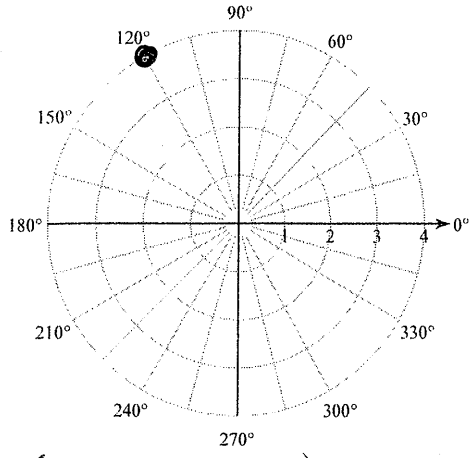
Pre-Calculus Name Key ID: 1

Warmup - Polar Points and Polar Parabolas

Find all pairs of polar coordinates that describe the same point as the provided polar coordinates.

1) $(4, 120°)$

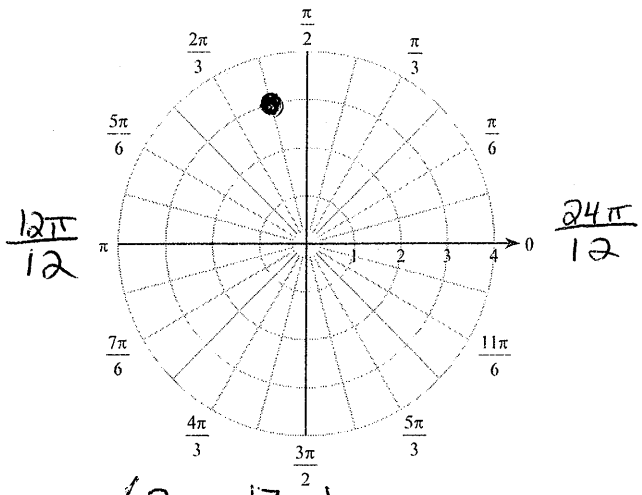


$(4, -240°)$

$(-4, 300°)$

$(-4, -60°)$

2) $\left(3, \frac{7\pi}{12}\right)$



$(3, -\frac{17\pi}{12})$

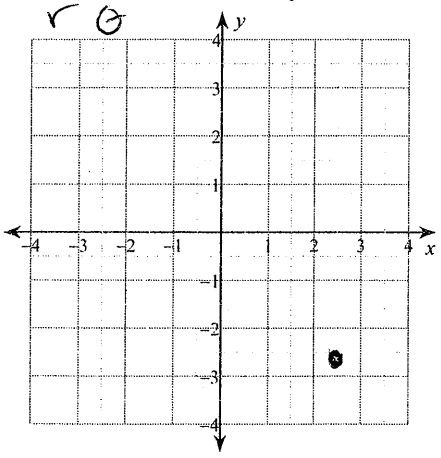
$(-3, -\frac{5\pi}{12})$

$(-3, \frac{19\pi}{12})$

Convert each pair of polar coordinates to rectangular coordinates.

3) $(4, 315°) \longrightarrow (2\sqrt{2}, -2\sqrt{2})$

r θ

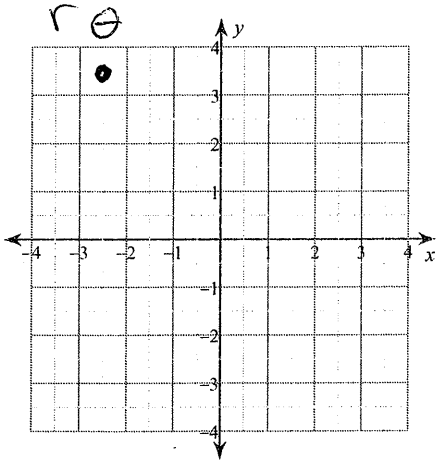


$X = 4 \cdot \cos(315) = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$

$Y = 4 \cdot \sin(315) = 4\left(\frac{-\sqrt{2}}{2}\right) = -2\sqrt{2}$

4) $\left(-4, \frac{2\pi}{3}\right) \longrightarrow (2, -2\sqrt{3})$

r θ



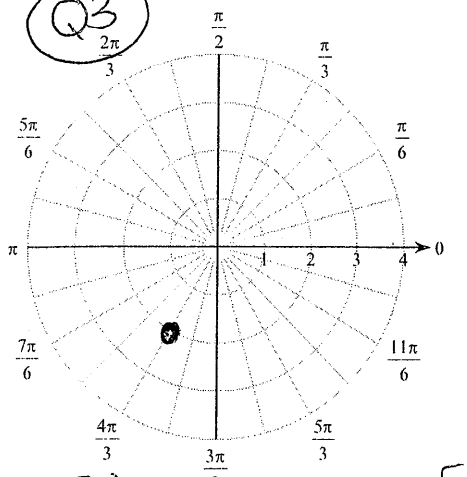
$X = -4\cos\left(\frac{2\pi}{3}\right) = -4\left(\frac{-1}{2}\right) = 2$

$Y = -4\sin\left(\frac{2\pi}{3}\right) = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

Convert each pair of rectangular coordinates to polar coordinates where $r > 0$ and $0 \le \theta < 2\pi$.

5) $(-1, -\sqrt{3}) \rightarrow \left(2, \frac{4\pi}{3}\right)$

Q3



$(-1)^2 + (-\sqrt{3})^2 = r^2$

$1 + 3 = r^2$

$2 = r$

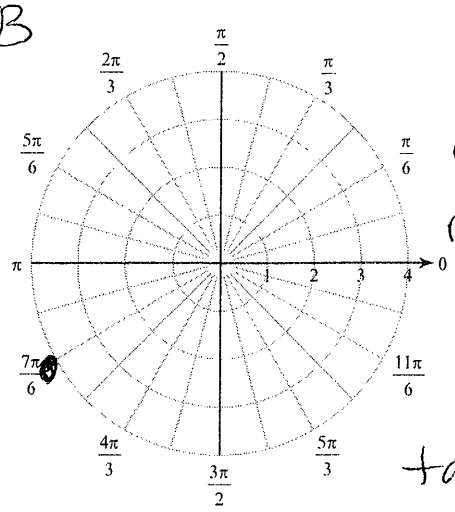
$\tan\theta = \frac{-\sqrt{3}}{-1}$

$\theta = \tan^{-1}(\sqrt{3})$

$\theta = \pi/3$ or $4\pi/3$

6) $\left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right) \rightarrow \left(3\sqrt{2}, \frac{7\pi}{6}\right)$

Q3



$r^2 = \left(-\frac{3\sqrt{3}}{2}\right)^2 + \left(-\frac{3}{2}\right)^2$

$r^2 = \frac{27}{2} + \frac{9}{2} = \frac{36}{2}$

$r^2 = 18$

$r = 3\sqrt{2}$

$\tan\theta = \frac{\left(-\frac{3}{2}\right)}{\left(-\frac{3\sqrt{3}}{2}\right)} = \frac{\sqrt{3}}{3}$

$\frac{-3}{2} \cdot \frac{-2}{3\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\theta = \frac{\pi}{6}$ $\frac{7\pi}{6}$

Q1 or Q3

Each polar equation describes a conic section with a focus at the origin. Find the equation of the directrix associated with the focus at the origin, classify the conic section, and graph the polar equation.

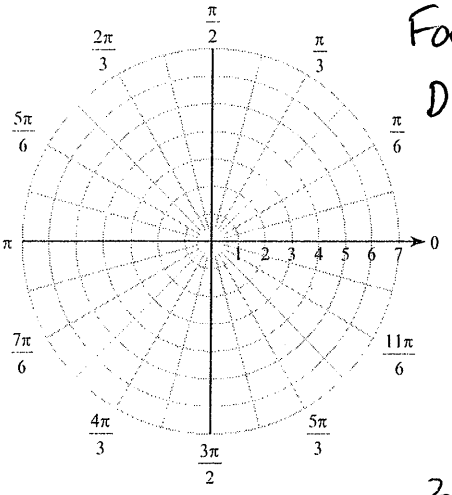
7) $r = \frac{6}{1 + \cos\theta}$

$\frac{1}{12} = \frac{1}{4c}$ $c = 3$

Vertex $(3, 0)$

Focus $(0, 0)$

Directrix $x = 6$



$r + r\cos\theta = 6$

$\sqrt{x^2+y^2} + x = 6$

$\sqrt{x^2+y^2} = 6 - x$

$x^2 + y^2 = 36 - 12x + x^2$

$y^2 = 36 - 12x$

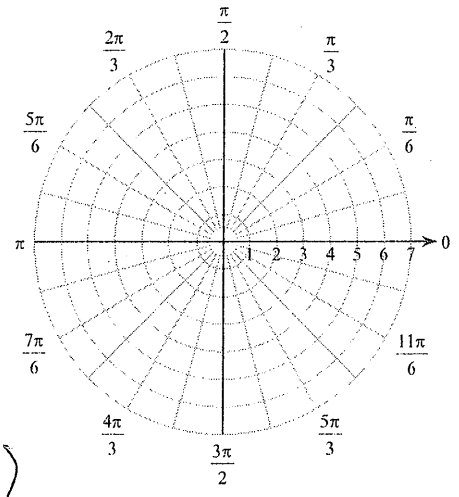
$\frac{y^2 - 36 = -12x}{-12}$

$\boxed{-\frac{1}{12}y^2 + 3 = x}$

Horizontal Parabola

Opens left

8) $r = \frac{1}{1 - \sin\theta}$



$\frac{1}{2} = \frac{1}{4c}$

$c = \frac{1}{2}$

Vertex $\left(0, -\frac{1}{2}\right)$

Focus $(0, 0)$

Directrix $y = -1$

$r - r\sin\theta = 1$

$\sqrt{x^2+y^2} - y = 1$

$\sqrt{x^2+y^2} = 1 + y$

$x^2 + y^2 = 1 + 2y + y^2$

$x^2 = 1 + 2y$

$\frac{x^2 - 1 = 2y}{2}$

$\boxed{\frac{1}{2}x^2 - \frac{1}{2} = y}$

Vertical Parabola

Opens up