Summer, 2024

Dear Eager Calc BC Student,

I hope that you enjoy your summer and that you are excited for the challenge ahead! If any of you do not know me, I am Mr. Dolce, and I will be your teacher this coming year for AP Calculus BC. I am looking forward to working with you! This is a rigorous class and can be very demanding. There will be high standards and high expectations: problems will require you to apply concepts in clever ways, sometimes you'll need to take some initiative and seek out information on your own, and you definitely won't have a review sheet for every test.

The material that we cover will require you to have a firm grasp of many topics that you have worked on in past years. With that in mind, I have created a review assignment for you. All of the questions should be answered without the use of a calculator. Do the problems in what will become your homework notebook for the class. When class begins, I'll ask that you maintain all of your homework in a dedicated notebook (or folder or binder) so that it can be regularly checked.

I know it's not a short assignment, but my hope is that some of it is easy for you so that you can focus your energy more on the parts that aren't as simple. I have attached my answer key to the assignment in the hopes that it will help you resolve some of your questions along the way. It is not being graded, but its *satisfactory* completion will count as an assignment for the first marking quarter. It is a tool to help you prepare for the course. The topics covered on the assignment will be included within upcoming units and tests. If you put in the effort, you will find it to be helpful in the long run. However… if you wait until the night before school starts, it is likely you will feel overwhelmed. On the other hand, if you complete the assignment at the very beginning of the summer, it will be a good idea to refresh yourself before September.

Please feel free to e-mail me at **cdolce@ccsdli.org** with any questions about the material or with any concerns you may have as you get ready for September. I'll get back to you as quickly as I can, but be aware that I do not check my school email every day during the summer. If you're looking for a little more help than I can give in an email, we can set up an extra help session during the last weeks of August for individuals or a small group (if you prefer).

Don't be scared. Really. It's going to be a fun year! ☺ I know that the work can seem intimidating, but if you are willing to put in the time and energy, I assure you that this will be a positive and rewarding experience! I am always willing to talk with you and help you in any way I can!

Sincerely,

Mr. Dolce

Name ______________________________ Summer 2023

Getting Ready for AP Calculus BC

Complete the problems in your notebook. Do not use a calculator.

I. Solving Equations

IA. Linear & Quadratic

Starting off simple… You'll need some factoring and the Quadratic Formula.

1) $2x-7(x+3)=5-3x$

2) $x^2-4x+3=0$

3) $x^2+6x=1$

4) $6x^2+11x-10=0$

IB. Absolute Value

Remember, if $|x|=a$, then $x=a$ or $x=-a$.

Ex: $|3x-1|=5$

$3x-1=5 \quad \vee \quad 3x-1=-5$

$$\boxed{x=2 \quad \vee \quad x=-\frac{4}{3}}$$

5) $|2x-5|=11$

6) $|4-x|=3$

IC. Handling Fractions in Equations

You can use Least Common Denominators to simplify your equations.

Multiply each term by the LCD.

Ex: $\frac{3}{x}-\frac{x-2}{4}=\frac{3x-1}{4x}-2$

$4\cancel{x}\cdot\frac{3}{\cancel{x}}-\cancel{4}x\cdot\frac{x-2}{\cancel{4}}=\cancel{4x}\cdot\frac{3x-1}{\cancel{4x}}-4x\cdot 2$

$12-x(x-2)=3x-1-8x$

Cross Multiplying is great when at least one fraction is isolated like

$$\frac{A+Bx}{C}=Dx+E$$

$$\frac{A+Bx}{C} \times \frac{Dx+E}{1}$$

$$A+Bx=C(Dx+E)$$

7) $\frac{5x}{4}+\frac{1}{2}=x-\frac{1}{2}$

8) $\frac{x}{5}-\frac{x}{2}=3+\frac{3x}{10}$

9) $\frac{5}{3}x-\frac{1}{5}=\frac{3}{2}x+\frac{2}{5}$

10) $\frac{2}{3}(12x-6)-(12-2x)=4\left(\frac{1}{2}x+3\right)$

11) $\frac{3}{4}(z-4)=6$

12) $\frac{3x-2}{2}=\frac{-x}{2x+1}$

13) $\frac{4}{3-x}=-2$

ID. Equations with Exponents

Sometimes you can see the same base on both sides.

Ex: $3^x = 9^{x-1}$

$3^x = \left(3^2\right)^{x-1}$

$x = 2x - 2$

$\boxed{2 = x}$

Otherwise, you need to use logarithms to solve.

Ex: $5^x = 43$

$\ln 5^x = \ln 43$

$x \ln 5 = \ln 43$

$\boxed{x = \dfrac{\ln 43}{\ln 5}}$

14) $2^x = 64$

15) $5^{x-3} = 125^x$

16) $3^{x^2} = 18$

17) $5^x = 128$

18) $e^{2x} = 144$

19) $10 = \dfrac{100}{1+e^{5x}}$

20) $40\left(1+3e^{-2x}\right) = 120$

21) I solved an equation and got the answer of $x = \frac{1}{2}\ln 16$. I double checked and everything! But the Answer Key says the answer is $x = \ln 4$. My friend says she got $x = 2\ln 2$ and she gets everything right. What's going on?

22) I solved an equation and got the answer of $x = \ln\frac{1}{2}$. But the Answer Key says the answer is $x = -\ln 2$. On the calculator, I can see that the two answers are equivalent. How can that be?

II. Inequalities

IIA. Linear

These are the easy ones. Just get x by itself.
Remember to reverse the inequality if you multiply or divide by a negative!

23) $4x - 5 < 7$

24) $3x + 4 \ge 6$

25) $5 - 3x \le 20$

IIB. Bigger Inequalities

Treat it like an equation instead of an inequality. For these, there are usually multiple solutions to the equation, and you will have to think critically about what intervals are included as solutions to the inequality.

Quadratics (and Absolute Value) are predictable.

$x^2 <$ will always have the solution set between the two solutions.

Ex: $x^2 < 9$

$\boxed{-3 < x < 3}$

$x^2 >$ will always have the solution set outside the two solutions.

Ex: $x^2 - x - 6 > 0$

$(x-3)(x+2) = 0$

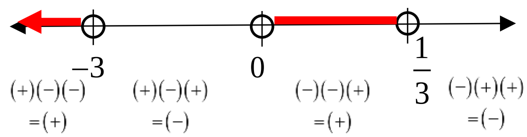
$x = 3, -2$

$\boxed{x < -2 \quad \vee \quad x > 3}$

For larger polynomials, test the intervals using Sign Analysis.

$$-3x(3x-1)(x+3)>0$$

Zeros: $0, \frac{1}{3}, -3$



Solution: $(-\infty, -3) \cup \left(0, \frac{1}{3}\right)$

26) $x^2 < 16$

27) $x^2 \geq 8$

28) $3x^2 + 1 > 4x$

29) $3x^2 + 7x - 6 \geq 0$

30) $3x^2 - 15x \leq 0$

31) $|2x+3| \leq 5$

32) $|3x-9| > 9$

33) $2x(x-3)(x+2)^2 \geq 0$

III. Algebra Techniques
IIIA. Polynomial Long Division

Be sure to line up the like terms and write neatly!

You can find some tutorials online if you need a review of the method.

34) Divide $6x^3 - 19x^2 + 12x - 7$ by $2x - 1$.

35) $(8x^3 - 14x^2 + 10x + 2) \div (4x - 3)$

IIIB. Complex Fractions

The best method for simplifying these is to multiply every term by the LCD.

Ex: $\dfrac{y - \dfrac{6}{y+1}}{1 + \dfrac{y^2-3}{3y+3}}$

$$= \frac{3(y+1)\cdot y \;-\; 3(y+1)\cdot \dfrac{6}{y+1}}{3(y+1)\cdot 1 \;+\; 3(y+1)\cdot \dfrac{y^2-3}{3(y+1)}} = \frac{3y(y+1)-18}{3(y+1)+y^2-3}$$

$$= \frac{3y^2+3y-18}{y^2+3y} = \frac{3(y+3)(y-2)}{y(y+3)}$$

$$= \boxed{\frac{3(y-2)}{y}}$$

36) Simplify: $\dfrac{\dfrac{2}{x-1}-\dfrac{2}{x+6}}{\dfrac{x}{x^2+5x-6}+\dfrac{1}{x-1}}$

37) Simplify: $\dfrac{\dfrac{x+y}{x}}{\dfrac{1}{x}+\dfrac{1}{y}}$

IV. Common Function Facts
IVA. Know Your Graphs

Pay attention to important points on the curve, the domain, any asymptotes, and the shape.

38) Match the function to its graph and write the domain of each.

Equation	i	ii	iii	iv	v	vi	vii	viii	ix
Graph									
Domain									

i. $y = x^2$

ii. $y = e^x$

iii. $y = \dfrac{1}{x}$

iv. $y = \ln x$

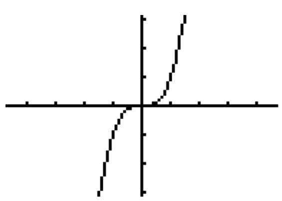
v. $y = \sqrt{x}$

vi. $y = x^3$

vii. $y = |x|$

viii. $y = \sin x$

ix. $y = \cos x$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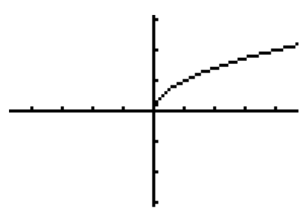
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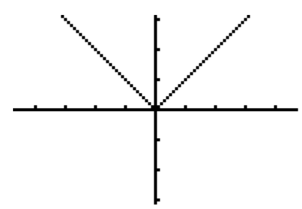
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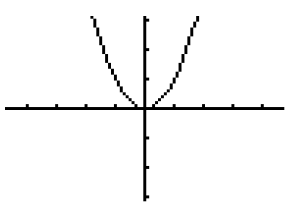
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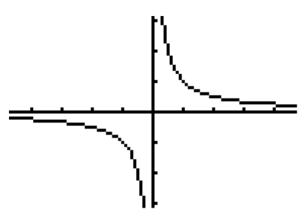
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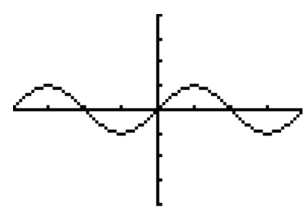
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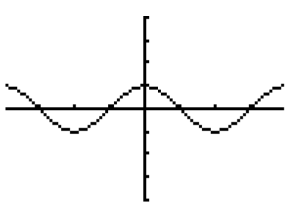
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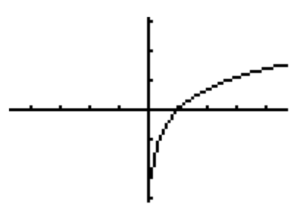
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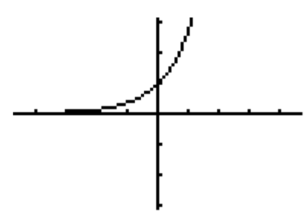
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IVC. Know Your Basic Transformations

Translations (Add/Subtract)

Up $f(x)+c$

Down $f(x)-c$

Left $f(x+c)$

Right $f(x-c)$

Reflections (Negate)

x-axis $-f(x)$

y-axis $f(-x)$

origin $-f(-x)$

Stretches/Compressions (Multiply)

Vertical Stretch $c\cdot f(x)$ where $c>1$

Vertical Compression $c\cdot f(x)$ where $c<1$

Horizontal Stretch $f(c\cdot x)$ where $c<1$

Horizontal Compression $f(c\cdot x)$ where $c>1$

39) Graph the function by thinking about transformations. You can also plug in a few numbers to be sure. Use graph paper (if available) or carefully make a graph in your notebook.

a) $y=\sqrt{x-3}$

b) $y=e^{x+2}$

c) $y=3+\sqrt{x}$

d) $y=4-x^2$

e) $y=e^{-x}$

f) $y=-e^{x}$

g) $y=-\ln x$

h) $y=|x+2|$

V. More About Lines

Slope-Intercept Form: $y=mx+b$

$m=$ slope
$b=$ y-intercept

Point-Slope Form: $y-y_1=m(x-x_1)$

$m=$ slope
(x_1,y_1) is any point on the line

40) Graph the line. Use graph paper (if available) or carefully make a graph in your notebook. State the coordinates of the x- and y-intercepts.

a) $y=\frac{2}{3}x-6$

b) $y=-\frac{1}{3}x+2$

c) $y+1=\frac{2}{3}(x-5)$

d) $y-2=3(x+3)$

41) Write the equation of the line:

a) Passing through $(-2,3)$ with slope $\frac{2}{3}$

b) Parallel to $y=-2x+3$ through $(4,0)$

c) Through the points $(-2,3)$ and $(2,-5)$

d) Passing through $(0,-3)$ with slope $\frac{3}{4}$

VI. A Tiny Bit of Geometry –Here are some formulas you should know.

Trapezoid

$$A = \frac{1}{2}(b_1 + b_2)h$$

Rectangular Prism

$$SA = 2lw + 2wh + 2lh$$

$$V = lwh$$

Cone

$$V = \frac{1}{3}\pi r^2 h$$

Circle

$$C = 2\pi r$$

$$A = \pi r^2$$

Cylinder

$$V = \pi r^2 h$$

Sphere

$$A = 4\pi r^2$$

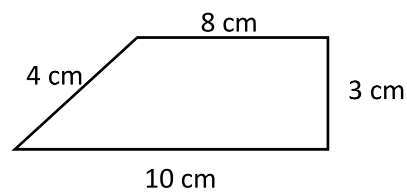
$$V = \frac{4}{3}\pi r^3$$

42) The circumference of a circle is 30π. Find the area of the circle.

43) A cylinder's height is 3 times its radius. Express the volume in terms of the length of the radius.

44) Find the surface area of a rectangular box with dimensions 20 in, 10 in, and 5 in.

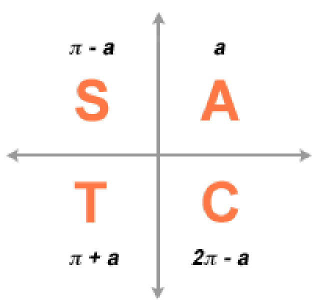
45) Find the area of the trapezoid:



46) A cone has a height of 6 and the circumference of the base is 16π. What is the volume of the cone?

VII. Trigonometry

VIIA. Basic Facts



where a is the reference angle.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

47) Fill in the chart with the exact values.

Degrees	30	45	60
Radians			
sin			
cos			
tan			

48) Draw and label the 4 intercept points on the Unit Circle.

49) Evaluate each using exact values.

a) $\tan\frac{2\pi}{3}$ c) $\cos\frac{7\pi}{4}$ e) $\csc\frac{4\pi}{3}$ g) $\sin\frac{\pi}{2}$

b) $\sin\frac{5\pi}{6}$ d) $\sec\frac{\pi}{3}$ f) $\sin\frac{5\pi}{4}$ h) $\cos\frac{3\pi}{2}$

VIIB. Trigonometric Equations

Ex: $\sin\theta = -\frac{\sqrt{3}}{2}$

ref $=\frac{\pi}{3}$ → 

(since $\sin\theta$ is neg) → $\boxed{\theta = \frac{4\pi}{3}, \frac{5\pi}{3}}$

50) Solve for θ on the interval $[0, 2\pi)$:

a) $2\sin\theta + 1 = 0$

b) $2\sin^2\theta + \sin\theta - 1 = 0$

c) $\sec\theta = \frac{2\sqrt{3}}{3}$

d) $2\cos^2\theta + 3\cos\theta + 1 = 0$

e) $2\cos^2\theta - \sqrt{2}\cos\theta = 0$

f) $\tan\theta = 1$

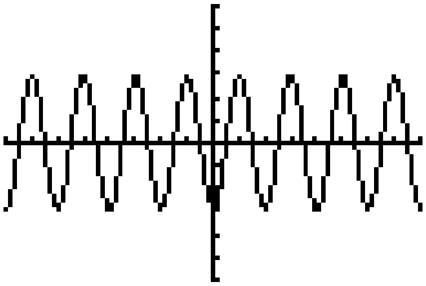
VIIC. Trigonometric Graphs

$y = A\sin Bx$
$y = A\cos Bx$ $|A| = \text{amplitude}$ $B = \text{frequency}$ $\frac{2\pi}{B} = \text{period}$

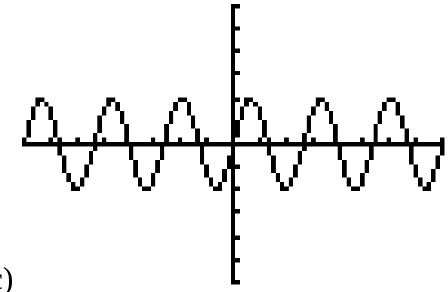
51) State the amplitude, frequency, and period of each equation. Use this information to sketch a graph of the function on the interval $[0, 2\pi]$. Use graph paper if available, otherwise make a good graph in your notebook.

a) $y = 4\sin 2x$ b) $y = 3\sin\frac{1}{4}x$ c) $y = -3\cos 2x$

53) State the amplitude and frequency of each of the graphs shown. (The window shows $-2\pi \le x \le 2\pi$)



a)



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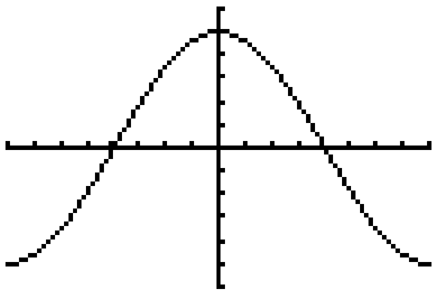

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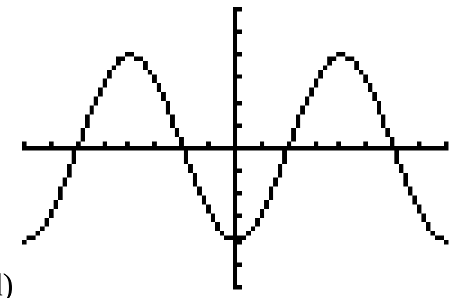
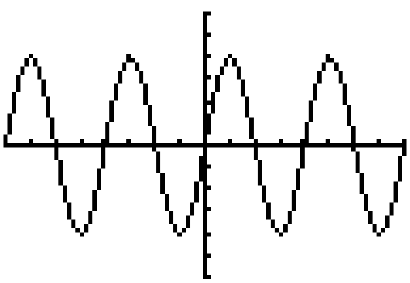


d)

54) Which equation fits the graph? (Window shows $-2\pi \le x \le 2\pi$)



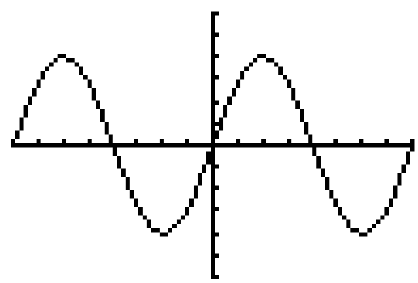
(A) $y = 4\sin x$

(B) $y = 4\sin 2x$

(C) $y = 4\cos x$

(D) $y = 4\cos 2x$

55) Which equation fits the graph? (Window shows $-2\pi \le x \le 2\pi$)



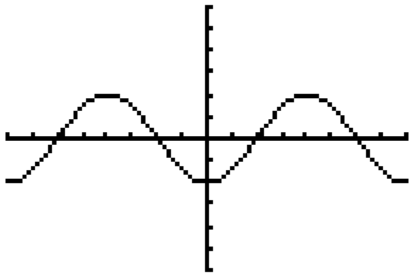
(A) $y = 4\sin x$

(B) $y = 4\sin 2x$

(C) $y = 4\cos x$

(D) $y = 4\cos 2x$

56) Which equation fits the graph? (Window shows $-2\pi \le x \le 2\pi$)



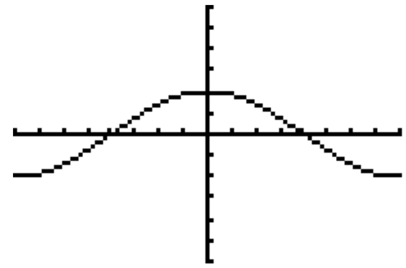
(A) $y = -2\cos x$

(B) $y = 2\cos\frac{1}{2}x$

(C) $y = -2\sin x$

(D) $y = 2\sin\frac{1}{2}x$

57) Which equation fits the graph? (Window shows $-2\pi \le x \le 2\pi$)



(A) $y = -2\cos x$

(B) $y = 2\cos\frac{1}{2}x$

(C) $y = -2\sin x$

(D) $y = 2\sin\frac{1}{2}x$

VIII. Intro to Calculus
VIIIA. Limits

You'll need multiple techniques:

- Think about the graphs of recognizable functions.
- Use factors/conjugates if $\frac{0}{0}$.
- Compare powers if $\frac{\infty}{\infty}$

58) $\lim_{x\to\infty} \ln x$

59) $\lim_{x\to\infty} \frac{1}{x}$

60) $\lim_{x\to 0^+} \ln x$

61) $\lim_{x\to 0^+} \frac{1}{x}$

62) $\lim_{x\to -\infty} e^x$

63) $\lim_{x\to 3} \frac{x^2 - x}{2x - 2}$

64) $\lim_{x\to 2} \frac{x^2 - x - 2}{x - 2}$

65) $\lim_{x\to\infty} \frac{3x^7 - \frac{2}{3}x + 4}{x^3 - 24}$

66) $\lim_{x\to -\infty} \frac{5x^4 - x}{x^3 - 10}$

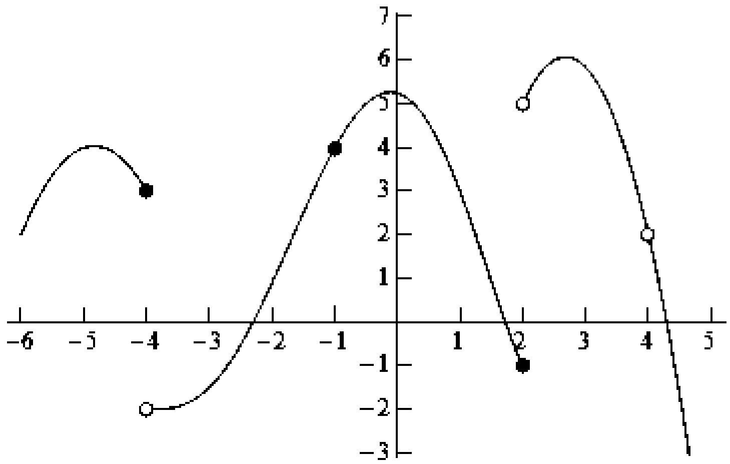
67) .0 $\lim_{x\to 5^-} \frac{2x-1}{x-5}$

68) $\lim_{x\to -\infty} \frac{9x+7}{x^3 - 14}$

69) $\lim_{x\to\infty} \frac{6 - 5x^2}{3x + 4}$

70) $\lim_{x\to\infty} \frac{4x^2 + 1}{2x^2 - 1}$

71) A function, $y = f(x)$, is shown here. Find the limits.



a) $\lim_{x\to -4^-} f(x)$

b) $\lim_{x\to -4^+} f(x)$

c) $\lim_{x\to -4} f(x)$

d) $\lim_{x\to -1} f(x)$

e) $\lim_{x\to 2} f(x)$

f) $\lim_{x\to 4^-} f(x)$

g) $\lim_{x\to 4^+} f(x)$

h) $\lim_{x\to 4} f(x)$

VIIIB. Rational Functions

Some facts you'll need:

- Zeros of numerator are x-intercepts
- Zeros of denominator are vertical asymptotes
- Zeros of both are holes (Removable Discontinuities); Do the limit to find the y-coord
- Limits to infinity find end-behavior asymptotes:
  - $\infty \rightarrow$ Oblique (divide)
  - # $\rightarrow$ Horizontal

72) Graph the function. Use graph paper, if available, or make a GOOD graph in your notebook. Find all x-intercepts, vertical asymptotes, holes, and asymptotes.

$$y = \frac{3x^2 - 3x - 18}{x^2 - 2x - 3}$$

73) Find the coordinates of any removable discontinuities for the function $y = \dfrac{2x(x-3)(x+1)}{(x+5)(x+1)}$.

74) What is the equation of the horizontal asymptote for the function $y = \dfrac{2x^2 - 3x + 4}{x^2 - 4}$?

VIIIC. Derivatives

Definition of the Derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

You'll also need: Power Rule, Product Rule, Quotient Rule, Chain Rule, and the transcendental rules.

75) Simplify: $\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$

76) Evaluate: $\lim_{h \to 0} \frac{3(2+h)^3 - 3(2)^3}{h}$

77) Simplify: $\lim_{h \to 0} \frac{4e^{(x+h)^2} - 4e^{x^2}}{h}$

78) Find the derivative.

a) $f(t) = t^{-1}\left(6 + 8t^{-2}\right)$

b) $g(x) = 4e^x \sin x$

c) $y = \frac{3}{x+2}$

d) $f(x) = 3x^4 + 4x^2 - 2x$

e) $f(x) = \frac{x^2}{4x-1}$

f) $f(x) = \frac{x}{3} + \frac{x^2}{4}$

g) $y = \frac{3\sin x}{9x + \cos x}$

h) $f(x) = \frac{x^5}{25} - \frac{2}{x^3} + 4x^2$

i) $y = x^3 \sin^2(4x)$

j) $y = \cos\left(5x^2\right)$

k) $f(x) = \sqrt{6x^3 + 3}$

l) $g(x) = \sin^3 x$

m) $f(x) = e^{4x - x^3}$

n) $f(x) = 4\sqrt{x^3}$

o) $f(x) = 6\sqrt{x} - 3x^{-2} + 4x^3 - 7x + 5$

79) Find the equation of the line tangent to $y = 4\cos x + 3$ at the point where $x = \frac{\pi}{3}$.

80) Find the equation of the line tangent to the graph of $f(x) = \sqrt{3x-2}$ at $x = 1$.

81) Find the linearization of $f(x) = \sqrt{x^2 + 9}$ at $x = -4$ and use it to approximate the value of $f(-3.9)$.

VIIID. Continuity & Differentiability

To be <u>continuous</u> at $x=c$,

- $f(c)$ must exist,
- $\lim_{x\to c} f(x)$ must exist, and
- $\lim_{x\to c} f(x)=f(c)$.

(No Vertical Asymptotes, Holes (Removable Discontinuities), or Jumps.)

To be <u>differentiable</u>, a continuous function's derivative must be continuous:

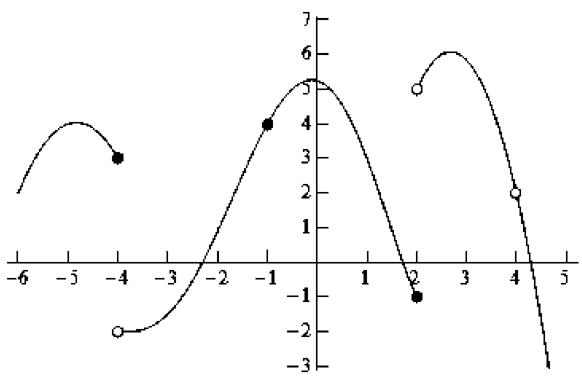
- The function must be continuous first and foremost!
- (No corners, cusps, or vertical tangent lines.)

82) For what values of x does the function fail to be continuous? Why? Describe the discontinuity.

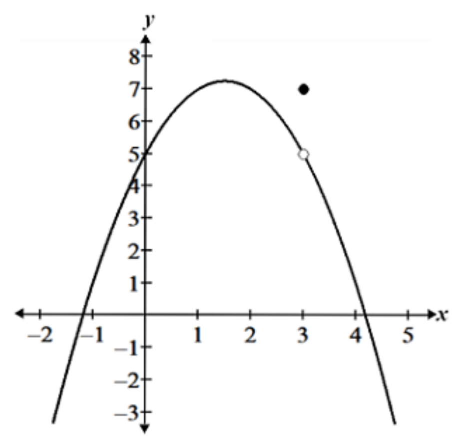
a) $f(x)=\dfrac{x^2-9}{x^2+4x+3}$

c) $f(x)=\begin{cases} x+3, & x<-2 \\ x^2-3, & -2\le x<2 \\ 2, & x\ge 2 \end{cases}$

b)



d)



83) The graph shown in part (d) above has a removable discontinuity at $x=3$. It can be expressed as $y=\begin{cases} -x^2+3x+4.75, & x\ne 3 \\ 7, & x=3 \end{cases}$. If the function is to be made continuous, the 7 must be replaced with what value?

84) Find the value of k that will make the following piecewise function continuous at $x=-1$.

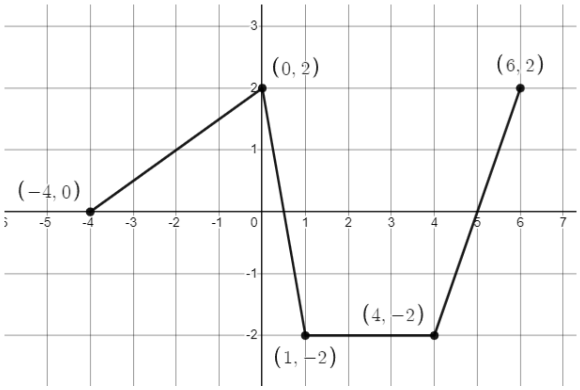
$$f(x)=\begin{cases} kx^2+1, & x\le -1 \\ 2x-k, & x>-1 \end{cases}$$

85) Find the value of k that will make the following piecewise function continuous at $x=2$.

$$f(x)=\begin{cases} 2x-3, & x\le 2 \\ x^2+k, & x>2 \end{cases}$$

86) Given the function $f(x)=\begin{cases} x^2+x, & x\le 1 \\ 3x-1, & x>1 \end{cases}$, is $f(x)$ continuous for all values of x? Is $f(x)$ differentiable for all values of x?

87) The graph of $y=f(x)$ is shown here. It is made up of line segments.



a) What is the value of $f'(-2)$?

b) What is the value of $f'(3)$?

c) At what values on the open interval $(-4,6)$ is the function not differentiable?

88) $y=x\sqrt{8-x^2}$

a) Find the domain.

b) Find the x-coordinates of any points of non-differentiability.

c) Find the x-coordinates of all critical points.

89) For what values of x does the function have a point of non-differentiability? Explain.

a) $y=\begin{cases} \sin x, & x<\pi \\ \cos x, & x\ge \pi \end{cases}$

b) $f(x)=3x^2-\dfrac{5}{x}$

c) $y=|x-3|$

d) $y=\begin{cases} 2x^3-1, & x\le 1 \\ 3x^2-2, & x>1 \end{cases}$

e) $g(x)=4\sqrt{x}-3x$

IX. Function Analysis

Increasing: $f'>0$ **Decreasing**: $f'<0$ **Horizontal**: $f'=0$

Rel Min: f' pos from left endpt, f' neg to right endpt, f' changes neg to pos

Rel Max: f' neg from left endpt, f' pos to right endpt, f' changes pos to neg

Conc Up: $f''>0$ **Conc Down**: $f''<0$

Pt of Inflec: f'' changes sign

90) For what values of x is the function $f(x)=2x^2-8x+3$ increasing?

91) For what values of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

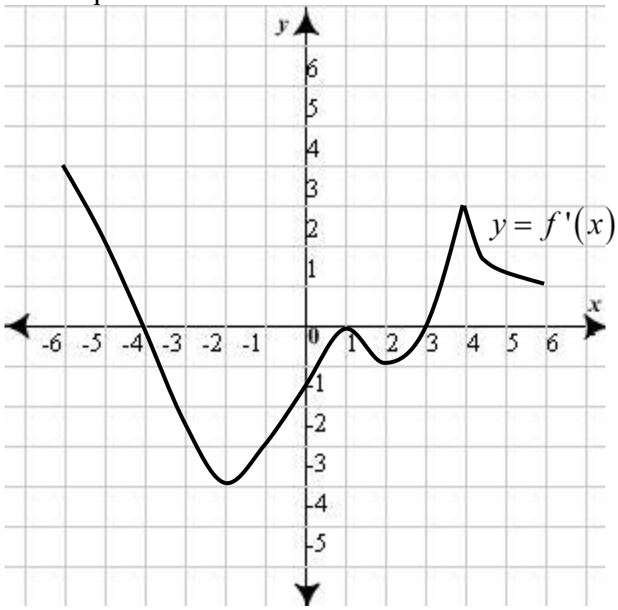
92) The function $f(x)=11+36x^2-2x^4$ is defined on the interval $[-5,5]$. Find the x coordinates of all relative minimum points.

93) For what values of x does the function $f(x)=4x\sqrt{x+2}$ have a relative minimum?

94) Given the function $g(x)=2x^4-26x^3+90x^2+18x-22$, find all intervals where the function is concave up.

95) Find all values of x for which the function $y=\frac{x^6}{15}+\frac{x^5}{20}-\frac{5x^4}{4}+10x+10$ has points of inflection.

96) The graph of the derivative of a function f is shown here on the domain $[-6,6]$. Justify your answers to the questions below.



a) For what values of x is $f(x)$ increasing?

b) For what values of x does $f(x)$ have a horizontal tangent line?

c) For what values of x does $f(x)$ have a relative minimum?

d) For what values of x is $f(x)$ concave up?

e) For what values of x does $f(x)$ have a point of inflection?

f) For what values of x is $f''(x)$ undefined?