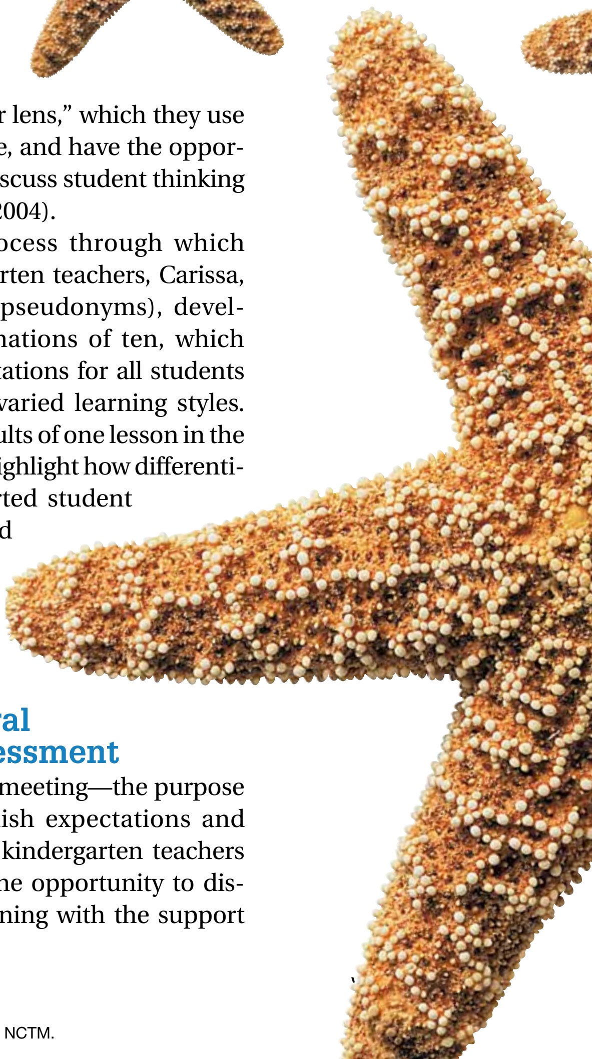
Making My

Young students, new to formal mathematics, come to school with various number experiences. To meet their needs, teachers must understand each student's thinking and tailor instruction appropriately. Differentiation is a vehicle for making this possible. According to Tomlinson (2001), differentiated instruction is student-centered; rooted in ongoing assessment; provides multiple access points; and allows for several approaches to problem solving. Unlike ability grouping, differentiation encourages teachers to maintain high expectations for *all* students while recognizing individual needs (Murata forthcoming). This type of instruction can be challenging, especially for first-year teachers who are focused on classroom management and learning new curricula (Levin, Hammer, and Coffey 2009).

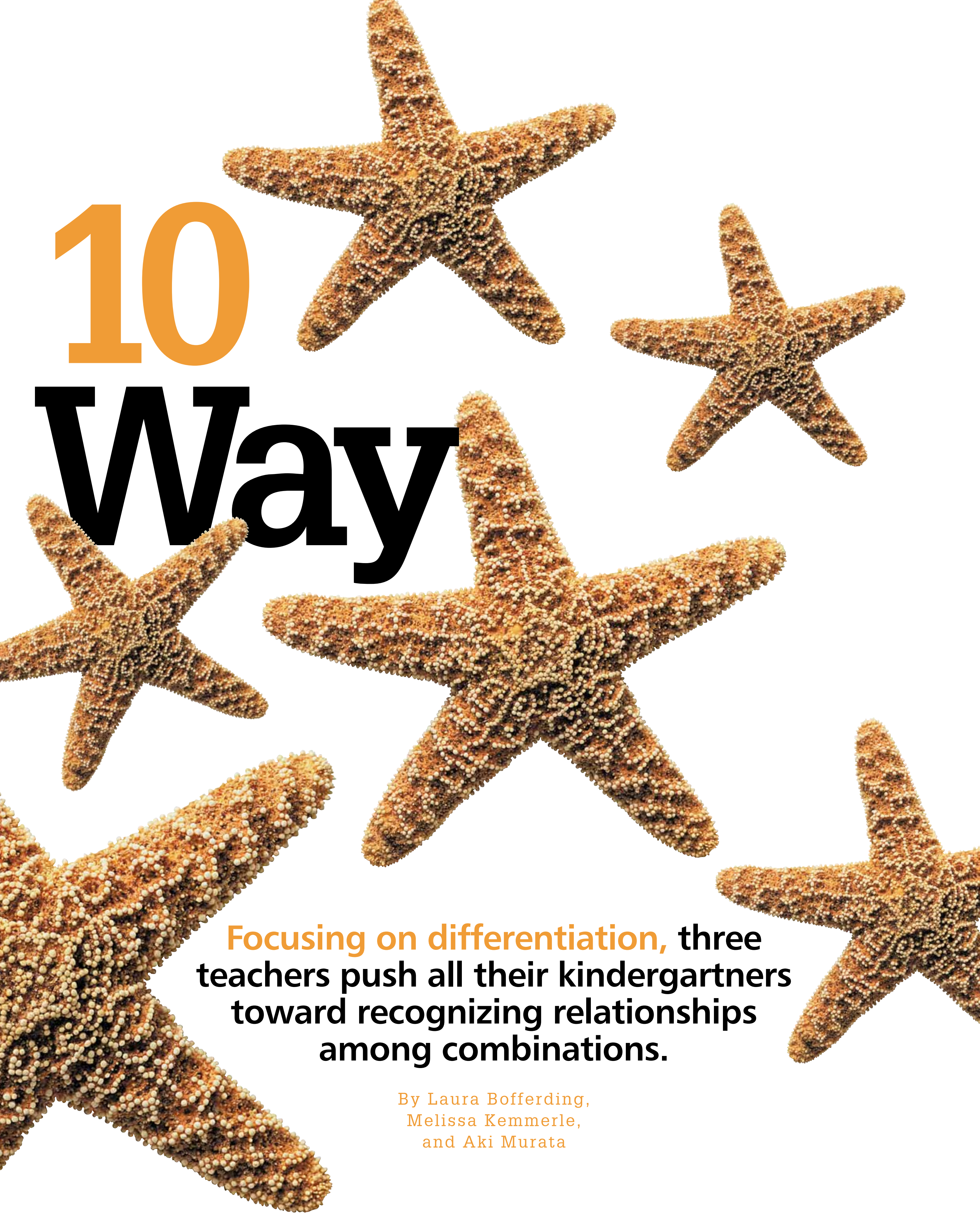
Lesson study is one structure that supports teachers' efforts to improve their lessons. Through lesson study, teachers plan a lesson or unit, observe one another teaching it, and continually reflect on students' learning in relation to the lesson supports (Lewis, Perry, and Murata 2006; Murata 2011). Additionally, teachers develop a "researcher lens," which they use to examine their practice, and have the opportunity to focus on and discuss student thinking (Choksi and Fernandez 2004).

We describe the process through which three first-year kindergarten teachers, Carissa, Rowland, and Naomi (pseudonyms), developed a unit on combinations of ten, which emphasized high expectations for all students while addressing their varied learning styles. We then focus on the results of one lesson in the lesson-study cycle and highlight how differentiated instruction supported student access and learning and how the process influenced planning and promoted teacher growth.



Meeting 1: General topic and preassessment

At the first lesson-study meeting—the purpose of which was to establish expectations and make initial plans—the kindergarten teachers exuded enthusiasm at the opportunity to discuss their students' learning with the support



10 Way

Focusing on differentiation, three teachers push all their kindergartners toward recognizing relationships among combinations.

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TABLE 1

At their first lesson-study meeting, kindergarten teachers and math education researchers created addition and subtraction problem scenarios as a preassessment for the teachers to gain insight into their students' diverse thinking and learning needs.

Problem type	Question
Join (result unknown)	Robbie has 6 tomatoes, and then he gets 4 more. How many tomatoes does he have?
Compare (difference unknown)	Robbie has 10 tomatoes, and I have 6. How many more does Robbie have?
Separate (result unknown)	You have 10 tomatoes and use 6 in some salsa. How many tomatoes are still left?
Part-part-whole (part unknown)	You have 10 tomatoes: 6 are red, and the rest are green. How many of the tomatoes are green?
Part-part-whole (whole unknown)	Robbie has 4 green tomatoes and 6 red tomatoes. How many tomatoes does he have?

of math education researchers. On the basis of their curriculum map, the teachers decided to focus the lesson-study unit on addition and subtraction word problems. The group discussed research on join, separate, compare, and part-part-whole problems—where different quantities within the problems (e.g., result, difference) are unknown (Fennema et al. 1993)—and crafted problem scenarios for the teachers to administer to a handful of students so they might gain insight into their students' diverse thinking and learning needs (see **table 1**).

Meeting 2: Part-part-whole problems

Armed with data on kindergartners' thinking, the lesson-study group gathered again to examine students' responses on the preassessment. Through guided analysis of their students' work, Carissa, Rowland, and Naomi noticed that some students needed manipulatives to make sense of verbal problems, whereas others relied on known number combinations to attempt problems. On the basis of students' identified needs, the team decided to focus the lesson study around part-part-whole problems and combinations of ten. According to the National Research Council (2001), "learning to see the part-whole relations in addition and subtraction situations is one of [K–grade 2 students'] most important accomplishments in arithmetic" (p. 191). In fact, two of the Common Core State Standards for Mathematics (CCSSM) for kindergarten students encourage them to decompose numbers into pairs and determine which numbers, when added to one through nine, will make ten (CCSSI 2010). The teachers left the meeting excited to design lessons involving part-part-whole word problems that would engage all their students in rich mathematical thinking.

Meetings 3 and 4: Part-part-whole, two unknowns

Depending on whether students start with one part or the whole quantity in a part-part-whole problem, they might count all, count up from the part to the whole, or count down from the whole (NRC 2001). To create a unit that allowed for multiple approaches and emphasized how the various combinations of a whole relate to one another, teachers chose to make both parts in the part-part-whole problems unknown and encourage students to find *all* combinations that make ten. Furthermore, the teachers anticipated that each student would think about these relationships in his or her own way, which they could highlight through class discussion. Thus, to maintain overall high academic expectations and allow space for individual needs to be met (see **table 2** on **p. 170**), the lessons incorporated both individual problem solving and whole-group sharing.

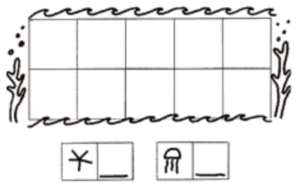
The goal of the second lesson (the one chosen for formal observation as part of the lesson study) was to explore all possible combinations of ten using an open-ended, ocean-themed word problem:

> There are 10 ocean animals in the ocean. You can have starfish and/or jellyfish. How many of each could you have?

As the teachers considered their classes, they designed instructional materials to allow students to access the problem in multiple ways, depending on their needs. Because several students needed concrete materials to solve the preassessment problems, the teachers planned to offer two colors of Unifix® cubes (one color for

FIGURE 1

To help constrain students to ten and to offer a stronger visual image, teachers distributed combination ten-frame activity sheets.



each sea creature) so that students could make physical combinations of ten. The teachers also created a ten-frame activity sheet (see **fig. 1**) that Carissa indicated could help constrain students to ten and provide a stronger visual image. They left spaces on the activity sheet for the totals of starfish and jellyfish, which they could use to highlight students' combinations and connect to the corresponding addition sentences during whole-class discussions. Because 4 + 6 could represent two combinations (depending on whether the 4 corresponds to starfish or jellyfish), students could find eleven different combinations in which the first number represents starfish. The teachers anticipated that all the students would find a combination and that students who knew their number facts would be challenged to evaluate if they had found them all. Rowland further hypothesized that the problem would "enable some students [who were already comfortable with addition] to think about subtraction if they want."

Focal lesson teaching event

In each classroom, students began the lesson sitting on the carpet. Carissa, Rowland, and Naomi reminded them of the ocean math stories from the previous day and had them think about which combinations of starfish and jellyfish make ten. Using three starfish and seven jellyfish as a model, the teachers combined Unifix cubes to represent this combination and filled in the ten-frame pictures and totals to record the combination.

Following the demonstration, each teacher asked her students to choose a workspace and challenged them to find as many combinations of starfish and jellyfish that make ten as they could. Then they were to record each combination on a separate ten-frame. Students were expected to complete their own combination activity sheets but were allowed to talk with one another. Most students worked near others, participating in parallel problem solving; they narrated their process but did not solve the problem together:

> ***Student Y:*** [*putting together six green cubes*] I'm doing six and six [*continuing to add six yellow cubes; recounting them all. Getting twelve, he removes two green cubes before recording them*].
> ***Student K:*** [*putting together three green "sea star" cubes and seven yellow "jellyfish" cubes to make a tower of ten*] Three starfish and one, two, three, four, five, six, seven [*counting each yellow cube and recording the combination before starting a new combination*]. One, two [*putting together two green "sea star" cubes*]. Next time I'll start with one sea star.

After 20–30 minutes, the class returned to the carpet to show ten-frames for the different combinations and discuss their findings. The teachers used this time to introduce interesting strategies and patterns through targeted student sharing. After student H explained how he found the opposites 9 + 1 = 10 and 1 + 9 = 10, another student clarified, "Instead of nine and one, he's going to do one and nine." Later, student X presented 1 + 9 again and claimed, "It's the same [as 1 + 9 already shown], but it's not the same," because the pictures were in different spots on the ten-frame. These comments opened up several avenues of discussion for the class to pursue in subsequent lessons.

Sea creature combinations

Because 4 + 6 could represent two combinations—depending on whether the 4 corresponds to starfish or jellyfish—students could find eleven different combinations where the first number represents starfish:

0 + 10	4 + 6	8 + 2
1 + 9	5 + 5	9 + 1
2 + 8	6 + 4	10 + 0
3 + 7	7 + 3	

Kindergartners worked together to find new combinations of ten.

AKI MURATA (2)

Lesson-study debrief

After all three teachers had taught the lesson, the lesson-study group gathered to discuss their cumulative observations. Because differentiation was a lesson goal, this theme arose in teachers' reflections on their students' material use, combinations, and strategies.

The role of materials in student solutions

The lesson materials afforded different strategies and access to students, and the level of student engagement in the exploration excited the teachers. The trio noticed that students who used the Unifix cubes counted multiple times. First, they took some of one color and counted them. Next, they counted on, adding a cube of a new color for each count, until they reached ten. Finally, they counted the number of each color to record the combination on their ten-frames. The teachers discussed that one purpose for using the ten-frame was to help students make sure the total was ten, and they found that students who had trouble making ten with cubes were able to use the ten-frame as a scaffold by placing one cube in each space or by more clearly seeing ten. Some students filled in the ten-frame and then counted to determine the number of starfish and jellyfish they drew; other students first drew and recorded the number of starfish and then drew and recorded the number of jellyfish. Whether students had to use a count-all strategy or could use a count-on strategy, they were all successful.

The problem context challenged students who already knew their number facts. Students who knew a combination wrote their numerical answers first and checked them as they drew sea creatures in the ten-frame. Referring to one such student, Rowland pointed out,

> She knows her facts up to twenty … but she had six jellyfish and three starfish, and she said that is ten.… So it is an example of knowing—memorizing—the fact but not having a contextual understanding.

Carissa and Naomi agreed and added that some of the more advanced students from their traditional instruction were challenged by the openness of the problem, whereas those who normally struggled were willing to try different approaches and thus found several combinations. Rowland reflected on the students' use of the materials:

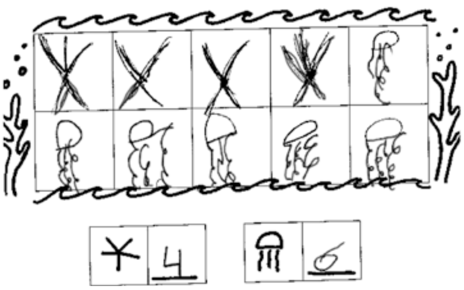
> Different kids bring these things together in different ways to figure it out.… It was interesting how many access points there were in this lesson for different kinds of learners. It allowed a lot of kids to be successful.

Strategies kindergartners use

Aside from how students used the materials, the teachers were interested in which strategies students used to find new combinations. The teachers noticed that the students were most likely to pick or draw a number of starfish and then build or draw from there to determine the number of jellyfish. For about half the students (29/61) this process resulted in duplicate combinations. In some cases, the ten-frames corresponding to these combinations looked identical; in other cases, students used the same number of ocean creatures but put them in different locations on their ten-frames (see **fig. 2**). As was the case for

FIGURE 2 Sometimes students put the same number of ocean creatures in different locations on their ten-frames and considered it a new combination.

(a)



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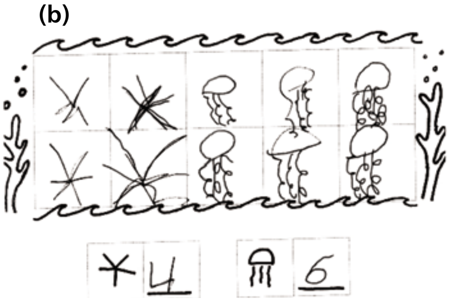
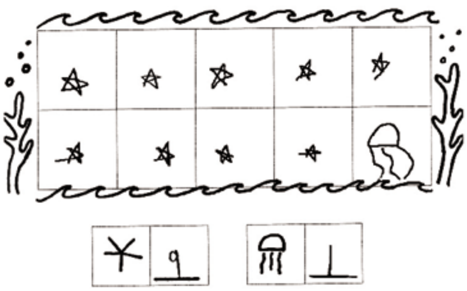


FIGURE 3 About a third of the students found at least one commutative pair: "It's almost the same, just the opposite."

(a)



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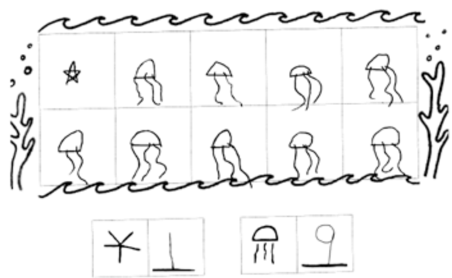

student X, these rearranged pictures constituted a new combination to some children.

Besides randomly choosing a starting number, other students moved toward thinking that was more systematic by *planning* to start with "more jellies" or "less jellies" the next time. Naomi stated,

> It's really interesting to see them realize that if I find a *different* starting number, it will have a different other number to make the combination. I thought that was really neat, because I hadn't really thought of that.

Additionally, some students used the strategies anticipated during the lesson planning (adding or taking away one starfish each time and finding the opposite—or commutative pair—for a combination). As illustrated in the lesson description, student K found new combinations by subtracting one starfish each time. Another student stated this explicitly, "It's easy, I have eight and two…. To get another combination, one goes up and one goes down." When discussing affordances of the Unifix cubes, Naomi suggested, "It's easier to take one off or add one more." A third student compared one combination to another (such as 7 + 3 and 3 + 7) and said, "It's almost the same, just the opposite." In fact, about a third of the students (20/61) found—whether intentional or not—at least one opposite pair (see **fig. 3**).

Implications for instruction

Overall, students found an average of three unique combinations each; although a few students found only one combination, others found up to eight. Furthermore, students used a range of strategies to determine new combinations. Because students' collective combinations spanned the eleven possibilities, the teachers were confident that they could facilitate a discussion based on patterns in the combinations to push all students toward more strategic thinking. Their goal was for students to be able to

TABLE 2

To differentiate instruction, the lessons in the lesson study unit included individual problem solving as well as whole-group sharing.

Lesson no.	Description	Materials	Goal
1	Gather students together and tell them, "Once there were three starfish hanging out on a rock. They thought their rock was the coolest rock in the ocean, so they invited their friends the jellyfish to come and play. Seven jellyfish friends showed up to make ten friends in the ocean." Draw the ocean creatures, and write the story as you tell it. Next, ask students to think about and share with their neighbors new stories where the number of ocean friends adds up to ten. Ask students to return to their workspaces. Then, give them a few ten-frame activity sheets on which to draw starfish and jellyfish to make stories with ten ocean friends. After they create stories, bring students back together and ask them to share their ocean stories with the class.	An ocean scene drawn on the board (with a rock on it) Ten jellyfish cutouts Ten starfish cutouts Ten-frame activity sheets (3–4 per student)	Introduce the ocean context and the idea of number stories.
2 (a focal lesson)	Gather students together and review the ocean context. Remind students of the three starfish and seven jellyfish combination and fill in a ten-frame activity sheet for this combination. Show students how to use the Unifix cubes to make this three–seven combination. Have students work individually with ten-frames and cubes to find as many different combinations of starfish and jellyfish that add up to ten as possible. After 20–30 minutes, bring students back together and work as a class to put up ten-frames for all possible combinations. Begin talking about how to know if you have found them all.	Ten-frame activity sheets (6–8 per student) Unifix cubes, sorted by color Board or wall space on which to tape student ten-frame activity sheets	Explore combinations of ten (using the ocean context and the ten-frame activity sheet) and begin thinking about finding *all* combinations of ten.
3	Gather students together to discuss strategies to find all possible combinations of ten. Ask, "How do you know if you've found all possible combinations?" Provide a new context, perhaps involving two farm animals, and challenge students to use one of the strategies to find all combinations of ten. Have students use the ten-frames and cubes again to help them. After 20–30 minutes, bring students back to share the combinations and discuss how they know if they have found them all.	Ten-frame activity sheets Unifix cubes, split into colors	Think about strategies for finding all combinations of ten. (How do you know you have found *all* of them?) Use a strategy to find all combinations of ten in a new context.
4	Ask students to make up their own stories about combinations of twelve. For example, a student may write a story about seven bananas and five monkeys or a story about two cats and ten dogs. Bring students back together to share their stories and record their combinations on the front board. Discuss how they know if they have found all possible combinations of twelve.	Unifix cubes, split into colors; a place to record their combinations; space on which to put their stories and pictures (if desired)	Think about strategies to find *all* combinations of twelve; create new stories or contexts for different combinations of twelve

systematically find all possible combinations of ten and to practice talking about their thinking. When asked about the next lesson, Carissa stated,

> I just really want to focus on the [opposite] strategy … because the strategy was already starting to come out, but not in a conscious way. I want to highlight the kids that were starting to use a strategy and see if other students can employ this.

In the context of ocean animals, the combination of three starfish and seven jellyfish *is* different than its “opposite,” the combination of seven starfish and three jellyfish; but the commutative property states that $3 + 7 = 7 + 3$. The teachers suggested that a further step in this exploration could involve having students explore when the order of the combinations matters (such as in contextual versus noncontextual problems). Similarly, students could explore whether the order in which they drew their creatures matters, as with student X.

In terms of logistics, the teachers agreed that the visuals were powerful for helping students see the different combinations and patterns among them. However, they also concluded that to reduce distraction, students should not bring their work to the carpet but turn it in. This would give the teachers a better opportunity to select key work to discuss.

Reflections

This lesson-study experience gave teachers time and space to think carefully about the diversity of their students’ thinking. When discussing the benefits of lesson study, Naomi stated, “It’s really unusual to have five different adults in the room to notice the little things that kids do.”

Rowland agreed with her. “This is an important reminder for me to sit back and watch my

problems and other ways of working around the number ten.

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The youngsters were proud of combinations that they found.

students learn sometimes rather than always being ready to teach them."

Carissa added,

> I think we often get caught up in many other aspects of teaching, like assessing and meeting certain standards, and this [lesson-study process] really provides a refreshing focus on the developmental process and how students think about and approach math.

The experience enabled the three teachers to notice that once students chose a number to start with, their problem became a traditional part-part-whole problem with one part unknown. Students primarily counted up from the first part rather than taking away from the whole, suggesting that this context might also be helpful for promoting the counting-up strategy for subtraction or for supporting students' thinking about missing-addend problems. The teachers were excited to use their newfound insights in their classrooms to better promote strategizing and sense making, which in turn, helped them learn and progress as teachers.

Carissa stated,

> It helped me to reflect upon my own practice and be more aware of different ways of approaching instruction and management.... While we focused on part-part whole combinations, students are now far more confident with missing-addend addition problems and other ways of working around the number ten.

Overall, the lesson study furnished the teachers with experiences and support to better understand their students' individual thinking and tailor their instruction through differentiation. Although differentiated instruction may take different forms, important in all of it is that teachers maintain high expectations for all students while recognizing individual learning needs. Students learn from one another by sharing ideas and strategies during whole-class discussions. Especially for young students, problem solving that takes advantage of students' diverse ideas helps build a foundation for their future understanding of what mathematics learning is. Instead of separating students according to their differences, balancing between group and individual learning in classrooms can help create effective differentiation.

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