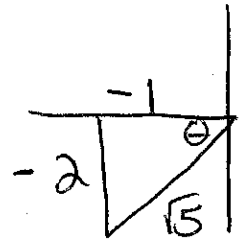
Honors Pre-Calculus Name Key

Unit Circle - Unit Review

Find the value of the trig function indicated.

1) Find sec θ if tan $\theta = 2$ and the triangle is in quadrant 3.

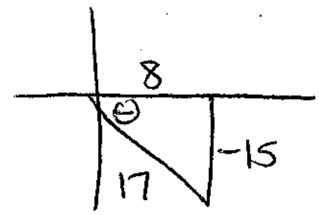
$\boxed{-\sqrt{5}}$



$\sec\theta = \frac{\sqrt{5}}{-1}$

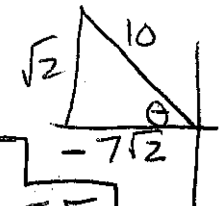
2) Find cot θ if sin $\theta = -\frac{15}{17}$ and the triangle is in quadrant 4.

$\boxed{-\frac{8}{15}}$



3) Find the other five trigonometric ratios if sin $\theta = \frac{\sqrt{2}}{10}$ and the triangle is in quadrant 2.

$\sqrt{98}$
$7\sqrt{2}$

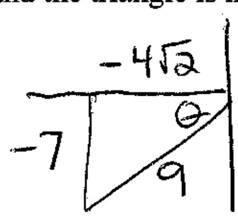


cos $\theta = \frac{-7\sqrt{2}}{10}$ tan $\theta = \frac{-1}{7}$ cot $\theta = -7$ sec $\theta = -\frac{5\sqrt{2}}{7}$ csc $\theta = 5\sqrt{2}$

$\frac{\cancel{\sqrt{2}}}{-7\cancel{\sqrt{2}}}$ $\frac{-7\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$ $\frac{10}{-7\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{10\sqrt{2}}{14}$ $\frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2}$

4) Find tan θ if csc $\theta = -\frac{9}{7}$ and the triangle is in quadrant 3.

$\boxed{\frac{7\sqrt{2}}{8}}$



$\tan\theta = \frac{-7}{-4\sqrt{2}} = \frac{7\sqrt{2}}{8}$

Solve each equation for $0 \le \theta < 360$.

5) $-3 + \sin\theta = \frac{-6 + \sqrt{3}}{2}$

$\boxed{\{60, 120\}}$

$\sin\theta = \frac{-6+\sqrt{3}}{2} + \frac{6}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

6) $4\cos\theta = -2$

$\boxed{\{120, 240\}}$

$\cos\theta = \frac{-1}{2}$

$\theta = \cos^{-1}\left(\frac{-1}{2}\right)$

Find the exact value of each trigonometric function.

7) $\csc -\frac{5\pi}{3} = \csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin(\pi/3)} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}$

$\boxed{\frac{2\sqrt{3}}{3}}$

8) $\cot 150° = \frac{\cos 150}{\sin 150} = \frac{(-\sqrt{3}/2)}{(1/2)}$

$\boxed{-\sqrt{3}}$

$-\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$

9) $\cos -585° = \cos(135)$

$\boxed{-\frac{\sqrt{2}}{2}}$

10) $\sec 180° = \frac{1}{\cos(180)} = \frac{1}{(-1)}$

$\boxed{-1}$

11) $\sec -\frac{\pi}{2} = \sec\left(\frac{3\pi}{2}\right) = \frac{1}{\cos\left(\frac{3\pi}{2}\right)} = \frac{1}{0}$

$\boxed{\text{Undefined}}$

12) $\sec -\frac{2\pi}{3} = \sec\left(\frac{4\pi}{3}\right) = \frac{1}{\cos\frac{4\pi}{3}} = \frac{1}{\left(-\frac{1}{2}\right)}$

$\boxed{-2}$

13) $\csc -855° = \csc(225) = \frac{1}{\sin(225)} = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)}$

$\boxed{-\sqrt{2}}$

$\frac{1}{1} \cdot -\frac{2}{\sqrt{2}} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{-2\sqrt{2}}{2} = -\sqrt{2}$

14) $\cot -330° = \cot(30) = \frac{\cos(30)}{\sin(30)} = \frac{(\sqrt{3}/2)}{(1/2)}$

$\boxed{\sqrt{3}}$

$\frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \sqrt{3}$

Find the value of each expression below. Give angles in degrees AND radians.

15) $\sin^{-1} \frac{1}{2}$

Q1 $\frac{\pi}{6}$ 30 Q2 $\frac{5\pi}{6}$ 150

16) $\tan^{-1}(-\sqrt{3})$

Q2 $\frac{2\pi}{3}$ 120° Q4 $\frac{5\pi}{3}$ 300

17) $\tan^{-1} -\frac{\sqrt{3}}{3}$

Q2 $5\pi/6$ 150° Q4 $11\pi/6$ 330

18) $\tan^{-1} -1$

Q2 $3\pi/4$ 135 Q4 $7\pi/4$ 315

19) $\cos^{-1} \frac{\sqrt{2}}{2}$

Q1 $\frac{\pi}{4}$ 45 Q4 $\frac{7\pi}{4}$ 315

20) $\csc^{-1}(-\sqrt{2})$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Q3 $\frac{5\pi}{4}$ 225° Q4 $\frac{7\pi}{4}$ 315

Find the exact value of each using a sum/difference formula.

21) cos 165

$$\boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

22) sin 195

$$\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

23) tan 195

$$\boxed{2-\sqrt{3}}$$

24) $\sin\frac{17\pi}{12}$

$$\boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

25) $\tan\frac{11\pi}{12}$

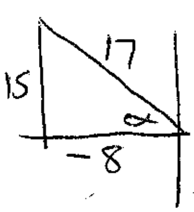
$$\boxed{\sqrt{3}-2}$$

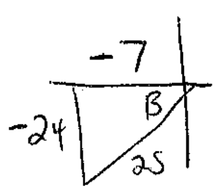
26) $\cos\frac{19\pi}{12}$

$$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

(# 21 – 26 → see work on a separate page!)

27) Suppose $\tan\alpha = -15/8$ and $\cos\beta = -7/25$ where $\pi/2<\alpha<\pi$ and $\pi<\beta<3\pi/2$ (set up a "cheat sheet!"). Then, find $\sin(\alpha+\beta)$.





$\sin\alpha = 15/17$ + $\sin\beta = \frac{-24}{25}$ −

$\cos\alpha = -8/17$ − $\cos\beta = -7/25$ −

$\tan\alpha = -15/8$ − $\tan\beta = 24/7$ +

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\left(\frac{15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{-8}{17}\right)\left(\frac{-24}{25}\right)$$

$$\frac{-105}{425} + \left(\frac{192}{425}\right)$$

$$\boxed{\frac{87}{425}}$$

Find a positive and a negative coterminal angle for each given angle.

1) $-195°$

$\boxed{165° \text{ and } -555°}$

2) $388°$

$\boxed{28° \text{ and } -332°}$

3) $\frac{4\pi}{5}$

$\boxed{\frac{14\pi}{5} \text{ and } -\frac{6\pi}{5}}$

4) $\frac{13\pi}{12}$

$\boxed{\frac{37\pi}{12} \text{ and } -\frac{11\pi}{12}}$

Find a coterminal angle between 0 and 2π for each given angle.

5) $\frac{51\pi}{10}$

$\boxed{\frac{11\pi}{10}}$

6) $-\frac{146\pi}{45}$

$\boxed{\frac{34\pi}{45}}$

Find the reference angle.

7) $\frac{5\pi}{9}$ Q2

$\boxed{\frac{4\pi}{9}}$

8) $220°$ Q2

$\boxed{40°}$

9) $\frac{19\pi}{12}$ Q4

$\boxed{\frac{5\pi}{12}}$

10) $285°$ Q4

$\boxed{75°}$

Convert each degree measure into radians.

11) $-555°$

$\boxed{-\frac{37\pi}{12}}$

12) $250°$

$\boxed{\frac{25\pi}{18}}$

13) $940°$

$\boxed{\frac{47\pi}{9}}$

14) $765°$

$\boxed{\frac{17\pi}{4}}$

Convert each radian measure into degrees.

15) $-\frac{17\pi}{9}$

$\boxed{-340°}$

16) $\frac{53\pi}{18}$

$\boxed{530°}$

17) $\frac{9\pi}{4}$

$\boxed{405°}$

18) $\frac{17\pi}{9}$

$\boxed{340°}$

21) $\cos(165) = \cos(135+30)$

$\cos 135 \cos 30 - \sin 135 \sin 30$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

22) $\sin(195) = \sin(150+45) = \sin 150 \cos 45 + \cos 150 \sin 45$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

23) $\tan(195) = \tan(150+45) = \frac{\tan(150) + \tan(45)}{1 - [\tan 150 \tan 45]}$

$$\frac{\left(-\frac{\sqrt{3}}{3}\right) + (1)}{1 - \left[\left(-\sqrt{3}/3\right)(1)\right]} = \frac{\left(\frac{-\sqrt{3}+3}{3}\right)}{1 - \left(-\frac{\sqrt{3}}{3}\right)} = \frac{\left(\frac{-\sqrt{3}+3}{3}\right)}{\left(\frac{3+\sqrt{3}}{3}\right)}$$

$$\frac{-\sqrt{3}+3}{\cancel{3}} \cdot \frac{\cancel{3}}{3+\sqrt{3}} = \frac{-\sqrt{3}+3}{3+\sqrt{3}} \frac{(3-\sqrt{3})}{(3-\sqrt{3})}$$

$$\frac{-3\sqrt{3} + 3 + 9 - 3\sqrt{3}}{9-3} = \frac{12-6\sqrt{3}}{6} = \boxed{2-\sqrt{3}}$$

24) $\sin\left(\frac{17\pi}{12}\right) = \sin\left(\frac{9\pi}{12} + \frac{8\pi}{12}\right) = \sin\left(\underset{\alpha}{\frac{3\pi}{4}} + \underset{\beta}{\frac{2\pi}{3}}\right)$

$\sin(3\pi/4)\cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{3\pi}{4}\right)\sin(2\pi/3)$

$$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$$

25) $\tan\left(\frac{11\pi}{12}\right) = \tan\left(\frac{9\pi}{12} + \frac{2\pi}{12}\right) = \tan\left(\overset{\alpha}{\frac{3\pi}{4}} + \overset{\beta}{\frac{\pi}{6}}\right)$

$$\frac{\tan\left(\frac{3\pi}{4}\right) + \tan\left(\pi/6\right)}{1 - \left[\tan\left(\frac{3\pi}{4}\right)\tan\left(\pi/6\right)\right]} = \frac{\left(-1\right) + \left(\frac{\sqrt{3}}{3}\right)}{1 - \left[\left(-1\right)\left(\sqrt{3}/3\right)\right]} = \frac{\left(\frac{-3+\sqrt{3}}{3}\right)}{1 + \frac{\sqrt{3}}{3}}$$

$$\frac{\left(\frac{-3+\sqrt{3}}{3}\right)}{\left(\frac{3+\sqrt{3}}{3}\right)} = \frac{-3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3+\sqrt{3}} = \frac{-3+\sqrt{3}}{3+\sqrt{3}} \cdot \frac{(3-\sqrt{3})}{(3-\sqrt{3})}$$

Conjugate

$$\frac{-9+3\sqrt{3}+3\sqrt{3}-3}{9-3} = \frac{-12+6\sqrt{3}}{6} = \boxed{-2+\sqrt{3}}$$

26) $\cos\left(19\pi/12\right) = \cos\left(\frac{10\pi}{12} + \frac{9\pi}{12}\right) = \cos\left(\underset{\alpha}{\frac{5\pi}{6}} + \underset{\beta}{\frac{3\pi}{4}}\right)$

$$\cos\left(\frac{5\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right) - \sin\left(\frac{5\pi}{6}\right)\sin\left(\frac{3\pi}{4}\right)$$

$$\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$