events
- Conditional probability
- Independent and dependent events
- Bayes' Theorem

B. Unit Assignment(s):

After completing this unit, students will participate in a performance task by tracking the percentages of a population that are afflicted by infections. Critical skills include modeling real-world situations and apply knowledge of overlapping events. Student can see how epidemiologists use mathematics on the job. Students are given information about two different infectious agents. Each agent must be treated differently and with a new treatment if the patient has been infected with both agents. Students will determine the probability that people will be healthy, have the first affliction, have the second affliction, or have both afflictions.

Mathematical Practices used in Unit 8:

MP.2 - Students will reason abstractly and quantitatively by decontextualizing data into Venn Diagrams, tree diagrams, tables, Punnett Squares, equations and probability notation.

MP.4 - Students will use manipulatives to model a situation as well as notation to model a real-world probability example. Students will relate the set notation used for P(A or B) and

P(A and B), as well as how they are represented on Venn Diagrams for both mutually exclusive and overlapping events.
MP.4 and 5 - Students will use a random number generator (internet or calculator) to simulate probability.
MP.6 - Throughout the unit, students will use precision in vocabulary and notation to discuss set theory, unions, intersections, subsets, complements, etc., and their meanings as defined in a given set.
MP.8 - Students will look for and express regularity in repeated reasoning by following the pattern in events that are independent. By finding P(A), P(B), and P(A and B) and comparing the values, students will determine that P(A and B) = P(A) * P(B) if and only if A and B are independent events.

Unit 9 Measurement and Modeling in Two and Three Dimensions *(approximately 14 instructional days)*

A. Content Standards: G-GMD.1, G-GMD.2, G-GMD.3, G-GMD.4, G-GMD.5, G-MG.1

Students will learn about cross sections and solids of rotation. They will utilize formulas to calculate the surface area of prisms, cylinders, pyramids, cones, and spheres. Students will work with geometric probability and scale factor. They will explore the differences between Euclidean and spherical geometry.
Major Topics:

- Cross sections
- Solids of rotation
- Surface area
- Modeling and density
- Problem solving with constraints

B. Unit Assignment(s): At the end of this unit, students will complete a Math in Careers task relating to a scale model of a sphere. Critical skills include finding a scale factor and applying knowledge of spherical geometry. Students will experience how a model maker uses mathematics on the job. Students are to create a scale model of a sphere to have a given volume. Students will determine the required scale factor to use. They will then determine the area of a triangle drawn on the model given the angle measures.

<u>Mathematical Practices used in Unit 9:</u>
MP.1 - Students will make sense of problems when they use multiple formulas to find the surface area and/or volume of composite solids.
MP.2 - Students must reason abstractly to visualize the cross sections of solids. Students relate the properties of three-dimensional figures (faces, vertices, and intersections of planes) and their rotational symmetry to help identify cross sections of solids. Students will use their knowledge of surface area of spheres and scale to find actual distances on the globe.
MP.4 - Students represent real-world problems with mathematical models when they find the density of real-life objects as the weight or mass per unit of volume, and extend that idea to density to population density, or the population of a region per unit area of the region.

MP.8 - Students will decompose solids into two-dimensional shapes for find surface area and determine shortcuts (formulas) for finding surface area of prisms and cylinders. Students will apply their knowledge of two- and three-dimensional figures to describe the transformation of figures by scale factor a.

Unit 10 Polynomial Functions, Expressions and Equations *(approximately 15 instructional days)*
A. Content Standards: F-IF.4, F-IF.5, A-APR.1, A-APR.2, A-APR.5, A-SSE.2, A-APR.6
Students will learn about transforming function graphs and inverses of functions. They will perform operations on polynomials. Student will expand their ability to solve equations by finding rational and complex solutions.
Major Topics:

- Transformations and inverses of functions
- Graphing cubic and polynomial functions
- Binomial theorems
- Factoring and dividing polynomials
- Finding rational and complex solutions of polynomial equations

B. Unit Assignment(s):
After completing this unit, students will complete a performance task by evaluating and subtracting quadratic functions representing the labor force of the United States. Critical skills include evaluating polynomial functions and operations with polynomials. Using data from the U.S. Census Bureau, the students will be provided with two functions for approximating the labor force, one for the total number of workers and one for the number of female workers. First, students will use the functions to calculate the two estimates for the number of workers. Next, students will work in pairs to write a polynomial function that models the number of male workers and will explain to another pair how they found their function. Finally, students will discuss, as a class, alternative strategies for determining the number of males without using the function they wrote.
<u>Mathematical Practices used in Unit 10:</u>
MP.1, MP.3 - Students will make sense of problems and persevere in solving them and construct viable arguments and critique the reasoning of others as they discuss factored form of a polynomial, describe the graphs of polynomials, and develop their understanding of polynomial division.
MP.2, MP.7, MP.8 - Students will reason abstractly and quantitatively, look for and make use of structure, and look for and express regularity in repeated reasoning as they make connections between the transformations of quadratic functions and transformations of polynomial functions. Students will also understand the relationship between a function and its inverse.
MP.7 - Students will also look for and make use of structure as they draw graphs of polynomials, and use polynomial division to determine factors of polynomials and how those factors relate to the zeros of the function.

Unit 11 Rational Functions, Expressions and Equations *(approximately 8 instructional days)*

A. Content Standards: F-IF.7d, A-APR.7, A-REI.2

Students will learn about graphing rational functions. They will perform operations on rational expressions. Students will graph and solve rational equations.

Major Topics:

- Graphing simple and complex rational functions
- Adding, subtracting, multiplying and dividing rational functions
- Solving rational equations

B. Unit Assignment(s):

After finishing this unit, students will complete a Math in Careers task by writing, analyzing, and graphing a function representing the concentration of acid in a mixture. Critical skills include representing real-world situations using rational functions, determining domain and range, and interpreting asymptotes. Students will write a rule for a function that represents a specific acid and water mixture. Students will determine a reasonable domain for their function and explain to a partner. They will graph the function labeling the axes with the quantities they represent and indicate the axis scales. With a partner, a student will analyze the function's rule to determine the vertical and horizontal asymptotes and determine their relevance or irrelevance.

Mathematical Practices used in Unit 11:

MP.7, MP.8 - Students will look for and make use of structure as well as look for and express regularity in repeated reasoning as they connect multiplication and division of fractions to that of rational expressions, and make connections between adding and subtracting fractions and adding and subtracting rational expressions.

MP.7 - Look for and make use of structure as they graph rational functions with the parameters a, b, h, and k.

MP.3 - Students will construct viable arguments as they solve rational equations graphically to find the zeros of the function, and algebraically by rewriting an equivalent polynomial equation to solve the original rational equation.

Unit 12 Radical Functions, Expressions and Equations *(approximately 9 instructional days)*

A. Content Standards: F-BF.4a, F-IF.7b, N-RN.1, N-RN.2, A-REI.2

Students will learn about inverses of quadratic and cubic functions. They will graph square and cube root functions. Students will simplify and solve radical equations.

Major Topics:

- Inverses of simple quadratic and cubic functions
- Graphing square and cube root functions
- Radical expressions and rational exponents
- Simplifying radical expressions
- Solving radical equations

B. Unit Assignment(s):
Upon finishing this unit, students will do a performance task in which they will take on the role of a nutritionist. Students will find a quadratic function that models BMI data. Critical skills include fitting a function to data, finding the appropriate domain and range, and finding the inverse of a function. Given the median BMI measures for a group of boys, students will create a scatter plot for the data. They will find a quadratic regression model for the data and explain their model. Students will then work with a partner to determine the domain and its restrictions for both the data set and its inverse. Lastly, students will graph the inverse of the function and determine what it models.
Mathematical Practices used in Unit 12:
MP.1 - Students will make sense of problems and persevere in solving them as they use various strategies to solve radical equations, and to obtain extraneous solutions.
MP.4 - Students will explore how a function and its inverse can both model a given real-world situation.
MP.7 - Students will look for and make use of structure about how the various parameters, a, b, h, and k, affect the graph of a square-root and cubic function, in relation to a quadratic and cubic function respectively.

Unit 13 Properties of Circles *(approximately 11 instructional days)*
A. Content Standards: G-C.1, G-C2, G-C.3, G-C.5, G-GMD.1, G-GPE.1
Students will learn relationships among inscribed angles, radii, chords, secants and tangents. They will derive and apply the formula for arc length and sector area and convert degree to radian measure.
Major Topics:

- Central, inscribed and circumscribed angles
- Relationship of radii to chords and tangents
- Tangents to a circle
- Radian measure
- Arc length and sector area

B. Unit Assignment(s):
After finishing this unit, students will complete a Math in Careers task by using knowledge of the properties of circles in the context of an astronomical event. Critical skills include modeling real world situations and applying theorems about tangents, secants and arc measures in a circle. With a partner, students will create a diagram to make sense of the problem and label the given information. Students will determine the best method to calculate the degree of the arc where the eclipse may be observed. Once the measure of the arc is known, students can calculate the length of the arc.
Mathematical Practices used in Unit 13:
MP.4 - Students will model with mathematics as they apply what they know to solve problems involving circles. For example, students will find the distance between the Space Station and Earth's horizon.

MP.3 - Students construct viable arguments and critique the reasoning of others when they answer the question, “Is it possible for 50% of the Earth’s equator to be within range of a satellite’s signal?”
MP.8 - Students will look for and express regularity in repeated reasoning to understand that the radian measure is the constant of proportionality between the length of an arc of a circle and its radius.