Young Children's Ideas about

A toddler, after some experimentation, puts a square peg into a square hole. What does she know about shapes? What more will she learn in preschool and elementary school. What *might* she learn?


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Two kindergartners create a rhombus.

Educators have learned a great deal about young children's knowledge of shapes. Much of it is quite surprising. In this article, we describe research on young children's thinking about geometric shapes and draw implications for teaching and learning.

Douglas H. Clements and Julie Sarama

What Do Children Know about Shapes?

Children's levels of understanding

As children develop, they think of shapes differently. At the *prerecognition level*, children perceive shapes but are unable to identify and distinguish among many shapes. They often draw the same irregular curve when copying circles, squares, or triangles (Clements and Battista 1992b). At the next, *visual*, level, children identify shapes according to their appearance (Clements and Battista 1992b; van Hiele 1959/1985). For example, they might say that a shape "is a rectangle because it looks like a door." At the *descriptive* level, children recognize and can characterize shapes by their properties. For instance, a student might think of a rectangle as being a figure that has two pairs of equal sides and all right angles. Because progress in children's levels of thinking depends of their education, children may achieve this level in the intermediate grades . . . or not until college!

Children at different levels think about shapes in different ways, and they construe such words as *square* with different meanings. To the prerecognition thinker, square may mean only a prototypical, horizontal square. To the visual thinker, *squares* might mean a variety of shapes that "look like a perfect box" no matter which way they are rotated. To a descriptive thinker, a square should be a closed figure with four equal sides and four right angles. But even to this child, the square has no relationship to the class of rectangles, as it does for thinkers at higher levels. These levels can help us understand how children think about shapes. We might remind ourselves to ask what children *see* when they view a shape. When we say "square," they might seem to agree with us for many prototypical cases but still mean something very different. The levels can also guide teachers in providing appropriate learning opportunities for children.

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Geometric Shapes

Children's ideas about common shapes

Young children have many ideas about common shapes. Levels alone, however, do not give teachers sufficient detail. For this reason, we conducted several studies of young children's ideas about shapes to help complete the picture (Clements et al. 1999; Hannibal 1999). In all, we interviewed 128 children, ages 3 to 6, for a total of an hour over several sessions. Children identified shapes in collections of shapes on paper and manipulable shapes cut from wood. The shapes were presented in different settings. For example, sometimes we had the children handle the shapes, and other times we asked them to identify the shapes placed in various orientations in either a fixed rectangular frame or a round hoop. Our conclusions from these interviews regarding the basic shapes are described here.

Circles. Children accurately identify circles, although children younger than six years old more often choose ellipses as circles. Apart from these infrequent exceptions (only 4 percent incorrect on our paper task), early childhood teachers can assume that most children know something about circles.

Squares. Young children are almost as accurate in identifying squares (87 percent correct on our paper task) as in identifying circles, although preschoolers are more likely to call nonsquare rhombus "squares." However, they were just as accurate as older children in naming "tilted" squares.

Triangles. Young children are less accurate in identifying triangles (60 percent correct). They are likely to accept triangular forms with curved sides and reject triangles that are too "long," "bent over," or "point not at the top." Some three-year-olds accept any shape with a "point" as being a triangle.

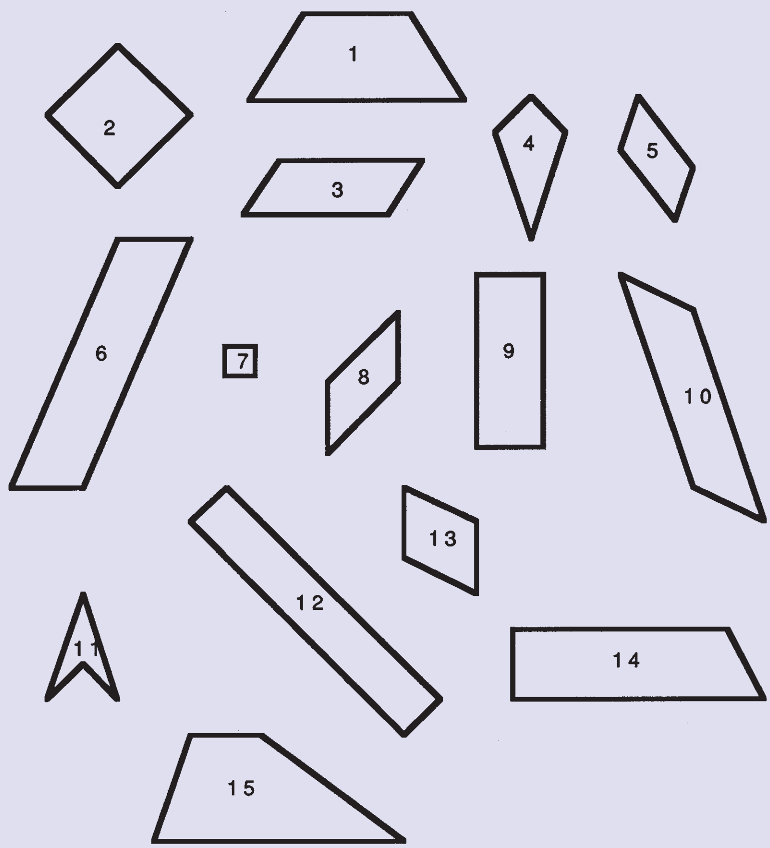
Rectangles. Again, children's average accuracy is low (54 percent). Children tend to accept "long" parallelograms or right trapezoids (see **fig. 1,** shapes 3, 6, 10, and 14) as rectangles. So children's image of a rectangle seems to be a four-sided figure with two long parallel sides and "close to" square corners. Only a small number of three- and four-year-olds seem not to have any ideas or images of rectangles or triangles.

Children's Often Surprising Thoughts about Shapes

We were impressed with how much preschoolers knew about shapes yet were more surprised by other findings.

FIGURE 1

A set of quadrilaterals



Not that much changes

It was surprising how *little* children learn about shapes from preschool to middle school. For example, our preschoolers identified about 60 percent of the triangles correctly. In a large study of elementary students doing the same task, scores ranged from 64 percent for kindergartners to 81 percent for sixth graders (Clements and Battista 1992a). Similarly, the preschoolers' scores were 54 percent on rectangles, and elementary school students' scores ranged from 63 percent to 68 percent. Although approximately eight years separate these groups, the scores on the *same tasks* increased only minimally. Perhaps we could be doing a better job teaching geometry in every year.

Individual differences can be dramatic

One of the six-year-olds in our study chose rectangles randomly and described them as "pointy." In comparison, one of the three-year-olds outscored all the six-year-olds on the rectangle task. Good opportunities to learn are more important than developmental level when it comes to children's learning about shapes.

Given good opportunities, children pose and solve interesting problems

A teacher in our study challenged her kindergartners to make various shapes with their bodies. Two boys tried to make a rhombus. They sat down facing each other and stretched their legs apart. With their feet touching each other, they made a very good rhombus. One of the children in the circle suggested that "if we put another child in the middle, we would make two triangles!" Immediately, the children called into service a boy named Ray, because he was the smallest, and they asked him to scrunch in the middle. The boys posed their own problems about decomposing shapes and devised their own solutions.

Computers help push back the boundaries of what children can do with geometric shapes. For example, preschooler Tammy was working with a computer program that presented a fixed set of shapes. She overlaid two overlapping triangles on one square and colored selected parts of this figure. This process created a third triangle that had not existed in the program! Computers can also be powerful and flexible manipulatives (Clements and McMillen 1996). For example, Mitchell wanted to make hexagons using the pattern-block triangle. He started without the computer and used a trial-and-error approach, counting the sides and checking after adding each triangle. However, using Shapes, a computer program, he *began* by planning (Sarama, Clements, and Vukelic 1996). He first placed two triangles, dragging them and turning them with the turn tool. Then he counted with his finger around the center of the incomplete hexagon, visualizing the other triangles. "Whoa!" he announced. "Four more!" After placing the next one, he said, "Three more!" Whereas without the computer Mitchell had to check each placement against a physical hexagon, his intentional and deliberate actions on the computer led him to form mental images. That is, he broke up the hexagon in his mind's eye and predicted each placement.

What Is Tricky for Children?

Different settings, different decisions

Specific settings affect children's decisions. In our first setting, we included obvious nonexamples, such as circles, in with a group of triangular shapes. Children accepted more types of triangles and more "pointy" forms that were not triangles. Similarly, children accepted more ovals as circles when shapes were drawn inside each other. Thus, what children choose is strongly influenced by the shapes they are comparing.

In our second setting, we placed the wooden shapes inside a hoop. Children were less likely to notice or care about whether they were "long" or "upside down." That is, not having a rectangular frame of reference affected what shapes they accepted. In the third setting, we asked children to justify their choices. They often changed their decisions, usually to correct ones. For example, one five-year-old increased her score by 26 percentage points when asked to explain her choices.

Limited understanding of properties

Even though talking about the shapes often helped, children's knowledge often has limits. Four-year-olds would frequently state that triangles had "three points" or "three sides." Half of them were not, however, sure what a "point" or "side" was (Clements 1987). For example, a student was asked to stand on a triangle from among several taped outlines of shapes on the floor. The student immediately stood on the triangle and explained that "it has three sides and three angles." When asked what she meant by three angles, she said, "I don't know."

What Can Teachers Do to Help?

The studies' results that we have described have many implications for geometry instruction. The remainder of this article offers research-based suggestions for improving the teaching of basic shapes and for enhancing children's understanding of these shapes.

Reconsider teaching "basic shapes" only through examples

As we continually contrast squares and rectangles, do we convince children of their separateness? Does teaching shapes mainly by showing examples sometimes build unnecessarily rigid ideas? In one study (Kay 1987), a first-grade teacher initially introduced the more general case, quadrilaterals, or shapes with four straight sides. Then she introduced rectangles as special quadrilaterals and squares as special rectangles. That is, she discussed the attributes of each category of shapes and their relationships; for example, a rectangle is a special kind of quadrilateral. She informally used terms that reflected this relationship; for example, that "this is a square-rectangle." At the end of instruction, most of her students identified characteristics of quadrilaterals, rectangles, and squares, and about half identified hierarchical relationships among these classes, even though *none* had done so previously. This teacher concluded that the typical approach of learning by showing examples is appropriate only for shapes that have few such exemplars, such as circles and possibly squares. For other shapes, especially such hierarchical-based classes as triangles and quadrilaterals, this approach alone is inadequate. We might question how deep these first graders' understanding of hierarchical relations was (Clements and Battista 1992b). However, we might also question the wisdom of the traditional, show-examples approach—it may lay groundwork that must be overturned to develop hierarchical, that is, abstract-relational, geometric thinking.

Give children credit for what they know

By the time they enter kindergarten, most children have many ideas about shapes. Yet teachers often do not ask children to extend their ideas. In one study (Thomas 1982), about two-thirds of teachers' interactions required only that children parrot what they already knew in a repetitious format, such as the following.

Teacher: Could you tell us what type of shape that is?
Children: A square.
Teacher: OK. It's a square.

Most questions that teachers asked were closed-ended and dealt only with memory. Few children asked questions, but when they did, the teacher responded with silence. Instead, teachers should build on what children know and generate rich discussions.

Avoid common misconceptions

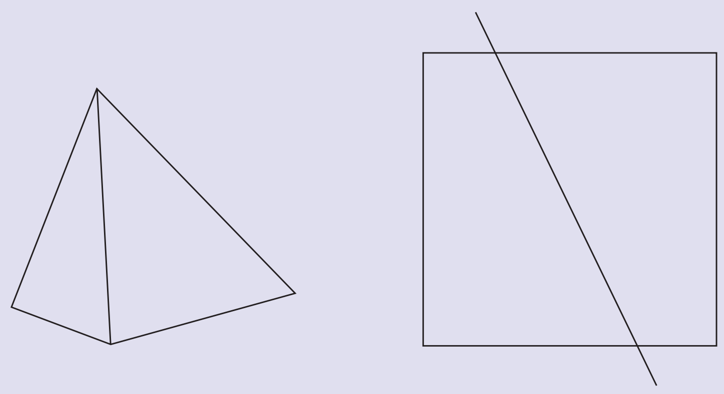
In the Thomas (1982) study, most teachers did not add new content. When they did, however, the statements were often incorrect, such as saying, "All diamonds are squares." Others were, at best, unfortunate, such as telling a child who chooses a square rectangle, "No, find a rectangle." Other unhelpful statements included that "two triangles put together always make a square" and that "a square cut in half always gives two triangles." (See **fig. 2** for counterexamples.)

Expand the limited notions that are too often "taught"

Everywhere, it seems, unfortunate geometric ideas surround us! As part of our research, we closely examined toy stores, teacher-supply stores, and catalogs for materials on "shape." With few exceptions, these materials introduce children only to "best examples" of triangles, rectangles, and squares. Most triangles are equilateral or isosceles triangles with horizontal bases. Most rectangles are horizontal or vertical, between two and three times as long as wide. Most squares have horizontal bases. Curricular materials are also often limited. As just one example, Leah was working on a computer program that asked her to choose fish of various shapes. When asked to pick square fish, she chose one whose square body was oriented at 45 degrees. The program announced that she was wrong, saying that it was a "diamond fish"!

FIGURE 2

Triangles and squares



Two triangles put together do not always make a square, and a square can be cut in half many ways.

Help children break free of these limitations by using a wide range of good examples and nonexamples. To do so, try these tactics:

- Vary the size, material, and color of examples. For rectangles and triangles, vary their orientation and dimensions, such as the rectangles in **figure 1.** For triangles, vary their type, including scalene—no sides equal in length—and obtuse—one angle larger than 90 degrees.
- Compare examples with nonexamples to focus children's attention on the essential attributes (see **fig. 3**).
- Encourage children to change shapes dynamically.

FIGURE 3

Tricky Triangles

The Tricky Triangles sheet includes triangles (shapes 1–5) and nontriangles (shapes 6–10) that are paired to encourage contrasts. For example, shape 1 is a triangle, but shape 6 is not a triangle because it is not closed. Shape 2 does not have a horizontal base and is not symmetric, but it is a triangle; however, shapes 7 and 8 are not triangles, because they have four sides.

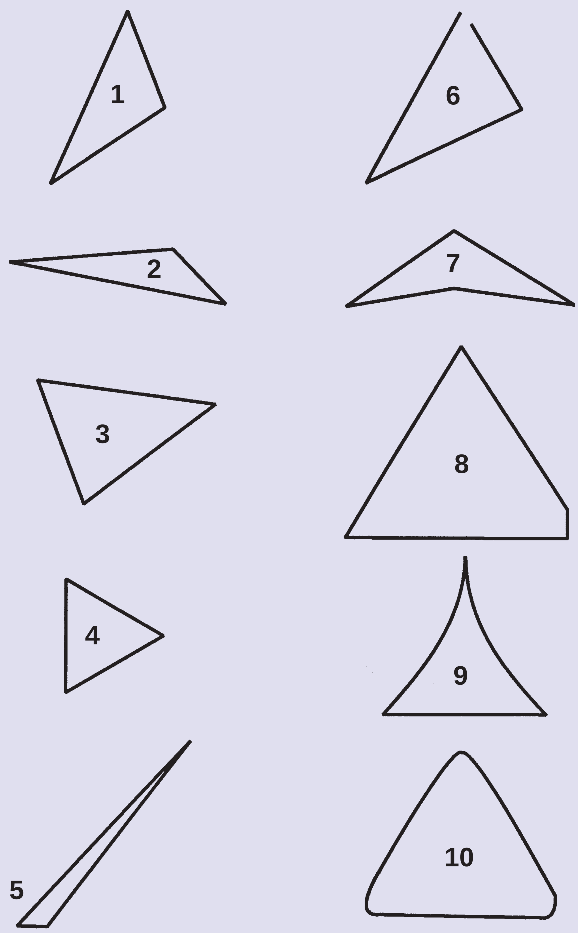
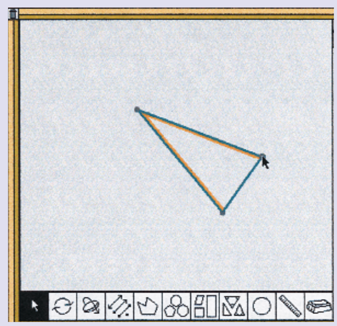


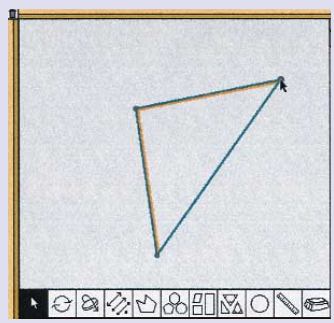
FIGURE 4

Manipulating a category of shape in Math Van

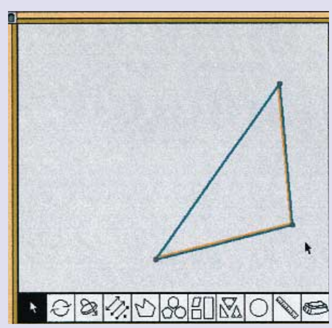
Math Van (McGraw-Hill School Division 1997) allows children to draw and manipulate a certain kind, or category, of shape. Here children defined an isosceles triangle. The double-color on the two equal sides represents their equality (a). Children can slide, turn, or flip this triangle. They can also distort it, but no matter how they do, it remains an isosceles triangle [(b) and (c)].



(a)



(b)



(c)

For example, some computer programs allow children to manipulate a figure, such as an isosceles triangle, in many ways, but it always stays an isosceles triangle (see **figs. 4a, 4b,** and **4c**).

Matching activities to children's level of thinking about shapes

Children at the prerecognition level might best work with shapes in the world. Help these children understand the relevant and irrelevant—for example, orientation, size, color—attributes of shapes with such activities as the following:

- Identifying shapes in the classroom, school, and community
- Sorting shapes and describing why they believe that a shape belongs to a group
- Copying and building with shapes using a wide range of materials

Children at the visual level also can measure, color, fold, and cut shapes to identify their attributes. For example, they might engage in the following activities:

- Describing why a figure belongs or does not belong to a shape category
- Folding a square or rhombus in various ways to determine its symmetries and the equality of its angles and sides
- Drawing shapes with turtle geometry, which helps children approach shapes actively but with precise mathematical commands and measurements; or having children give commands to a "child-turtle" who walks the path
- Using a "right-angle tester" to find angles in the classroom equal to, larger than, or smaller than a right angle
- Using computers to find new ways to manipulate **(fig. 5a)** and make **(fig. 5b)** shapes (Clements and Meredith 1998)
- Discussing such general categories as quadrilaterals, or "four-sided shapes," and triangles, counting the sides of various figures to choose the category in which they belong

Conclusions

An adult's ability to instantly "see" shapes in the world is the result, not the origin, of geometric knowledge. The origin is our early active manipulation of our world. Young children learn a lot about shapes. *Geometry instruction needs to begin early.* Our research and that of Gagatsis and Patronis (1990) show that young children's concepts about shapes stabilize by six years of age but that these concepts are not necessarily accurate. We can do a lot as teachers to supplement curricula that too often are part of the problem rather than the solution.

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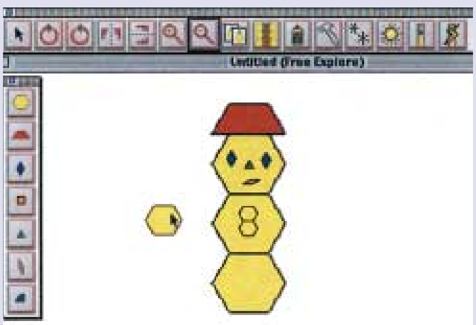
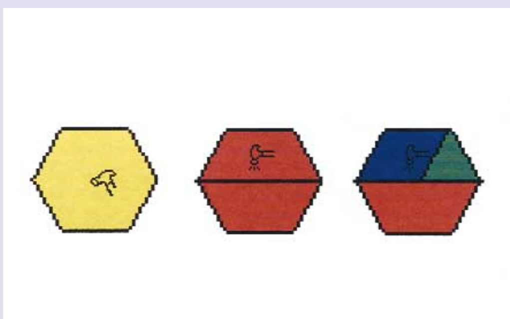
FIGURE 5

Manipulating shapes in the Shapes program

The Shapes program (Clements and Meredith 1998) allows children to manipulate shapes in ways not possible with usual pattern blocks, such as to define new units-of-units, which would turn and duplicate together. They can enlarge or shrink shapes, as Leah did in making her snow person (a). They can hammer shapes to decompose them into parts (b).



(a)



(b)

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