Experiences to Help Children Learn to Count On

wo children are playing a board game with their father. The game involves rolling two dice, finding the sum of the two numbers, and then moving that number of places on the board.

Josh, the almost-four-year-old, rolls the dice. He carefully counts the number of dots on each die and reports that he has a five and a four. Then he counts the dots together: "1, 2, 3, 4, 5, 6, 7, 8, 9." He moves nine spaces, one space at a time, to advance his piece on the game board.

Jody, the five-year-old, rolls the two dice, reports that she has a four and a six and counts on from the six—"7, 8, 9, 10"—to get her answer.

Josh is outraged. "She is cheating. She is only counting one of them. It's not fair." Jody patiently explains that she is not cheating and was including the dots on both dice.

She demonstrates to her brother that her answer was right by counting one by one on both dice.

Her explanation calms Josh down for the moment, but on the next turn, Jody again counts on from the larger number to get the answer. Again, Josh protests vigorously.

This anecdote illustrates a key aspect of the development of number sense in the early child-hood years—learning to count on. Fosnot and Dolk (2001) identify *counting on* as a landmark strategy that children discover from their experiences in

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Edited by Andrew M. Tyminski, atyminski@purdue.edu, an assistant professor of mathematics education at Purdue University, West Lafayette, IN 47907. "Early Childhood Corner" addresses the early childhood teacher's need to support young children's emerging mathematics understandings and skills in a context that conforms with current knowledge about the way that children in prekindergarten and kindergarten learn mathematics. Readers are encouraged to send submissions to this department by accessing tcm.msubmit.net. Manuscripts should not exceed eight double-spaced typed pages.

the world and in school. In kindergarten and early in first grade, young children learn to combine two small collections. For example, when adding 4 and 5, counting on makes it possible to start at one numeral and add on the second (Fuson 1982). Learning to count by tens and ones depends on the counting-on strategy. To count three tens and four ones, the child learns to say, "10, 20, 30, 31, 32, 33, 34." Later, in first grade and in second grade, the ability to combine larger collections efficiently relies on counting on. Therefore, the child who has not acquired the counting-on idea by second grade is at a serious disadvantage in learning meaningful arithmetic.

The counting-on strategy is not something that teachers should present to children as a rote procedure. Trying to force children to follow a counting-on rule may be detrimental to the development of mathematical thinking (Kamii 2000). In the anecdote above, Josh cannot understand what Jody is doing to figure out the total rolled on the dice, despite her clear attempt to explain and demonstrate what she has discovered. His concept of number is not sufficiently developed to see why counting on works and why it is an efficient method of operating. Although early childhood teachers should not force this understanding on young children, they can be purposeful in creating experiences that encourage counting on and providing opportunities for children to develop an understanding of this concept at their own developmental pace.

Any meaningful activities that require children to count contribute to building number concepts. The learning experiences described here were selected because they can help young children focus on counting on and provide them with access to the activities even when they are not yet ready to use that strategy.

Daily Routines

Classroom routines have great potential for engaging young children in mathematical thinking. By being deliberate about the situations they create and the questions they ask, teachers can push children's understanding. As Fosnot and Dolk (2001) note, "Teachers need to turn these experiences into real, open dilemmas...." (p. 42).

Calendar questions

Routines based on the calendar and attendance are commonplace in early childhood classrooms. Teachers who ask children to predict what the date will be in three more days or when the class will be taking a trip to a bakery are prompting children to count on using the numerals on the calendar (see fig. 1). If Marta volunteers that her birthday is on the 17th, the natural question is, "How can we figure out how many days it is to your birthday?" By starting at the date on the calendar, Marta counts on the additional days by using the calendar numbers. Calendar questions have real meaning for children, and the calendar's structure encourages counting on as a way to answer these questions.

Many standards-based mathematics programs and textbooks include classroom routines that provide young children with rich counting experiences. Everyday Mathematics (University of Chicago School Mathematics Project 2004) and Investigations in Number, Data and Space (TERC 1998) use techniques such as the counting jar, "today's question," number lines, "number of the day," pocket charts, and hundreds charts to regularly pose counting tasks and problems. As part of their daily routine, children, on arriving in their classroom, should count the number of objects in the counting jar, find a way to represent the information by using pictures or numbers, and then create another collection of objects of the same size. Variations such as increasing the size of the numbers, working with one more or one less than the specified number, having children estimate before counting, and delegating responsibility for filling the jar with a number of their choice lead young children to invent counting strategies such as counting on (Economopoulos and Murray 1998, pp. 68–70).

Snack time

Class helpers are often asked to figure out how many snacks are needed for the students in the class. However, when the snacks are presorted, the class helpers can just distribute them without analyzing the situation. However, as Fosnot and Dolk (2001, pp. 42–44) suggest, having the children model the situation in advance—by using representations for the snacks and the number of children in the class—expands the potential for

Figure 1

A student uses the class calendar to practice counting on.



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Figure 2

When counting out the number of muffins needed at snack time, a student uses muffin cups to represent the muffins.



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meaningful mathematics. If children have to figure out whether there are enough boxes of raisins to serve each child, they are challenged to organize their counting—by using cubes or pictures as representations, for example—and to use more efficient procedures (see fig. 2).

Transitions

When children are walking down the hall or up the stairs, they often count steps on their own. Teachers can capitalize on this natural activity. For example, after the children have walked and counted five steps, stop them and ask how many steps they have gone so far. When the children begin walking again, help them remember that they stopped at five and have them continue counting from five as they move forward. Walking to the playground, cafete-

ria, and bathroom provide numerous opportunities for meaningful counting.

Clean-up time

During clean-up time, many teachable moments arise. Clean-up routines lend themselves to counting experiences in which children can count both objects picked up and time elapsed during cleaning. When the teacher notes that there are six trucks in the bin and asks a child how many trucks there will be when the two trucks the child is holding are put in, the child is encouraged by the question to start counting on from six. Of course, the child may still count the six trucks before counting the additional ones.

Games and Activities

Most early childhood classrooms have a small-group activity time or mathematics learning centers. The following ideas could be used in both contexts.

Dice games

Any activity that requires a child to add randomly generated numbers encourages counting on. Young children learn through experience to visually evaluate the pattern of dots on dice (or dominoes or num-

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ber cards). When a child names the number on one die without counting, he or she is more likely to incorporate the dots of the second by counting on. For the child who recognizes number symbols, this possibility can be maximized by using one die labeled with numerals and one labeled with dots. Children who have not yet invented the counting-on strategy, however, will count the pattern of dots as well as "count" the written symbol. "At first, they may picture in their minds the objects [they are counting]," says

Ginsburg (1989, p. 65). For example, if the number 3 is rolled, a child who does not yet count on says, "1, 2, 3," as she touches three places on the numeral or imagines the dots in her head.

The teacher can further structure the situation by using one regular die and one with only one or two dots on all faces. If the child throws the regular die first and identifies the number thrown, she can more easily count on the one or two dots on the second die. Counting on from small numbers to larger numbers is often the first evidence of the use of the counting-on strategy (Fuson 1992, p. 248). By gradually increasing the number of dots on the

second die, the teacher is supporting the extension of the counting-on strategy to larger numbers.

Using one die with the rule to double the roll is another constraint that may help push the child's discovery. In this case, the dots for both addends must be counted on the same die and so may encourage the child to begin counting at the roll. However, teachers should have counters available for children who need to represent both addends at once.

Bead games

During "choice time," young children are naturally drawn to beads. The teacher's role is to encourage problem solving as the children interact with the materials. By having children string beads in multiples by color, size, or shape, the teacher can encourage counting on. If a child has strung five red beads and five blue beads, the teacher might ask, "How many beads are there altogether? Show me the red beads." Such questioning may lead the child to count on from the red beads. In this case, because young children are so comfortable with fives, the child is more likely to count on. Then the teacher could move to a string with six red beads and six blue beads and so on. By sequencing the questions, the teacher is helping the child generalize the strategy.

Counting money

When young children count pennies, the "pennies" could be any countable object. However, when nickels and dimes are introduced into a problem, the relationship between five pennies and a nickel, for example, is abstract. The use of one coin to represent a group of pennies is challenging for children because they cannot count one by one using a nickel without trading—that is, without substituting five pennies for the nickel: "The coins can become a 'constraint' to counting by ones and therefore may facilitate the development of counting on..." (Fosnot and Dolk 2001, p. 9).

The counting-on strategy makes combining nickels and pennies so much more efficient than trading the nickels and counting all the pennies beginning with one. A young child identifies the value of the nickel in much the same way that he or she visually evaluates the pattern on a domino. If children know the nickel represents five, they start with the five and say, "5, 6, 7, 8" to combine the nickel with three pennies. Because of the real-world significance of coins, children are eager to practice problem solving with coins. By capitaliz-

ing on children's interest and the constraints of the material, the teacher can influence the discovery of the counting-on strategy.

Using a calculator

As with many real-life tools, young children enjoy using calculators. Once a child can recognize written symbols, the numerals on the calculator encourage counting on. For example, the teacher can ask the child to count on from eight on the calculator by asking the child to enter 8 and then +, 1, and = and then say the numeral that shows as he continues to enter the equals sign. The calculator's repeat operation function shows what happens when the child counts on by ones. Although the child may be using only the pattern to count on, the structured, scaffolded experience may help him see the value of the strategy when counting objects.

Hidden or unknown addends

Hiding part of a collection provides a limitation that may lead a child to count on. A teacher tells a child that she is going to hide five counters in a cup, although she does not count out the counters or even show the child the counters. The child knows the cup represents five. The teacher then asks the child to begin at the five and add three more—that is, to count on "6, 7, 8." If the child cannot do this, the teacher allows the child to take the counters out of the cup to count them sequentially ("1, 2, 3, 4, 5") and add on "6, 7, 8." This approach encourages the child to start counting at the five. Using a number card to record the number in the cup may also prompt the child to count on.

A game that is an appropriate algebra activity for the early childhood years—How Many in the Cup?—is an extension of this activity. The game models the missing addend situation. An unknown number of counters is in the cup. The child sees some counters outside the cup (3), knows that the total number of counters inside and outside the cup is (5), and needs to figure out the unknown amount—how many counters are in the cup (2). One child makes a guess and counts on from the number in the cup to the total. Another child counts or quickly evaluates the visible ones and counts on to the given total. Using a recording sheet to keep track of the number of counters in the cup, the number outside the cup, and the total may contribute to the child's learning that counting on is a meaningful strategy (Van de Walle 2007, p. 124).

Conclusion

Counting on is a strategy that young children construct over a period of years as a result of diverse and numerous counting experiences anchored in the world around them. During the early childhood years, their understanding of number is fragile, and learning to count on is not an all-or-nothing proposition. Teachers may see children counting on with small collections and find that this strategy is not yet generalized to larger collections. Children may count on using one concrete material but not others. The context also may determine whether a child uses the counting-on strategy. Sometimes, a child will count on in one activity and then, in the next activity, need to count one by one from the beginning to confirm the number.

Teachers must keep in mind the big ideas and strategies related to number sense as they build on children's natural interests. As Sarama and Clements (2006) note, "High-quality learning is often incidental and informal, but not unplanned or unsystematic" (p. 39). By carefully observing children's counting strategies in various activities, teachers can systematically intervene to extend children's learning. The activities presented in this article are examples of the kind of learning experiences teachers can provide that meaningfully encourage children's development of number concepts in general and the counting-on strategy specifically. As Josh, the younger child in the anecdote, continues to engage in mathematical problem solving over the next few years, he will be ready to understand his sister's shortcut for counting the dice.

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