

Voting Review

Key

1.

(a) How do you decide the winner of an election using the Plurality Method?

Candidate with most 1<sup>st</sup> place votes

(b) How do you decide the winner of an election using the Borda Count Method?

Assign points 4-1<sup>st</sup> place 3-2<sup>nd</sup> place 2-3<sup>rd</sup> place 1-4<sup>th</sup> place

(c) How do you decide the winner of an election using the Sequential Runoff Method?

Eliminate candidate with least 1<sup>st</sup> place - recount votes  
Then eliminate candidate with least 1<sup>st</sup> place - recount again, etc

(d) How do you decide the winner of an election using the Method of Pairwise Comparisons?

How do you know how many pairs?  $\frac{n(n-1)}{2}$  pairs

Pair every candidate head to head -

2.

(a) What is a plurality candidate? Does every election have a plurality candidate?

Most 1<sup>st</sup> place

(b) What is a majority candidate? Does every election have a majority candidate?

Most 1<sup>st</sup> place - more than 50%

(c) What is a Condorcet candidate? Does every election have a Condorcet candidate?

Candidate who beats all others head to head

3. An election is to be decided using the Borda count method. There are four candidates (A,B,C,D) in this election.

(a) How many points are given out by one ballot?  $\textcircled{10}$   $4+3+2+1$

(b) If there are 110 voters in the election, what is the total number of points given out to the candidates?

$$110 \cdot 10 = \boxed{1100 \text{ points}}$$

(c) If candidate A gets 320 points, candidate B gets 290 points, and candidate C gets 170 points, how many points did candidate D get?

$$1100 - 320 - 290 - 170 = \boxed{320 \text{ left for D}}$$

(d) Who is the winner of this election using the Borda Count Method?

A and D tie

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4. Consider the following preference schedule:

Number of Voters	14	10	8	4	1	
1 <sup>st</sup> choice	4	A	C	D	B	C
2 <sup>nd</sup> choice	3	B	B	C	D	D
3 <sup>rd</sup> choice	2	C	D	B	C	B
4 <sup>th</sup> choice	1	D	A	A	A	A

(a) Find the winner of the election using the Plurality Method. **A with 14**

(b) Find the winner of the election using the Borda Count Method.

$A: 14(4) + 10(1) + 8(1) + 4(1) + 1(1) = 79$       $D: 14(1) + 10(2) + 8(4) + 4(3) + 1(3) = 81$   
 $B: 14(3) + 10(3) + 8(2) + 4(4) + 1(2) = 106$   
 $C: 14(2) + 10(4) + 8(3) + 4(2) + 1(4) = 104$

**B wins** (violates Condorcet)

(c) Find the winner of the election using the Sequential Runoff Method.

Eliminate B     Eliminate C  
 $A=14$     $D=12$       $A=14$     $D=23$   
 $B=-$   
 $C=11$

**D wins** (Violates Condorcet)

(d) Find the winner of the election using the Method of Pairwise Comparisons.

$\frac{4-1}{2} = 6$  pairs

$A-B$  14-23      $B-C$  18-19  
 $A-C$  14-23      $B-D$  28-9  
 $A-D$  14-23      $C-D$  25-12

$\frac{A}{0}$     $\frac{B}{11}$     $\frac{C}{11}$     $\frac{D}{1}$

**C wins** (also a Condorcet Candidate)

(e) Find the winner of the election using the Basic Runoff Method.

$A$  vs  $C$  14-23     **C wins**

5. Consider the following preference schedule:

Number of Voters	10	6	5	4	2	
1 <sup>st</sup> choice	4	A	B	B	C	D
2 <sup>nd</sup> choice	3	C	D	C	A	C
3 <sup>rd</sup> choice	2	B	C	A	D	B
4 <sup>th</sup> choice	1	D	A	D	B	A

= 27

(a) Is there a Condorcet candidate in this election?

$A-B$  14-13      $A-D$  19-8      $B-D$  21-6     **Yes - C**  
 $A-C$  10-17      $B-C$  11-16      $C-D$  19-8

(b) Is there a majority candidate in this election?

No - need 14 votes

(c) Find the winner using the Plurality-with-Elimination Method. (Sequential Runoff)

Eliminate D     Eliminate C  
 $A$     $B$     $C$       $A$     $B$   
 10   11   4     14   13  
**A wins**

(d) Suppose that B drops out of the race. Find the winner of the recount when B is removed using the Plurality-With-Elimination method.

$\frac{A}{10}$     $\frac{C}{9}$     $\frac{D}{8}$      Eliminate D      $\frac{A}{10}$     $\frac{C}{17}$      **C wins**

(e) You have just shown that Plurality-With-Elimination violates several fairness criteria. Which ones? Explain your answer.

Plurality with Elimination / Sequential Runoff violates the Condorcet Criterion

Plurality with Elimination violates the Independence of Irrelevant Alternatives

Voting Review

6. An election is to be decided using the Borda count method. There are four candidates (A,B,C,D) in this election.

Duplicate

- (a) How many points are given out by one ballot?
- (b) If there are 110 voters in the election, what is the total number of points given out to the candidates?
- (c) If candidate A gets 320 points, candidate B gets 290 points, and candidate C gets 170 points, how many points did candidate D get?
- (d) Who is the winner of this election using the Borda Count Method?

7. For this question, use the following preference schedule:

# of Voters	5	3	2	4	1	= 15
1st choice	3	A	A	B	C	C
2nd choice	2	B	C	A	A	B
3rd choice	1	C	B	C	B	A

- (a) How many votes were cast in the election? 15
- (b) How many votes are needed for a majority? 8
- (c) Who is the plurality winner? A
- (d) Does the plurality winner have a majority? Yes
- (e) In the Borda Count Method, how many points does candidate C get?  $5(1) + 3(2) + 2(1) + 4(3) + 1(3) = 28$
- (f) Between candidates B and C, who is preferred head-to-head? B-C  
7-8 C wins
- (g) In the first round of plurality with elimination, is there a winning candidate? If not, which candidate is eliminated first? There is a winner  $\rightarrow$  A! A has a majority

8. Find the Banzhaf Power Index

[10: 8, 6, 3]

$P_1, P_2, P_3$

$$\frac{P_1, P_2}{P_1, P_3} \quad P_1 = \frac{3}{5} \quad 60\%$$

$$\frac{P_1, P_2, P_3}{P_1, P_2, P_3} \quad P_2 = \frac{1}{5} \quad 20\%$$

$$P_3 = \frac{1}{5} \quad 20\%$$

Find the Shapley-Shubik Power Index

[10: 8, 6, 3]

$$\begin{matrix} P_1, P_2, P_3 & P_2, P_1, P_3 & P_3, P_1, P_2 \\ P_1, P_3, P_2 & P_2, P_3, P_1 & P_3, P_2, P_1 \end{matrix}$$

$$P_1 = \frac{4}{6} \quad P_2 = \frac{1}{6} \quad P_3 = \frac{1}{6}$$

$$66.7\% \quad 16.7\% \quad 16.7\%$$

**Voting Review**

9. What is the total number of pairwise comparisons in an election among 6 candidates? 15 pairs

$$\frac{6(5)}{2} = 15$$

10. "If there is a choice that has a majority of the first-place votes in an election, then that choice should be the winner of the election." This fairness criterion is called the

- A monotonicity criterion
- B Condorcet criterion
- C majority criterion
- D independence of irrelevant alternatives criterion
- E None of the above

11. "If in an election there is a Condorcet candidate, then such a candidate should be the winner of the election." This statement is another way to phrase the

- A independence of irrelevant alternatives criterion
- B Condorcet criterion
- C monotonicity criterion
- D majority criterion
- E None of the above

12. An election is held among four candidates (A, B, C, and D). Using a voting method we will call X, the winner of the election is candidate A. Due to an irregularity in the original vote count a recount is required. Before the recount takes place, candidate B drops out of the race. In the recount, still using voting method X, candidate D wins the election. Based on this information, we can say that voting method X violates the

- A majority criterion
- B monotonicity criterion
- C Condorcet criterion
- D independence of irrelevant alternatives criterion
- E None of the above

13. Arrow's Impossibility Theorem implies

- A that in every election, no matter what voting method we use, at least one of the four fairness criteria will be violated.
- B that every voting method can potentially violate each one of the four fairness criteria.
- C that in every election, each of the voting methods must produce a different winner.
- D that it is impossible to have a voting method that satisfies all four of the fairness criteria.

14. The results of a hypothetical election are summarized in the table below. An "X" indicates that the voter approves of the candidate.

Candidates	VOTERS							
	Richard	Sally	Thomas	Uma	Vera	Walter	Yvette	Zoe
ADAMS	X		X	X	X		X	X
BARNES			X		X	X		
COLLINS	X				X	X	X	

6  
3  
4

a) Who is the approval winner?

Adams