

Addition problems in creative ways

During one of my small-group math lessons, I decided to challenge a group of students. Our entire class had been working on basic missing-addend problems throughout the week. The problems had seemed easy for these six students. I had not yet instructed my class on two-digit addition and subtraction, but I gave this small group some two-digit, missing-addend problems. I handed each of them a whiteboard and a marker and told them they could use any strategy they wanted to find the correct answer.

OK, boys and girls. Since those one-digit, missing-addend problems were too easy for you, I'm going to give you some problems that are really going to make you think. Use your whiteboard to help you. The first problem I want you to figure out is thirty-four plus what number equals sixty-two? [I noticed each student writing the problem $34 + \underline{\quad} = 62$ on their whiteboards. Then they drew, counted, and used number lines to help them solve the problem. Reid spoke first.]
I'm done! The answer is twenty-three.

Show me what you did to get this answer, Reid.

Well, I first wrote the problem on my whiteboard. Then, I put thirty-four in my head and counted up on my fingers to sixty-two.

can you double-check your answer? I want you to look at your answer to figure out how to check yourself while I work with another student [moving over to listen to Caeden's strategy].

I didn't get twenty-three; I got twenty-eight. I used a hundred chart to count my jumps. I started at thirty-four [and] circled it blue. Then I jumped down to forty-four, which was ten; jumped down to fifty-four, which was twenty; and counted up to sixty-two. [See fig. 1.]

Caeden, this is great; but what happens when you don't have a hundred chart?

I can draw just thirty-four, forty-four, and fifty-four, (that's twenty) on my board and then count up to sixty-two.

Morgan, can you show us how you solved the problem?

I turned this into a subtraction problem. I said, "Sixty-two minus what number equals thirty-four?" I counted backward: sixty-two to fifty-two [is] ten; fifty-two to forty-two [is] twenty; forty-two to thirty-two, which is thirty. Then I knew that I was supposed to stop at thirty-four, so I added thirty-two plus what number equals thirty-four. Two. So thirty minus two is twenty-eight.

(continued online)

FIGURE 1

Caeden's method works well when a hundred chart is available.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

How did you have enough fingers to count [that high]?

I decided to place a mark on my paper when I got to ten. I had two marks and three fingers up.

Is there any way you could have made a mistake when you were counting? How



The teacher reflection and more student dialogue accompany the online version of "back talk" at www.nctm.org/tcm.

Laken B. Harrell, lharrell@cityschools.com, teaches second-grade mathematics at Hugh Mercer Elementary School in Fredericksburg, Virginia. She is interested in problem solving and investigative thinking and is pursuing a career as a math specialist. Megan Kelly Murray, mek2v@cms.mail.virginia.edu, can be contacted at the University of Virginia in Charlottesville. E-mail your insights into a student's mathematical thinking to tcm@nctm.org. Include "back talk" in the subject line.

More addition problems in creative ways

So, for you, it is easier to count backward than forward. Caeden and Morgan, what do you notice about both of your strategies?

[Morgan] Mine is the opposite of his. Caeden added, and I subtracted. They both got the same answer, though.

[Jacory] Mrs. Harrell, I did it a different way. I wrote the problem on my board [demonstrating $34 + \underline{\quad} = 62$]. Then I punched thirty-four on my hand. I drew tallies and counted out loud until I got to sixty-two. [See fig. 2.]

How did you know to start at thirty-four and punch it on your hand?

Because I already knew that was the first number because it is in the problem. I wanted to save time, so I didn't start at zero. I just counted up, and so I wouldn't lose count, I wrote the tallies.

[Reid] I figured out what I did, Mrs. Harrell. I lost

track of what I was doing. The answer is twenty-eight. I decided to check myself. I drew circles. I put thirty-four in my head, and then I counted up to sixty-two. I drew a circle for every number I said.

[What I had wanted to occur happened exactly as I desired. I knew by having Reid check her work, she would come up with the correct answer. As she counted, she drew the circles that were missing, which was a way of self-checking to solve the answer.] Reid, you and Jacory are thinking the same way now. One of you drew circles, and the other drew tallies. OK, Abe and Nayra, do you have a different way to solve the problem?

[Abe] I wrote the numbers 34 and 62 on my whiteboard. I wanted the easiest way to solve this, so I drew the sixty-two in base-ten blocks. [See fig. 3.] I know I have to take away thirty-four,

FIGURE 2

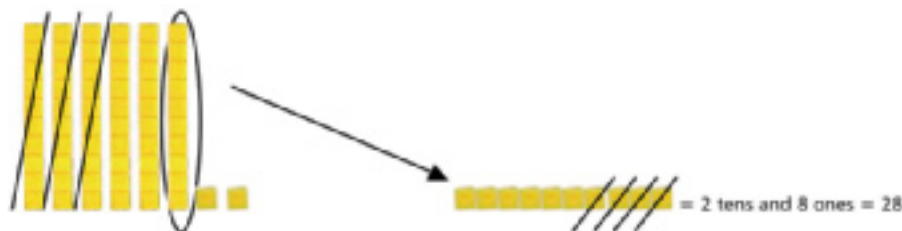
Jacory counted up and used tallies in a one-to-one correspondence.

34 (punched in hand), 35, 36, 37, 38, 39 40, 41, 42, 43, 44 45, 46, 47, 48, 49 50, 51, 52, 53, 54 55, 56, 57, 58, 59 60, 61, 62



FIGURE 3

Abe's use of base-ten blocks showed improvement from prior understanding, but his teacher was still unclear whether he connected the concrete tools and his strategy with the standard algorithm.



so I took away the thirty, first, since I had three rods. Now, I don't have four, so I have to break up a ten. Now I could take the four away.

Abe, this is the second time you have shown me something in base-ten blocks. You really like to work with these. How did you know that there are ten cubes in one rod?

I counted the rod and found out there are ten cubes, so I traded for ten cubes.

Great. Now we have five different ways to solve this problem. Nayra, how did you solve it?

I wrote the problem on my whiteboard. Then I drew sixty-two circles and crossed out thirty-four of them to find out how many were added to get the answer. I counted, and twenty, six, and two [are] left. Twenty-eight is the answer.

Teacher reflection

I was pleased that each student came up with a different way to solve the problem. Even though I had never taught them two-digit addition, all the students were able to determine the correct answer. From the varying methods, I gained much insight into each child's thinking.

Reid had a great start figuring out the answer. She wanted a visual representation, so she drew tally marks. However, she seemed confused with the tally marks, and she stopped. So I gave her more time to either correct her original strategy or think of a different way to solve the problem.

Jacory's strategy was similar to Reid's, and if I were to have called on him immediately after Reid's presentation, this might have caused Reid to stop thinking about the problem. Thus, I chose to continue with various other students' strategies before allowing Jacory to discuss his strategy. I believed that an alternate method would help Reid get the correct answer and that using another method might encourage her to verify her original answer and identify her error.

When I continued with Caeden, I learned that he can count by tens and ones, with or without a number chart. This revealed a strong understanding of our base-ten system. Like Reid, Caeden started counting at thirty-four, holding that number in his thoughts. Caeden used the number chart to help him skip count to forty-four, to fifty-four, and then begin counting by ones.

Jacory counted by individual ones and then grouped the tallies to keep track of how many he added on.



LAKEN B. HARRELL

Similarly, Morgan used this skip-counting method with groups of tens and ones, but she counted backward by tens without the aid of a hundred chart. She was aware of what she was doing, counting back correctly (not always an easy task for a child of this age) and adding back the two that was subtracted from the final ten. Some children use this compensation method; clearly Morgan had a deep understanding of the basic facts and mental-mathematics

Nayra used friendly numbers and a semi-concrete method to demonstrate her thinking.



LAKEN B. HARRELL

strategies. She seemed flexible in her thinking, and she began to move to a more abstract way of approaching subtraction.

When it came to Jacory, I was quite pleased that he knew how to use tally marks as an easy way to show groups of five. As he wrote a tally, he said the number aloud. He was proud of himself and even wanted to show the students in the other two groups how to implement counting in a one-to-one correspondence with tally marks. By using a counting-up method, Jacory connected the concrete pictorial representation of a tally mark to the abstract idea of taking away a group of objects. He did not use groups of values. Instead, he counted by ones individually, then grouped the tally marks to keep a record of how many he had added on so that he could calculate the difference of the joint problem with the unknown difference.

As I examined Abe's work, I noticed that he demonstrated how he could use base-ten blocks to model and solve this problem, marked growth from previous experiences. It appeared that he was gaining a better understanding of the standard algorithm, and he used the base-ten blocks to support this algorithm. However, I was unsure whether he was connecting these concrete tools and this strategy with the standard algorithm. In future lessons, when we really delve into sub-



LAKEN B. HARRELL

traction, I want to support Abe with activities to help him make this connection between the manipulative tools and the standard algorithm. I also want to provide all the students with various strategies to support their mental-mathematics understanding.

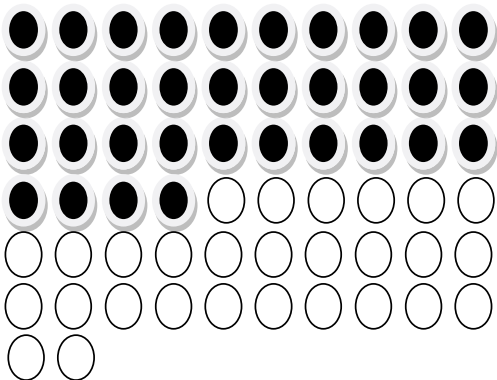
Although Nayra took a while to figure out a solution to this problem, she arrived at a correct visual representation of the problem. Nayra appeared to be thinking of friendly numbers, because each row of her drawing had ten circles (see fig. 4). When I asked Nayra about this, she said she wanted an easy way to count how many she had drawn. She was using friendly numbers through a semi-concrete method of a pictorial representation to demonstrate her thinking. She also used a counting-up method, but she began by drawing thirty-four items, then counted up or added on to thirty-four until she got sixty-two. To determine the solutions, she changed from a closed circle to an open circle and went back to count how many open circles she had added on. Again, her invented strategy supported her understanding of subtraction and allowed me to continue this process so I can help her, as well as the other students, better understand grouping, adding larger numbers on, and the standard algorithm.

This interview has me questioning whether teaching two-digit addition and subtraction will be easy once it is time to be taught as designated

Although Caeden used a hundred chart, further investigation revealed that he counts by tens and ones with or without the chart—evidence of a solid grasp of the base-ten number system.

FIGURE 4 Nayra's invented strategy supported her understanding of subtraction and allowed her teacher insight to help develop Nayra's understanding of grouping and the standard algorithm.

$$34 + \underline{\quad} = 62$$



in the curriculum/pacing guide. Moreover, this experience makes me question whether there is a right time to teach it, since these children are obviously ready for this material. My past instruction has always been more direct, but each of these children was able to come up with a strategy that worked for him or her without my providing explicit instruction on how to solve the problem. I saw students experiment with their math knowledge and learn from the strategies of another child, especially in Reid's case. The conversations that took place in the group of six students helped Reid realize her mistake and correct it.

The next step I am going to take is to have my entire class try completing a problem similar to this one. If all the different strategies do not arise in the larger, whole-class discussion, I will let the higher-level students show the strategies to the others. Students will then have a pocketful of strategies to choose from when solving two-digit, missing-addend problems. I hope that if some of my lower-level students have trouble, my higher-level students may be able to help support them in their understanding by demonstrating their thinking and

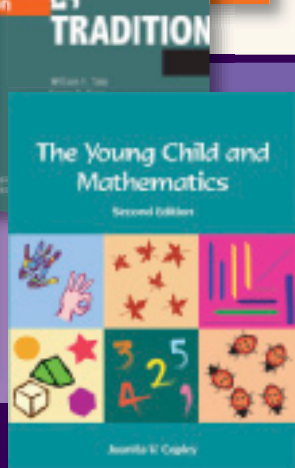
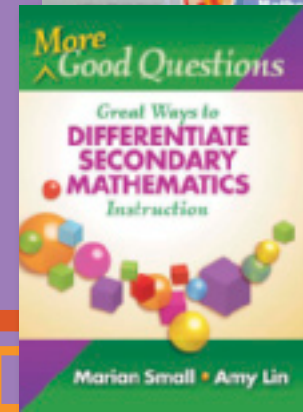
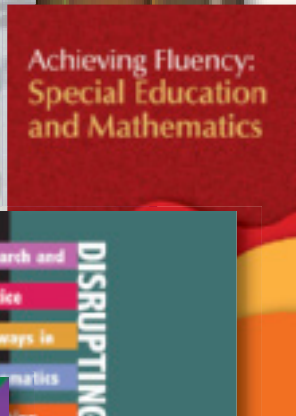
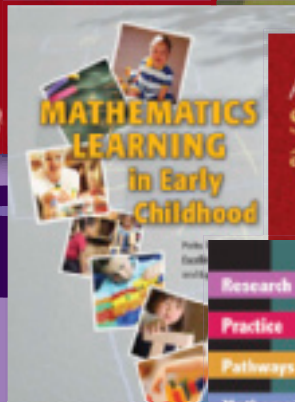
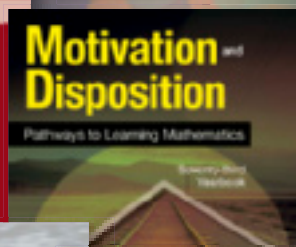
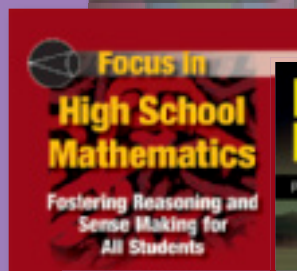
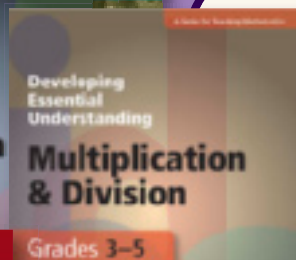
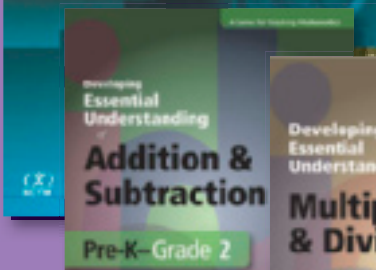
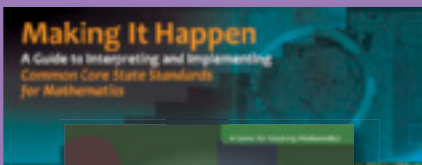
strategies. I want to allow students to correct their own thinking, as Reid was able to do in our small-group interview. I am curious as to how well my lower-achieving students will succeed, and if they have difficulties, whether they will learn from the methods of stronger mathematics students. I will be cognizant of students' conversations so I can point out similarities in methods, help students make connections, and clarify their thinking. By questioning students and allowing them to communicate their thinking verbally, in writing, and in small-group and whole-group discussions, I will ascertain that the children understand and can use other strategies.

Finally, as a result of this work, I will make the time to discuss two-digit addition and subtraction, allowing my students to become familiar with various subtraction strategies. Each child will be able to adopt the approach that he or she is most comfortable with. However, I will also coach students to use strategies that may be easier for other problems. Assessing them in this manner will help me know how to facilitate their development of understanding the standard algorithm.

New Books from NCTM

Shop online where you can view tables of contents and sample pages!

www.nctm.org/catalog



Making it Happen: Executive Summary (includes eBook!)

Stock #: 13789
List Price: \$49.95
Member Price: \$19.95

Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Prekindergarten—Grade 2

Stock #: 13792
List Price: \$30.95
Member Price: \$24.76

Developing Essential Understanding of Multiplication and Division for Teaching Mathematics in Grades 3—5

Stock #: 13795
List Price: \$30.95
Member Price: \$24.76

Focus in High School Mathematics: Fostering Reasoning and Sense Making for All Students

Stock #: 13989
List Price: \$29.95
Member Price: \$23.96

Motivation and Disposition: Pathways to Learning Mathematics, 73rd Yearbook

Stock #: 13864
List Price: \$55.95
Member Price: \$44.76

Disrupting Tradition: Research and Practice in Mathematics Education

Stock #: 13515
List Price: \$29.95
Member Price: \$23.96

Achieving Fluency: Special Education and Mathematics

Stock #: 13783
List Price: \$34.95
Member Price: \$27.96

More Good Questions: Great Ways to Differentiate Secondary Mathematics Instruction

Stock #: 13782
List Price: \$29.95
Member Price: \$23.96

Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity

Stock #: 13904
List Price: \$49.46
Member Price: \$39.57

The Young Child and Mathematics, 2nd Edition

Stock #: 13861
List Price: \$32.00
Member Price: \$25.60

Need it now?

Purchase and download eBooks and individual chapters of select titles. Visit www.nctm.org/ebooks for details.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

(800) 235-7566 | WWW.NCTM.ORG

For more information or to place an order, please call
(800) 235-7566 or visit www.nctm.org/catalog.

