Glendale Unified School District

School Level
High School

Date
*(Meeting date will be typed in **after** Board Approval)*

Department: Mathematics

Course Title: Integrated Mathematics II/IIIA Accelerated A/B

Course Code: 3511D/3512D

School(s) Course Offered: Clark Magnet High School, Glendale High

UC/CSU Approved: Yes, mathematics "c"

Course Credits: 10, Full Year

Recommended Prerequisite: Integrated Math I A/B

Recommended Textbook:

CPM Core Connections
Integrated II
Leslie Dietiker, Michael Kassarjian
CPM Educational Program

CPM Core Connections
Integrated III
Judy Kysh, Evra Baldinger, Michael Kassarjian, Karen Wootton

Course Description
Course overview:
Integrated Mathematics II/IIIA Accelerated is part one of a three-part compacted math series. This course provides students with instruction in all of the course content for Integrated Math II and half of the content of the Integrated Math III. It is designed as the single point of acceleration at the high school level as recommended by the California Mathematics Framework. Students who enrolled in middle school in an accelerated pathway may continue to accelerate into high school. Students who did not accelerate in middle school may choose to do so with this course after completing Integrated Math I. This course is aligned to the California Common Core State standards for high school mathematics and supports the

Standards for Mathematical Practice. With this course, students will develop a deep conceptual understanding of the mathematical relationships and concepts needed to succeed in higher level math courses.

- Key components of Integrated Mathematics II/IIIA Accelerated include:
- A balance of mathematical understanding and skill proficiency
- Problem solving that underscores logical thinking and effective strategies
- Communication and justification of ideas and logical arguments
- Mastery over time so students build broad understanding that deepens multiple ways of seeing and thinking about math tasks
- Effective teamwork so that students practice talking and listening about mathematics

Assessment strategies and tools will include

- Individual Tests
- Team Tests
- Participant Quizzes
- Student Presentations
- Class Observations
- Portfolios
- Homework

Course content:

Semester A

Unit 1: Exploring Algebraic and Geometric Relationships *(approximately 11 instructional days)*

A. *Content Standards:* F-BF.1a, F-IF.4, A-SSE.1a, A-SEE.1a, A-SSE.3a, A-APR.1, G.CO.9, G-CO.10, G-GMD.6

The first unit is an introduction to the course on several levels. The chapter is designed to accomplish several objectives including the development of interest in the students in the study of mathematics and to give students an overview of several of the concepts they will learn in this course, and lastly to review vocabulary, mathematical representations, and ways of organizing data that will assist students throughout the course. Students begin the course by reviewing essential prerequisite concepts and skills involving graphing linear equations, rigid transformations in the coordinate plane, and basic probability. In this chapter, students will be asked to prove relationships, beginning the development of a set of algebraic and geometric theorems that they will build upon and use throughout the course.

Progression of Content: In Unit 1, students will investigate triangle angle and side relationships that will be continue as a thread throughout the course. Students will develop a strong understanding of angle and side relationships through the study of triangle congruence, triangle similarity, trigonometry and special right triangles. Students will also build on the work with area models and diamond problems to factor quadratic expressions using area models. They will complete a quadratic web, moving among tables, equations, graphs and descriptions of quadratic functions. Students will use congruent triangles as a tool to investigate relationships among sides and angles of special quadrilaterals.

B. Unit Assignment(s):

Mathematical Practices used in Unit 1

Attend to Precision as they describe polygons using geometric vocabulary, write complete descriptions of graphs, and identify and justify angle pair relationships.
Look for and Make Use of Structure as they categorize polygons by shared attributes.
Use Appropriate Tools Strategically as they make and test predictions, and represent algebraic expressions using algebra tiles.
Make Sense of Problems and Look for and Express Regularity in Repeated reasoning as they generalize linear and nonlinear patterns, and write algebraic expressions in equivalent forms.
Construct Viable Arguments and Critique the Reasoning of Others as they demonstrate why certain angle relationships are always true.

Sample Activities:

Polygon Bucket - Each group of students will be given a collection of polygons to sort. They will begin with a Venn Diagram with the titles "Quadrilateral" and "Equilateral." After discussing the definitions of the two terms, students will sort their polygons by placing them into the correct spot on a large Venn Diagram. Students will then be given a resource page with the polygons already on the Venn Diagram. Based on the polygons on each circle, students will label the diagram. This activity will culminate with students coming up with their own titles for a new Venn Diagram and placing their polygons in it. Students will exchange their diagrams with other groups and determine the titles of the other team's diagram.
Tiling - Students will be given a resource page with a tiling pattern composed of parallelograms. Using tracing paper, and rigid transformations, students will determine which angles are congruent. Students will color all angles that have equal measure the same color. Then, by zeroing in on a small portion of the tiling pattern, students will see that the corresponding angles are always congruent. Students will further examine corresponding angles formed when lines are not parallel to form a conjecture regarding congruent corresponding angles. Furthermore, students will be given a second tiling pattern with diagonals drawn for each parallelogram. Once again, students will color all angels that have equal measure the same color. After examining a small portion of the tiling pattern, they will make a conjecture and use it to prove the Triangle Angle Sum Theorem.

Unit 2: Justification and Similarity *(approximately 10 instructional days)*

A. Content Standards: G-SRT.1a, G-SRT.2, G-SRT.3, G-SRT.4, G-SRT.5, G-CO.9

This unit reviews the theorems for triangle congruence that students learned in the previous course and then uses dilation to start the study of similar shapes. In the first section, students will review and apply triangle congruence theorems (SSS, SAS, ASA, AAS and HL). Students will focus on the special relationships between corresponding angles and sides of similar figures. Students apply similar triangles to solve everyday problems and then work through more complex proofs.

Progression of Content:
Unit 2 continues to build on the question, "What can I figure out about a triangle?" The study will continue in Unit 6 when students investigate special right triangles and Pythagorean triples. Students will then have a set of tools for solving for unknown parts of triangles. In later units, students will use these tools as they investigate properties of regular polygons, calculate volume and surface area of prisms and pyramids, and solve various application problems.
B. Unit Assignment(s):
Mathematical Practices used in Unit 2
Construct Viable Arguments as they investigate the conditions that prove triangles are congruent; determine whether conditional statements and their converses are true; use proof by contradiction as they prove converses of parallel line theorems; and work with similar triangle proofs.
Attend to Precision as they name congruence relationships and describe transformations, perform dilations and determine missing side lengths of similar figures.
Use Appropriate Tools Strategically as they test for triangle similarity.
Make Sense of Problems and Persevere in Solving Them as they apply properties of similar triangles to model with mathematics.
Sample Activities:
Warm-up Stretch – Each group of students will draw a polygon on a white board and make a point near the polygon. Putting one end of the connected rubber bands on the point, and using the connection of the rubber bands as a "trace" over the original polygon, students will draw a new polygon that is the same shape, not the same size. They will then answer a series of questions regarding the new figure and will make conjectures about their figures. Using paper so they can measure more precisely, students will draw more dilations and make conjectures about similar figures.
George Washington's Nose – After proving the similarity conditions and learning about similarity ratios, students will be asked to find the length of George Washington's nose on Mount Rushmore. On a resource page, they will be given that the measurement of the carving of his face is 60 feet long from his chin to the top of his head. The problem is written so that students are forced to look for information not given in the problem. Some students will use the measurements of their own face, others may use measurements from pictures of Mount Rushmore. The purpose of the problem is to be creative in finding useful data to produce equivalent ratios to solve for an unknown.

Unit 3: Probability and Trigonometry *(approximately 10 instructional days)*
A. Content Standards: G-SRT.6, G-SRT.8, S-CP.1, S-CP.7, S-MD.6
In this unit, students will develop prediction skills as they extend their understanding of probability. Students will use tree diagrams and area models as ways to represent probabilities and sample spaces. Students will formulize methods for computing probabilities of unions, intersections, and complements of events. They will play games of chance to calculate expected value. They will also investigate how the side ratios in a right triangle (called trigonometric

ratios) can serve as powerful mathematical tools that allow them to solve for missing side lengths and missing angle measures in any right triangle.

Progression of Content: In Unit 3, students will extend their understanding of simple probability they learned in middle school. Students will use tree diagrams and area models as ways to represent probabilities and sample spaces. Students will formalize methods for computing probabilities of unions, intersections and complements of events. Students will calculate expected value of random variables.

B. Unit Assignment(s):

Math Practices in Unit 3:

Model with Mathematics as they look at everyday problems and predict outcomes; decide which tool is better for modeling different probabilistic situations—a tree diagram, a systematic list, or an area model; draw diagrams and use trigonometry to solve everyday problems.
Reason Abstractly and Quantitatively as they create representations of problems involving probability.
Make Sense of Problems involving the probabilities of independent events and analyze and make conjectures about the outcomes.
Attend to Precision as they differentiate between unions and intersections.
Look for and Make Use of Structure as they look for patterns around expected value, make conjectures about outcomes, look for patterns in slope triangles and trig ratios, and find patterns involving angles and ratios.

Sample Activities:

The Rat Race – After learning how to set up and use area diagrams to determine probability, students will be given a diagram of a maze for a rat to run. There are three different rooms that the rat can end up in and only one will have cheese in it. The students will first determine the probability of ending in each room. They will then answer questions about the rat and his chances of ending up in each room to determine if he has gotten "smarter" after each run.
How Tall Is It? – After learning the tangent ratio and its connection to slope triangles, students will use a clinometer and tangent ratios to determine the height of the school flagpole, memorial torch and a local mountain.

Unit 4: Factoring and More Trigonometry *(approximately 8 instructional days)*

A. Content Standards: A-SSE.2, A-SSE.3a, A-APR.1, G-SRT.6, G-SRT.7, G-SRT.8

In this unit, students will begin their studies of quadratic functions and expressions; and will also build on their knowledge of linear and exponential relationships. Students will learn how to factor quadratic expressions by reversing the process of multiplying binomials. They will also learn methods for factoring the difference of two squares and perfect square trinomials. Another part of this chapter is using the relationships of the legs and the hypotenuse of right triangles to develop the sine and cosine ratios. Students will model real-world situations using right triangles and solve problems using the trigonometric ratios. They will also be introduced to inverse trigonometric ratios to solve for missing angles.

Progression of Content:
In Unit 4, students will develop a strong understanding of angle and side relationships through the study of triangle congruence, triangle similarity, trigonometry and special right triangles. Students will build on their understanding of similar triangle properties to begin their study of trigonometry. Students use trigonometry throughout the rest of the course as they solve complex problems involving area, perimeter, and surface area and as they develop an algorithm for determining the area of regular polygons.
Students will investigate quadratic functions and develop their ability to move from one representation (table, graph, equation, situation) to another. They will create a web of multiple representations of quadratic functions. They will solve quadratics by factoring and using the Zero Product Property, completing the square, and the Quadratic Formula.
B. Unit Assignment(s):
Mathematics Practices used in Unit 4:
Use Appropriate Tools Strategically as they use algebra tiles and area models to identify ways to rewrite expressions and look for shortcuts and strategies to solve problems.
Look for and Express Regularity in Repeated Reasoning as they generalize a process for factoring trinomial expressions and factoring more complicated quadratic expressions.
Construct Viable Arguments as they justify why some expressions can be factored in multiple ways and consider whether polynomials are a closed set under different operations.
Look for and Make Use of Structure as they discover and justify techniques for factoring special cases.
Model with Mathematics to everyday life as they use two new trigonometry ratios (sine and cosine), and apply their trigonometric tools to everyday problem situations.
Reason Abstractly and Quantitatively as they work backwards to solve problems.
Sample Activity:
Special Quadratics – After learning how to factor a polynomial using generic rectangles, students will be given a set of "special quadratics" that they will factor completely and look for patterns. The special quadratics will include the difference of two squares and perfect squares. They will also determine the characteristics of polynomials that cannot be factored.

Unit 5: Quadratic Functions *(approximately 11 instructional days)*

A. *Content Standards:* F-IF.4, F-IF.5, F-IF.7a, F-IF.8a, F-IF.9, A-CED.2, A-REI.4a, A-REI.4b, A-SSE.1a, A-SSE.1.b, A-SSE.2, A-SSE.3a, A-APR.1, N-CN.1, N-CN.2, N-CN.7, N-CN.8

In this unit, students will learn to make different representations of quadratic functions. These representations will be in the forms of equations, graphs, tables, and situations. They will find efficient ways to make sketches of graphs by exploring the relationships between the graphs and their standard forms. Students will learn to solve quadratic equations through the processes of factoring, Zero Product Property, completing the square, and the Quadratic Formula. Through the study of these different forms of quadratic equations, students will see how these can be used to model everyday situations.

Progression of Content:
In Unit 5, students will broaden their understanding of functions to include quadratic functions and extend their ability to solve one-variable equations to solve quadratic equations. Later in the course, students will extend their study of quadratics by focusing on transformations of functions. They will complete the square to change a quadratic from a standard form to graphing form. They will model and solve real-world problems, such as situations involving projectile motion.

B. Unit Assignment(s):

Math Practices used in Unit 5:

Attend to Precision as they use appropriate vocabulary to describe features of quadratic functions and observe similarities and differences among them.
Reason Abstractly and Quantitatively as they make sense of quadratic relationships represented in graphs, tables, equations, and words.
Look for and Make Use of Structure as they use the Zero Product Property and the features of parabolas to develop methods for creating quick and accurate sketches of quadratic graphs, work backward to determine a quadratic equation given a table of values, make connections between representations of quadratic functions, solve quadratic equations and use the properties of squared quantities to determine the number of real solutions.
Construct Viable Arguments to justify the closure of polynomials or to disprove closure through counterexamples.
Use Appropriate Tools Strategically as they use their knowledge of perfect square trinomials and algebraic properties to rewrite quadratic equations in perfect square form and solve quadratic equations using a variety of methods.
Express Regularity in Repeated Reasoning as they generalize a method for completing the square.

Sample Activities:

Water Balloon Contest – As an introductory lesson to parabolas, students will be given data from a water balloon launching competition using multiple representations. For each set of data, students will make a table and create a graph to represent the path of the balloon. After comparing the tables and graphs, students will determine the winner of both the Longest Distance and the Highest Launch.
Quality Control – Students become a Quality Assurance Representative of a Function Factory. In their position, they need to respond to complaint letters sent to the company from customers who have requested specific details of a parabola and have not received the correct parabola. Students are required to determine the equation of the parabola in standard form that is either described in the letter or graphed in the letter. Using the information given, students will write an equation based on intercept and orientation and will need to rewrite the equation in the requested form.

Unit 6: More Right Triangles *(approximately 9 instructional days)*

A. Content Standards: G-SRT.4, G-SRT.5, G-SRT.6, G-SRT.8, G-SRT.8.1, F-TF.8, N-RN.1, N-RN.2, N-RN.3, N-CN.1, N-CN.8, N-CN.9, A-CED.2, A-SSE.3c, A-REI.4, A-REI.4a, F-IF.8b, F-BF.1a, F-IF.4, F-IF.5, S-CP.5, S-MD.7

In this unit, students will continue to study the powerful similarity and side ratio relationships in right triangles by using the Pythagorean Theorem. They will explore relationships in special right triangles that can give them more ways to determine missing side and angle measures using the 30º-60º-90º and 45º-45º-90º triangles. Students will complete several projects and activities that will help them synthesize their understanding and make connections between different concepts they have learned so far.

Progression of Content:
In Unit 6, students will develop a set of tools to determine missing side lengths and angle measures of triangles when sufficient information is given and the triangle is not ambiguous. In later units, students will use these tools as they investigate properties of regular polygons, calculate volume and surface area of prisms and pyramids and solve different application problems.

B. Unit Assignment(s):

<u>Mathematics Practices used in Unit 6:</u>
Look for and Make Use of Structure as they use patterns to discover relationships in special right triangles and use special right triangle relationships to find exact values for trig ratios.
Attend to Precision as they communicate these new relationships.
Express Regularity in Repeated Reasoning as they look for patterns to identify Pythagorean Triples as well as generalize and apply patterns for special right triangles and extend the properties of integer exponents to fractional exponents.
Model with Mathematics as they apply similarity and inverse trigonometric ratios to problems in everyday life.
Make Use of Structure as they apply the technique of completing the square to a general quadratic equation and investigate the closure of rational numbers under different operations.

<u>Sample Activity</u>:
At Your Service - Students will apply similarity and inverse trigonometric ratios to solve a real world problem involving a tennis serve. A player is determined to figure out the height at which she must hit a tennis ball so that it crosses over the net and lands in the service box. Students are given the dimensions of a tennis court, the height of the net, and the height of the tennis player. They will need to decide if it reasonable for the player to hit the ball so that it will just clear the net and land in the service box. They will also determine the angle at which the ball hits the ground.

Unit 7: Proof and Conditional Probability *(approximately 9 instructional days)*

A. Content Standards: G-SRT.4, G-SRT.5, G-SRT.6, G-SRT.8, G-SRT.8.1, F-TF.8, N-RN.1, N-RN.2, N-RN.3, N-CN.1, N-CN.8, N-CN.9, A-CED.2, A-SSE.3c, A-REI.4, A-REI.4a, F-IF.8b, F-BF.1a, F-IF.4, F-IF.5, S-CP.5, S-MD.7

The first section of this unit weaves together the concepts of congruent triangles, proof and quadrilateral
models, and two-way tables as methods of displaying probabilities.

Progression of Content:

In Unit 7, students will develop and complete a set of justification tools so that they can create proofs for future geometric discoveries. Students will use their understanding of congruent triangles to find the areas of regular polygons and discover and prove new facts about circles.

B. Unit Assignment(s):

Math Practices used in Unit 7:

Construct Viable Arguments as they apply congruent triangle properties to prove characteristics of quadrilaterals; analyze, conjecture, and plan solution pathways for proofs about rhombi; investigate conditional probability, independence, and association; and explain their thinking about independent situations.

Make Sense of Problems as they analyze geometric figures and make conjectures.

Attend to Precision as they use the vocabulary of probability.

Reason Quantitatively as they make the transition from two-way tables with frequencies (counts) to relative frequency (probability) two-way tables.

Use Appropriate Tools Strategically as they decide which probability tools to use to solve problems.

Sample Activity:

Explore-Conjecture-Prove – Using the model of Explore-Conjecture-Prove, students will learn and prove properties of kites and rhombi and the Triangle Midsegment Theorem. Through this model, students will move from flowchart proofs to two-column proofs.

Semester B

Unit 8: Polygons and Circles *(approximately 9 instructional days)*

A. Content Standards: G-SRT.8, G-C.1, G-C.3, G-C.5, G.CO.10, G-GMD.1, G-GMD.5

This unit extends students' work with quadrilaterals and triangles to focus on the angles, area, and perimeter of regular polygons with any number of sides. This work will lead to circles by informally considering the limit of a regular polygon as the number of sides as they approach infinity. The first section begins with constructions. By creating and justifying the constructions, students are required to connect back to concepts from several earlier chapters. They will build a concrete understanding of the role that a radius plays in a circle. In the second section, students will use triangle angle sums and linear pair relationships to make discoveries about the interior and exterior angles of polygons. Using these new tools, students will develop a strategy to calculate the area of a regular polygon with any number of sides. In the last two sections, students examine the relationships between areas of similar figures leading to the

discovery that the ratio of the areas between similar figures is equal to the square of the linear scale factor. This leads to the development of the area and circumference formulas for a circle.

Progression of Content: Students will work with constructions to review and reinforce prior knowledge about triangles, while preparing for investigations of circles, sectors, and arcs. Students will develop a strong understanding of the circle as the set of all points in a plane equidistant from a fixed point. This work lays the foundation for students to write equations of circles in the coordinate plane and study properties of angles, arcs, chords, secants and tangents in more depth.

B. Unit Assignment(s):

Mathematics Practices used in Unit 8:

Students will Use Appropriate Tools Strategically as they work with constructions.
Construct Viable Arguments as they are asked to explain, justify and prove their answers.
Attend to Precision as they describe their findings using new vocabulary.
Use Repeated Reasoning to generalize a process for determining the sum of the interior angles of a polygon.
Look for and Make Use of Structure to calculate individual interior and exterior angle measures in regular polygons; generalize a process for calculating the area of any regular polygon; investigate the area of regular polygons and think about what happens as the number of sides approaches infinity; and investigate the area of sectors and length of arcs using understanding of area and circumference of a circle.
Express Regularity in Repeated Reasoning as they look for patterns and discover the r : r^2 relationship.
Reason Abstractly and Quantitatively as they make sense of given conditions and apply their reasoning to other situations.
Model with Mathematics as they apply their knowledge of perimeters and areas of similar shapes to everyday situations.

Sample Activities:

Go, Rowdy Rodents! – After developing a method to calculate the measures of interior and exterior angles in any regular polygon, students will be given a task that allows them to explore how to find the area of any regular polygon. They are told that the principal has asked them determine the amount of glass needed to make a stained-glass window, with a picture of the school mascot, whose shape is a regular octagon. They will also determine the cost of the window, which involves determining the cost of the glass as well the cost of the polished brass trim that will surround the window.
The Cookie Cutter –
diameter is used. Comparing their results, students will write a letter describing their results.

Unit 9: Modeling with Functions *(approximately 12 instructional days)*

A. *Content Standards:* A-SSE.1a, A-SSE.3a, A-SSE.3b, A-CED.1**,** A-CED.2, A-CED.3**,** A-CED.4, F-IF.4, F-IF.5, F-IF.6, F-IF.7a, F-IF.7b, F-BF.1a, F-BF.1b, F-BF.3, F-BF.4a, F-LE.3, A-REI.7

In this unit, students will build on what they learned in previous chapters about quadratics and solve quadratic equations and develop fluency with different representations of quadratic functions. They will extend their understanding of linear systems and inequalities to solve systems, and inequalities involving quadratic relationships. Finally, they will focus on the characteristics of different functions, build new functions and explore the rate of growth of different functions.

Progression of Content:
Students will complete the square to rewrite the equations of circles not centered at the origin in graphing form. Students will learn the geometric definition of a parabola and graph parabolas using the focus and directrix.
B. Unit Assignment(s):
Mathematical Practices used in Unit 9:
Model with Mathematics as they write the equation of a function to fit data; use the function to make predictions and connect it to other mathematical ideas; represent situations with quadratic inequalities and use their models to answer questions; calculate and interpret the average rate of change in situations involving projectile motion; and move between real-world contexts, graphs, and equations.
Look for and Make Use of Structure as they experiment with transformations of parabolas and generalize their findings; apply the technique of completing the square to rewrite quadratic equations in graphing form; apply their knowledge of solving linear systems and graphing linear and quadratic functions to solve nonlinear systems of equations; and use graphing technology and algebraic skills to investigate the growth patterns of different functions.
Express Regularity in Repeated Reasoning as they apply the principles of transforming a function to absolute value functions.
Attend to Precision as they identify functions and specify the domains to write the equations of piecewise-defined functions.
Look for and Make Use of Structure as they investigate combinations of functions, and reverse the order of operations to create inverse functions.
Sample Activities:
Parabola Lab – As part of a Parabola Department of a Function Company, using technology, students will investigate how to move a parabola vertically and horizontally, change the orientation of the parabola and vertically stretch or compress the parabola. After determining the general equation of a parabola, students will extend the pattern to other families of equations.
Making Cranes – Two students have volunteered to make 1000 origami cranes for their teacher. One is experienced and works at a steady rate, the other is learning and getting faster as he makes more. To determine if they will meet their goal of making 1000 cranes during spring break, students will first write a piecewise function to model the number of cranes made over time by each person, then they will need to combine the functions to answer the final question.

Unit 10: Circles and More *(approximately 9 instructional days)*

A. Content Standards: A-REI.7, G-GPE.1, G.GPE.2, G-SRT.5, G-C.2, G-C.3, G-C.4, G-C.5

This unit builds on students' understanding of circles and triangles to develop the equation of a circle. Students also discover, prove and apply relationships among angles, arcs, chords, and tangent lines. Students will apply their geometric knowledge of circles to the studies of circles on a coordinate plane, leading them to use the Pythagorean Theorem to derive the equation of a circle.

Progression of Content:

Students will use a compass and the derivation of the circle equation to develop a strong understanding of a circle as the set of all points in a plane equidistant from another fixed point. This work lays the foundation for study of other conic sections in later courses.

B. Unit Assignment(s):

Mathematics Practices used in Unit 10:

Construct Viable Arguments and Critique the Reasoning of Others as they prove that a line bisecting any arc of a circle will pass through the center of that circle; discover the relationships between inscribed angles and the arcs they intercept; and prove relationships between parts of circles.

Look for and Express Regularity in Repeated Reasoning as they use the Pythagorean Theorem to determine the equation of a circle; prove relationships among segments, arcs, and angles formed by chords and tangents and then use these relationships to solve mathematical problems.

Look for and Make Use of Structure as they learn how to change the equation of a circle from one form to another and investigate parabolas.

Use Appropriate Tools Strategically as they construct special segments and discover other properties of chords and arcs.

Model with Mathematics as they apply their knowledge to solving problems about Earth.

Sample Activities:

The World's Widest Tree – In order to show the students the need to learn more about a circle, students are to take the role of an archaeologist who has just found a portion of the trunk of a tree. To determine the width of the tree, the diameter is needed, but since only a portion was found, students will need to use a chord to find the diameter of the tree trunk.

Eratosthenes' Remarkable Discovery - Students replicate Eratosthenes' method to calculate the earth's radius, and the circumference of Earth at a time when most people thought the world was flat, but he was convinced it was round. To do this, students use the properties of parallel lines along with the properties of circles learned so far.

Unit 11: Solids *(approximately 7 instructional days)*

A. Content Standards: G-GMD.1, G-GMD.3, G-GMD.5

In this unit, students will study three-dimensional solids and their volumes and apply knowledge of similar figures to three-dimensional figures. They will study surface area and volumes of prisms and cylinders. Students will also expand their study of solids to pyramids, cones, and spheres. They will apply their newly acquired mathematical knowledge to many real-world applications.

Progression of Content:

Students who continue on to calculus will use their knowledge of cross-sections to find the area of a two-dimensional cross sections and use it to calculate the volume of irregular solids using integration. Furthermore, the volume of solids is used in related rates problems in calculus. Once in calculus, students also do the derivation of the volume formulas through integration.

B. Unit Assignment(s):

Mathematical Practices used in Unit 11:

Look for and Make Use of Structure as they make connections to previous work with volume of prisms and connect those ideas to other ways to calculate surface area and volume; discover the patterns that will lead to the discovery of the r-r^2-r^3 relationship; consider their previous knowledge about pyramids and relate it to cones; and investigate and generalize relationships among volumes and surface areas of cones, spheres, and cylinders.

Attend to Precision as they collect information about volumes with different scale factors and consider units of measure in their solutions.

Model with Mathematics as they apply their knowledge of three-dimensional shapes to solve everyday problems and choose a tent shape based on desired attributes.

Reason Abstractly and Quantitatively as they compute the volume of pyramids and make sense of the meaning of the formulas for volume.

Sample Activities:

How Does the Volume Change? - After developing methods to find the area and volume of prisms and cylinders, students will be given several scenarios where they will first have to determine if they need to find volume or area of a solid, and then will need to use similar figures to determine the amount of paint needed to
paint a statue of Benjamin Franklin and how much oil can be stored in storage tanks.

Happy Birthday! – Given a sample party hat, students will need to determine the size and shape of the paper that forms the hat. After determining the amount of paper needed for one hat (in square inches), they will have to figure out the total amount of paper needed to make a hat for each person in the class.

Unit 12: Investigations and Functions *(approximately 10 instructional days)*

A. *Content Standards:* F-IF.4, F-IF.7b, F-BF.1, A-APR.1

This unit starts a focus on investigation and justification that continues throughout the course as students formulate and investigate mathematical questions and create logical and convincing

arguments to support their findings. Students use a graphing calculator to create multiple representations of a function, and review how to fully describe the graph of a function using precise mathematical language. Students are also introduced to the way a parent graph and parameters define a family of functions. Modeling mathematical problems is a big emphasis from the start.

Progression of Content:
The investigation strategies students have developed throughout the course, especially in this unit, will continue to be used and further elaborated in future units as they study logarithmic, inverse, polynomial, and trigonometric functions.
B. Unit Assignment(s):
Mathematics Practices used in Unit 12:
Look for and Make Use of Structure as they determine which inputs and outputs are possible for each type of function.
Construct Viable Arguments and Critique the Reasoning of Others as they determine the order for the function machines.
Use Appropriate Tools Strategically as they learn features of the graphing calculator.
Attend to Precision as they graph functions with asymptotes and use proper vocabulary when describing statistical data.
Look for Regularity in Repeated Reasoning as they investigate a family of functions by changing a parameter.
Reason Abstractly and Quantitatively as they model the relationship between height and volume of rectangular prisms.
Sample Activities:
Function Exploration - Each group of students will be given a radical function to fully investigate. They will make a complete graph and describe it using the following attributes: shape, line of symmetry, asymptotes, increasing or decreasing, x- and y-intercepts, domain and range, endpoints, maximum or minimum points, continuous or discrete, and whether or not it is a function. After all graphs have been presented to the class, a discussion will follow that will discuss the similarities and differences between the graphs.
Open Box - Modeling a geometric relationship, each group of students will be given six equally sized sheets of grid paper. They will cut the corners from the paper and fold it to make a box without a lid. After making several boxes, students will use multiple representations (table, diagrams and graph) to determine which box has the greatest volume. Students will then generalize their results by writing an equation to represent the volume with height x. Using technology, students will find the height of the box with the largest possible volume.

Unit 13: Transformations of Parent Graphs *(approximately 11 instructional days)*
A. *Content Standards:* F.BF.1, F.BF.3, F-IF.4, F-IF.5, F-IF.6, F-IF.7b, F-IF.7e, A-CED.2, A-SSE.1b, G-GPE.3.1

In this unit, students learn how to generate families of functions from parent functions. Students develop a general equation of form f(x) = a(x – h)^2 + k for the family of quadratic functions and learn to graph a parabola quickly by identifying its orientation, vertical stretch (or compression), and vertex. Students then continue to generalize families of functions by applying the same kinds of transformations to other parent functions, describing the role of the locator point (h, k) for each family of functions.

Progression of Content:
The idea of families of functions will be revisited several times throughout this course and the next. Each time students are introduced to a new parent functions (e.g., inverses and logarithmic functions in Unit 16), they will be asked to graph members of its family and write an equation in graphing form for the family. The members of a family of functions are all related to a parent function and to each other by a sequence of transformations. As students gain familiarity with the properties of new functions, they will build their ability to choose the appropriate function to model a particular relationship.
B. Unit Assignment(s):
Mathematical Practices used in Unit 13:
Look for and Make Use of Structure when they graph quadratic functions and rewrite the equations of quadratic functions from standard form to graphing form; make connections between the transformations of parabolas and other parent graphs; apply knowledge of parabolas and other parent functions to identify the locator point (h, k) for different families of functions; explore odd and even functions; and complete the square for equations of parabolas and circles and identify the vertex or center and radius.
Model with Mathematics as they write quadratic functions to represent relationships, check the reasonableness of their answers, and make predictions.
Look for and Express Regularity in Repeated Reasoning as they explore transformations of graphs that are not functions.
Sample Activities:
Transforming Other Parent Graphs - Having transformed quadratic equations earlier in the chapter, students will now discover the transformations of five other parent graphs. Each group will organize their work into a poster that clearly shows: each parent graph, examples of transformations and each equation in graphing form. As a challenge for the other groups, each poster will also show a graph for which other teams need to write the equations and will give an equation for each of the other teams to graph.

Unit 14: Solving and Inequalities *(approximately 9 instructional days)*

A. *Content Standards*: A-APR.4, A-REI.2, A-REI.11, A-SSE.1b, A-SSE.2, A-CED.2, A-CED.3, F-BF.1, F-IF.4

In this unit, students are asked to think about or visualize the kinds and number of solutions that an equation, inequality, or system of equations or inequalities might have. Another main focus is the application of equations, inequalities, and systems to solve problems. Students will

use graphing as a powerful method for solving equations and systems as well as for visualizing the solutions, then reverse the process, when given solutions and asked to visualize the graphs.

Progression of Content:
Students will return to the focus on solving and solutions in Integrated Math IIIB/PreCalculus Accelerated when they extend their ideas to solving systems of equations with three variables. One benefit of solving equations by graphing is that students soon face equations that they cannot solve using algebraic methods, so they need to use graphing to determine a solution. Graphing becomes a very powerful mathematical tool that students can use to solve polynomial and trigonometric equations in the next course.
B. Unit Assignment(s):
Mathematics Practices used in Unit 14:
Look for and Make Use of Structure as they choose methods for solving linear and nonlinear equations and inequalities.
Attend to Precision when determining and verifying solutions graphically and algebraically.
Model with Mathematics as they use systems of equations to model and analyze situations, including problems with constraints to determine an optimal solution.
Sample Activity:
How Tall is Harold? After learning to solve systems of equations both algebraically and graphically, students are given a scenario in which foods in a food fight hit Harold in the head. Given information about the flight of the food and Harold's distance from the food, students will model both graphically and algebraically and from their models, they will determine Harold's height.

Unit 15: Normal Distributions and Geometric Modeling *(approximately 12 instructional days)*
A. Content Standards: S-IC.1, S-IC.2, S-IC.3, S-IC.4, S-IC.5, S-IC.6, G-GMD.4, G-MG.1, G-MG.3
In this unit, students will begin their studies of the fundamentals of designing studies and experiments, as well as their limitations. The importance of random sampling for studies and random assignment for experiments is stressed. Students begin by looking at surveys, and gain experience with two common sources of bias in those surveys. Students will next perform an experiment to help them compare and contrast experiments with observational studies (such as opinion surveys), stressing distinctions between experiments and observational studies. Relative frequency histograms will record the data, and will be modeled with normal distributions. Finally, students focus on geometric modeling, looking at cross-sections and solids of revolution.

Progression of Content:
Designing studies and experiments and using normal density curves as models to calculate probabilities is important for unit 17. In unit 17, students begin to explore inferential statistics, using samples to make predictions about populations.

B. Unit Assignment(s):
Mathematics Practices used in Unit 15:
Construct Viable Arguments and Critique the Reasoning of Others as they write research questions and consider issues of bias, explore convenience sampling and try to incorporate some level of random selection into their own sampling for their survey, and explore the differences between observational studies, surveys, and experiments.
Model with Mathematics as they explore randomly selected samples and samples selected intentionally, explore the impact of a lurking variables, and compare relative frequency histograms to normal probability density models.
Use Appropriate Tools Strategically as they create relative frequency histograms and explore statistics with their calculator.
Reason Abstractly and Quantitatively as they continue to explore normal distributions and predict percentiles.
Sample Activities:
Conclusions from Studies - After learning about survey design, samples, randomness and bias, students will write their own survey questions. After gathering data, they will use histograms, percentile, and a normal probability density function to determine if their data is valid. They will then be given an opportunity to run an experiment, adjust the experiment, and re-evaluate their data.
How Can I Get That Cross-Section? - In order to show students that volumes of solids can be found using crosssections, they are first asked to slice a cube to get different cross-sections. Students will be given a glob of clay that they must first make into a cube. Using dental floss to slice the cube, students will slice the cube to get 4 different cross-sections: a square, a rectangle that is not a square, a triangle, and a hexagon.

Unit 16: Simulating Sampling Variability *(approximately 8 instructional days)*
A. Content Standards: S-IC.1, S-IC.2, S-IC.4, S-IC.5, S-IC.6, S-MD.6, S-MD.7
This unit introduces students to techniques for computing complex probabilities through simulations. Students then investigate the concept of natural variation in samples and how that variation can be modeled and controlled through sample size. Students also begin working with inferential statistics and statistical hypothesis testing using margin of error. Students will develop an understanding of counterintuitive probability problems by using area models or tree diagrams.

Progression of Content:
This unit provides an introduction to inferential statistics. Students will make statements about populations based on information obtained from a sample. They will study this topic more extensively if they take a formal course in statistics.

B. Unit Assignment(s):

Mathematics Practices used in Unit 16:

Model with Mathematics as they simulate the probability of a newborn being a boy or girl; simulate the number of streaks they can expect in a random process; take a random sample of candies to understand margin of error; explore sample-to-sample variability by conducting a hypothesis test; and use simulations to determine if a manufactured part is within typical quality specifications.

Attend to Precision as they evaluate the mean and margin of error from a data set, evaluate the results from their simulations, simulate the quality control process for a specific company.

Use Appropriate Tools Strategically as they compare the effects of two treatments in an experiment, evaluate results of simulations, analyze decisions and strategies in situations that are counterintuitive.

Sample Activity:

AIDS in South Africa - Students will showcase their understanding of estimating sample-to-sample variability and conducting a hypothesis test. Given the claim of a drug manufacturing company, students will simulate 100 samples of 125 residents. They will determine the mean, margin of error, and determine if they can support the claim of the drug company. Students are asked if their margin of error is reasonable and then tell what must be done to make the margin of error smaller.

Unit 17: Counting and Closure *(approximately 9 instructional days)*

A. *Content Standards:* A-CED.2, A-SSE.1b, F-IF.4, F-IF.5, F-IF.7a, F-IF.7b, S-CP.9, S-MD.6, S-MD.7, G-SRT.8

This unit is about closure for the year, making connections to appropriate tools to use in appropriate scenarios and the counting principle. Students will develop systematic counting methods based on the fundamental counting principle, permutations, combinations, and other counting methods for situations where repetition of outcomes is allowed. A key component of this chapter is requiring students to decide which tools, skills, and strategies to use in each situation above.

Progression of Content:

The content learned in this unit may be further learned and applied in future probability courses.

B. Unit Assignment(s):

Mathematics Practices used in Unit 17:

Look for and Make Use of Structure as they understand general counting problems, permutations, and factorials, and investigate the golden ratio.

Reason Quantitatively as they decide how many items are in a sample space, make sense of counting problem situations and their relationships, and consider different ways to count.

Construct Viable Arguments and Critique the Reasoning of Others as they work to defend their conjectures about the mathematical relationship between permutations and combinations.

Model with Mathematics as they apply their knowledge of functions to model everyday situations.
Make Sense of Problems and Persevere in Solving Them as they apply their knowledge about probability to solve everyday problems.
Sample Activities:
The Ice Cream Shop – In the middle of a heat wave, the local ice cream shop has only five flavors of ice cream remaining. In order to maximize the number of customers they can serve on Friday, customers may order only three scoops of ice cream, either in a cup or a cone. The following day, customers could still order three scoops of ice cream, but no more than one scoop of each flavor. Students will determine the total number of outcomes for this situation using various counting methods.
Zoe and the Poison Weed – Zoe the goat is tethered to a barn in a fenced pen. When tethered, she cannot enter the barn. There is one poison weed in the area where Zoe can graze. Given the rate at which Zoe eats, using geometry, students will determine the probability that she will eat the weed during the time she is in the yard.