Verifying Trigonometric Identities Notes

$$\sec(x)-\sin(x)\tan(x)=\cos(x)$$

$$\frac{1}{\cos(x)}-\left(\frac{\sin(x)}{1}\cdot\frac{\sin(x)}{\cos(x)}\right)$$

$$\frac{1-\sin^2(x)}{\cos(x)}$$

$$\frac{\cos^2(x)}{\cos(x)} = \cos(x) \checkmark$$

$$\tan(x)-\sin(x)\cos(x)=\tan(x)\sin^2(x)$$

$$\frac{\sin(x)}{\cos(x)}-\frac{\sin(x)\cos(x)\cos(x)}{\cos(x)}$$

$$\frac{\sin(x)-\sin(x)\cos^2(x)}{\cos(x)}$$

GCF $\frac{\sin(x)[1-\cos^2(x)]}{\cos(x)}$

$$\frac{\sin(x)}{\cos(x)}\cdot\frac{\sin^2(x)}{1} = \tan(x)\sin^2(x) \checkmark$$

$$\frac{\sin(x)}{1+\cos(x)}=\csc(x)-\cot(x)$$

multiply by the conjugate

$$\frac{\sin(x)[1-\cos(x)]}{(1+\cos(x))(1-\cos(x))}$$

$$\frac{\sin(x)[1-\cos(x)]}{1-\cos^2(x)}$$

$$\frac{\sin(x)[1-\cos(x)]}{\sin^2(x)} = \frac{1-\cos(x)}{\sin(x)} = \frac{1}{\sin(x)}-\frac{\cos(x)}{\sin(x)} = \csc(x)-\cot(x)$$

$\checkmark$

Verifying Trig Identities using the Pythagorean Relationships

1) $cot(x) \cdot [csc^2(x) - 1] = cot^3(x)$

$\cot(x) \cdot \cot^2(x)$

$\cot^3(x)$ ✓

2) $sec(\theta) \cdot \dfrac{sin(\theta)}{tan(\theta)} = 1$

$\left(\frac{1}{\cos\theta}\right) \cdot \left(\frac{\sin\theta}{1} \cdot \frac{\cos\theta}{\sin\theta}\right)$

$\left(\frac{1}{\cos\theta}\right) \cdot \left(\frac{\cos\theta}{1}\right)$

$\boxed{1}$ ✓

3) $csc(\theta) - sin(\theta) - cos(\theta)cot(\theta) = 0$

$\frac{1}{\sin\theta} - \frac{\sin\theta}{1} - \left(\frac{\cos\theta}{1} \cdot \frac{\cos\theta}{\sin\theta}\right)$

$\underbrace{\frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}} - \frac{\cos^2\theta}{\sin\theta}$

$\frac{\cos^2\theta}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta} = 0$ ✓

4) $6 - sin^2(x) = 5 + cos^2(x)$

$6 - (1 - \cos^2\theta)$

$6 - 1 + \cos^2\theta$

$\boxed{5 + \cos^2\theta}$ ✓

Verifying Trigonometric Identities Practice

$$\cos(x)\cot(x)-\csc(x)=-\sin(x)$$

$$\cos(x)\cdot\frac{\cos(x)}{\sin(x)}-\frac{1}{\sin(x)}$$

$$\frac{\cos^2(x)-1}{\sin(x)}=\frac{-\sin^2(x)}{\sin(x)}=\boxed{-\sin(X)}\ \checkmark$$

$$\frac{\sec(\theta)\csc(\theta)}{\tan(\theta)+\cot(\theta)}=1\ \frac{\left(\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}\right)}{\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)}=\frac{\left(\frac{1}{\cos\theta\sin\theta}\right)}{\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)}=\frac{\left(\frac{1}{\cos\theta\sin\theta}\right)}{\left(\frac{1}{\sin\theta\cos\theta}\right)}$$

$$=\boxed{1}\ \checkmark$$

$$[\sin(\theta)-\cos(\theta)]^2+[\sin(\theta)+\cos(\theta)]^2=2$$

$$=[\sin^2\theta-\sin\theta\cos\theta-\sin\theta\cos\theta+\cos^2\theta]+[\sin^2\theta+\sin\theta\cos\theta+\sin\theta\cos\theta+\cos^2\theta]$$

$$=2\sin^2\theta+2\cos^2\theta=2(\sin^2\theta+\cos^2\theta)=2(1)=\boxed{2}\ \checkmark$$

$$\cot(x)-\cos(x)\sin(x)=\cot(x)\cos^2(x)$$

$$\frac{\cos(x)}{\sin(x)}-\frac{\cos(x)\sin(x)\sin(x)}{\sin(x)}$$

GCF $$\frac{\cos(x)-\cos(x)\sin^2(x)}{\sin(x)}$$

$$\frac{\cos(x)[1-\sin^2(x)]}{\sin(x)}=\frac{\cos(x)}{\sin(x)}\cdot\frac{\cos^2(x)}{1}=\boxed{\cot(x)\cos^2(x)}\ \checkmark$$

$\tan^2\theta + 1 = \sec^2\theta$

Leave this as is!!

$$\frac{\tan(x)}{1+\sec(x)} + \frac{1+\sec(x)}{\tan(x)} = 2\csc(x)$$

$$\frac{\tan(x)\cdot\tan(x)}{\tan(x)[1+\sec(x)]} + \frac{[1+\sec(x)][1+\sec(x)]}{\tan(x)[1+\sec(x)]} = \frac{\underbrace{\tan^2(x)+1}_{\sec^2\theta} + 2\sec(x) + \sec^2(x)}{\tan(x)[1+\sec(x)]} = \frac{2\sec^2(x)+2\sec(x)}{\tan(x)[1+\sec(x)]}$$

$$= \frac{2\sec(x)[\sec(x)+1]}{\tan(x)[\sec(x)+1]} = \frac{2\sec(x)}{\tan(x)} = \frac{\frac{2}{\cos(x)}}{\frac{\sin x}{\cos x}} = \frac{2}{\cos(x)}\cdot\frac{\cos(x)}{\sin(x)} = \boxed{2\csc(x)}$$

$$\cot^2(x) - \cos^2(x) = \cot^2(x)\cos^2(x)$$

$$\frac{\cos^2(x)}{\sin^2(x)} - \frac{\cos^2(x)\sin^2(x)}{\sin^2(x)}$$

GCF

$$\frac{\cos^2(x)[1-\sin^2(x)]}{\sin^2(x)} = \frac{\cos^2(x)}{\sin^2(x)}\cdot\frac{\cos^2(x)}{1} = \boxed{\cot^2(x)\cos^2(x)}$$

$$\tan^2(x) - \sin^2(x) = \tan^2(x)\sin^2(x)$$

$$\frac{\sin^2(x)}{\cos^2(x)} - \frac{\sin^2(x)\cos^2(x)}{\cos^2(x)}$$

GCF

$$\frac{\sin^2(x)[1-\cos^2(x)]}{\cos^2(x)} = \frac{\sin^2(x)}{\cos^2(x)}\cdot\frac{\sin^2(x)}{1} = \boxed{\tan^2(x)\sin^2(x)}$$

$$\frac{\cot(x)-\tan(x)}{\sin(x)\cos(x)} = \csc^2(x) - \sec^2(x)$$

$$\frac{\frac{\cos(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)}}{\sin(x)\cos(x)} = \frac{\left(\frac{\cos^2(x)-\sin^2(x)}{\sin(x)\cos(x)}\right)}{\left(\frac{\sin(x)\cos(x)}{1}\right)} = \frac{\cos^2(x)-\sin^2(x)}{\sin(x)\cos(x)}\cdot\frac{1}{\sin(x)\cos(x)}$$

$$= \frac{\cos^2 x - \sin^2 x}{\sin^2 x\cos^2(x)} = \frac{\cos^2(x)}{\sin^2(x)\cos^2(x)} - \frac{\sin^2(x)}{\sin^2(x)\cos^2(x)} = \frac{1}{\sin^2(x)} - \frac{1}{\cos^2(x)} = \boxed{\csc^2(x) - \sec^2(x)}$$

Split into two fractions