

Rational Functions and their Properties

Identify the holes and vertical asymptotes of each.

$$1) f(x) = \frac{x-2}{4x^3 - 20x^2 + 24x} = \frac{1}{4x(x-3)}$$

$$4x(x^2 - 5x + 6)$$

$$4x(x-3)(x-2)$$

hole  $(2, \frac{1}{8})$  VA  $x=0$   
 $x=3$

$$2) f(x) = \frac{x^2 - 5x + 6}{2x^2 + 4x - 6} = \frac{(x-2)(x-3)}{2(x+3)(x-1)}$$

$$2(x^2 + 2x - 3)$$

No Holes VA:  $x=1$   $x=-3$

$$3) f(x) = \frac{x^3 + 3x^2 - 4x}{4x^2 - 36} = \frac{x(x^2 + 3x - 4)}{4(x^2 - 9)}$$

$$\frac{x(x+4)(x-1)}{4(x+3)(x-3)}$$

no holes  
VA  $x=3$   
 $x=-3$

$$4) f(x) = \frac{x^2 + 3x - 4}{-2x^2 + 2x + 12} = \frac{(x+4)(x-1)}{(x-3)(x+2)}$$

$$-2(x^2 - x - 6)$$

no holes  
VA  $x=3$   $x=-2$

Identify the horizontal asymptote of each.

$$5) f(x) = \frac{x-2}{4x+4} \quad y = \frac{1}{4}$$

$$6) f(x) = \frac{x-2}{x-1} \quad y = 1$$

$$7) f(x) = \frac{1}{2x} \quad y = 0$$

$$8) f(x) = \frac{-2x+4}{x^2-4} \quad y = 0$$

Identify the x-intercepts and the y-intercept of each.

$$9) f(x) = \frac{-2x+4}{x+3}$$

$x \rightarrow (2, 0)$   
 $y \rightarrow (0, \frac{4}{3})$

$$10) f(x) = \frac{x^3 + 2x^2 - 3x}{3x^2 - 3x - 6} = \frac{x(x^2 + 2x - 3)}{3(x^2 - x - 2)} = \frac{x(x+3)(x-1)}{3(x-2)(x+1)}$$

$x \rightarrow (0, 0) (-3, 0) (1, 0)$

$y \rightarrow (0, 0)$

$$11) f(x) = \frac{x^2 - 4x}{4x^2 + 8x - 12} = \frac{x(x-4)}{4(x+3)(x-1)}$$

$$4(x^2 + 2x - 3)$$

$x (0, 0) (4, 0)$

$y (0, 0)$

$$12) f(x) = \frac{x^2 + 2x - 8}{2x^2 - 12x + 16} = \frac{(x+4)(x-2)}{2(x-4)(x-2)} = \frac{x+4}{2(x-4)}$$

$$2(x^2 - 6x + 8)$$

$x (-4, 0)$

$y (0, -\frac{1}{2})$

Rational Expressions and Operations (Add/Subtract/Multiply/Divide/Simplify)

Simplify each expression.

$$1) \frac{49a-35}{14a+63} \cdot \frac{7(7a-5)}{7(2a+9)}$$

$$\boxed{\frac{7a-5}{2a+9}}$$

$$2) \frac{5p^2+9p-2}{7p^2+18p+8} \cdot \frac{(5p-1)(p+2)}{(7p+4)(p+2)}$$

$$\boxed{\frac{5p-1}{7p+4}}$$

$$3) \frac{2(7r^2+5r-2)}{14r^2+32r+18} \cdot \frac{14r^3+18r^2}{35r-10}$$

$$2(7r^2+16r+9)$$

$$\frac{2(\cancel{7r-2})(r+1)}{2(\cancel{7r+9})(r+1)} \cdot \frac{2r^2(\cancel{7r+9})}{5(\cancel{7r-2})}$$

$$\boxed{\frac{2r^2}{5}}$$

$$4) \frac{2n^2+19n+42}{14n+49} \cdot \frac{2n-10}{4n-20}$$

$$\frac{(\cancel{2n+7})(n+6)}{7(\cancel{2n+7})} \cdot \frac{2(n-5)}{4(n-5)}$$

$$\frac{2(n+6)}{28} = \boxed{\frac{n+6}{14}}$$

$$5) \frac{5m}{4m-8} \div \frac{25m(m+2)}{5m^2-5m-30}$$

$$5(m^2-m-6)$$

$$\frac{5m}{4(m-2)} \cdot \frac{5(m-3)(m+2)}{25m(m+2)}$$

$$\frac{25m(m-3)}{100m(m-2)} = \boxed{\frac{m-3}{4(m-2)}}$$

$$6) \frac{35n^2-76n+32}{n-6} \div \frac{35n^2-76n+32}{6n-24}$$

$$\frac{(\cancel{7n-4})(5n-8)}{(n-6)} \cdot \frac{6(n-4)}{(\cancel{7n-4})(5n-8)}$$

$$\boxed{\frac{6(n-4)}{n-6}}$$

$$7) \frac{a-1}{3a+15} + \frac{4}{3}$$

$$\frac{a-1}{3(a+5)} + \frac{4(a+5)}{3(a+5)}$$

$$\frac{a-1+4a+20}{3(a+5)} = \boxed{\frac{5a+19}{3(a+5)}}$$

$$8) \frac{2}{x-1} - \frac{6x}{x+2}$$

$$\frac{2(x+2)}{(x-1)(x+2)} + \frac{-6x(x-1)}{(x-1)(x+2)}$$

$$\frac{2x+4-6x^2+6x}{(x-1)(x+2)} = \frac{-6x^2+8x+4}{(x-1)(x+2)}$$

$$9) \frac{\frac{1}{4} + \frac{5}{2}}{\frac{x}{4} + \frac{16}{25}} = \frac{\frac{1}{4} + \frac{10}{4}}{\frac{25x}{100} + \frac{64}{100}} = \frac{\left(\frac{11}{4}\right)}{\left(\frac{25x+64}{100}\right)}$$

$$\frac{11}{4} \cdot \frac{100}{25x+64} = \boxed{\frac{275}{25x+64}}$$

$$10) \frac{\frac{x+2}{x} - \frac{3}{x+2}}{\frac{12}{x} - \frac{x}{9}} = \frac{\frac{(x+2)(x+2)}{x(x+2)} + \frac{-3(x)}{x(x+2)}}{\frac{12(9)}{9x} - \frac{x^2}{9x}}$$

$$\frac{(x^2+4x+4-3x)}{x(x+2)} = \frac{(x^2+x+4)}{x(x+2)}$$

$$\frac{\left(\frac{108-x^2}{9x}\right)}{\left(\frac{108-x^2}{9x}\right)}$$

$$\frac{x^2+x+4}{x(x+2)} \cdot \frac{9x}{108-x^2} = \boxed{\frac{9x^2+9x+36}{-x^3-2x^2+108x+216}}$$

$$11) \frac{\frac{1}{8} + \frac{x}{4}}{\frac{25}{x^2} - \frac{25}{16}} = \frac{\frac{1}{8} + \frac{2x}{8}}{\frac{25(16)}{16x^2} - \frac{25x^2}{16x^2}}$$

$$\frac{\left(\frac{1+2x}{8}\right)}{\left(\frac{400-25x^2}{16x^2}\right)} = \frac{1+2x}{8} \cdot \frac{2x^2}{400-25x^2}$$

$$\boxed{\frac{2x^2+4x^3}{400-25x^2}}$$

$$12) \frac{\frac{1}{2} - \frac{a-5}{5}}{\frac{5}{a^2} + \frac{25}{a^2}} = \frac{\frac{5}{10} + \frac{-2(a-5)}{10}}{\frac{30}{a^2}}$$

$$\frac{\left(\frac{5-2a+10}{10}\right)}{\left(\frac{30}{a^2}\right)} = \frac{-2a+15}{10} \cdot \frac{a^2}{30}$$

$$\boxed{\frac{-2a^3+15a^2}{300}}$$

# Partial Fraction Decomposition Practice

*Key*

Setup the partial fractions for each... do not solve.

$$1) \frac{1}{2x^2+x} = \frac{1}{x(2x+1)} \quad \boxed{\frac{A}{x} + \frac{B}{2x+1}}$$

$$2) \frac{x+1}{x^2+4x+3} \quad \boxed{\frac{A}{x+3} + \frac{B}{x+1}}$$

$(x+3)(x+1)$

$$3) \frac{x^2+12x+12}{x^3-4x} \quad \boxed{\frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}}$$

$x(x^2-4) = x(x-2)(x+2)$

$$4) \frac{x^2}{x^4-2x^2-8} \quad \boxed{\frac{A}{x+2} + \frac{B}{x-2} + \frac{Cx+D}{x^2+2}}$$

$(x^2-4)(x^2+2) = (x+2)(x-2)(x^2+2)$

$$5) \frac{x}{(x-1)(x^2+x+1)} \quad \boxed{\frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}}$$

$$6) \frac{x^2-4x+7}{(x+1)(x^2-2x+3)} \quad \boxed{\frac{A}{x+1} + \frac{Bx+C}{x^2-2x+3}}$$

$$7) \frac{6x^2+1}{x^2(x-1)^3} \quad \boxed{\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3}}$$

$$8) \frac{3x^2-7x-2}{x^3-x} \quad \boxed{\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}}$$

$x(x^2-1)$   
 $x(x+1)(x-1)$

Find the partial fraction decomposition of each.

1)  $\frac{3x-1}{x^2-x} = \frac{A}{x} + \frac{B}{x-1}$

$$\frac{1}{x} + \frac{2}{x-1}$$

2)  $\frac{-4x+3}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$

$$\frac{-\frac{5}{2}}{x+3} + \frac{-\frac{3}{2}}{x-3} \quad \text{or} \quad \frac{-5}{2(x+3)} + \frac{-3}{2(x-3)}$$

3)  $\frac{-7x^2-4x-14}{(x^2+1)(x-2)}$

$$\frac{Ax+B}{x^2+1} + \frac{C}{x-2}$$

$$\frac{3x+2}{x^2+1} - \frac{10}{x-2}$$

4)  $\frac{-3x-29}{x^2-4x-21} = \frac{A}{x-7} + \frac{B}{x+3}$

$$\frac{-5}{x-7} + \frac{2}{x+3}$$

5)  $\frac{11x-7}{2x^2-3x-2} = \frac{A}{2x+1} + \frac{B}{x-2}$

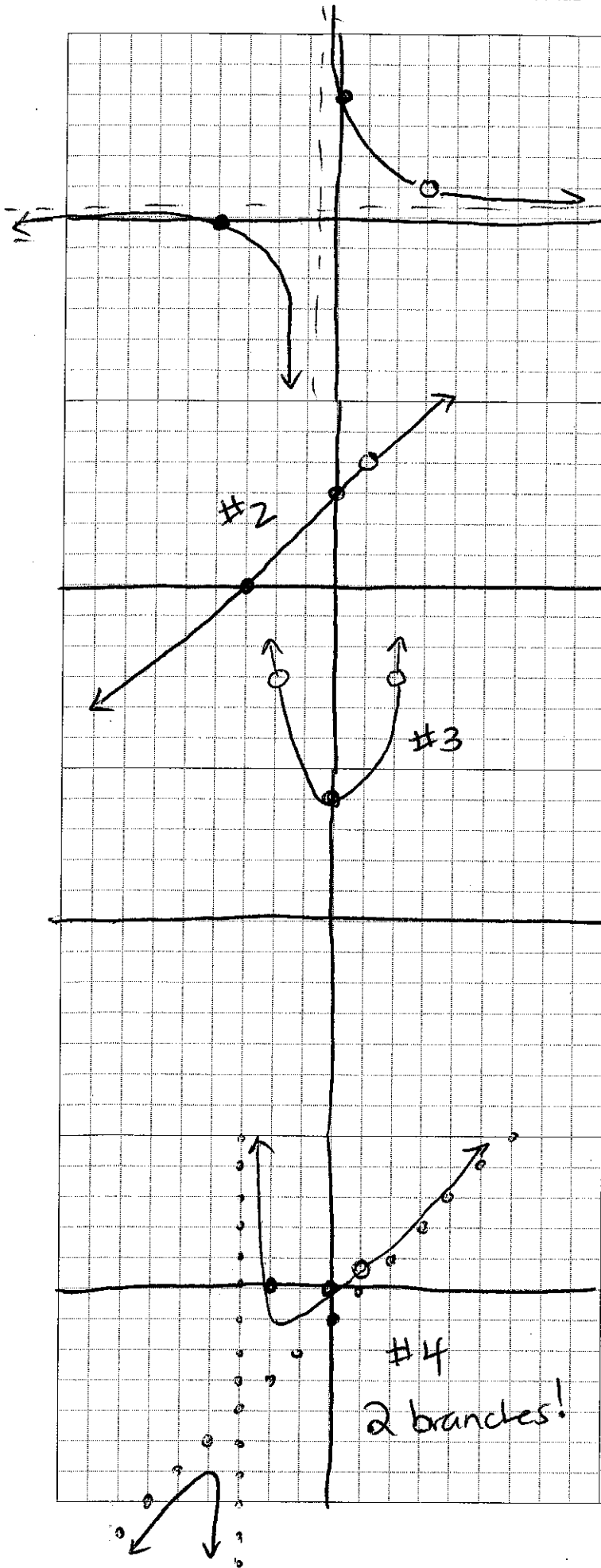
$$\frac{5}{2x+1} + \frac{3}{x-2}$$

6)  $\frac{-x^2+25x-74}{(x+1)(x^2-8x+16)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x+1}$

$$\frac{3}{x-4} + \frac{2}{(x-4)^2} - \frac{4}{x+1}$$

Rational Functions Review - Honors Pre-Calculus

Key



Graph:  $\frac{x^2+x-12}{2x^2-5x-3} = \frac{(x+4)(x-3)}{(2x+1)(x-3)} = \frac{x+4}{2x+1}$

Vertical Asymptotes:  $x = -1/2$

Horizontal Asymptote:  $y = 1/2$

Holes:  $(3, 1)$

Slant asymptote: \_\_\_\_\_

x-intercepts  $(-4, 0)$  y intercepts  $(0, 4)$

Graph:  $\frac{x^2+2x-3}{x-1} = \frac{(x-1)(x+3)}{x-1} = x+3$  (a line w/ a hole)

Vertical Asymptotes: \_\_\_\_\_

Horizontal Asymptote: \_\_\_\_\_

Holes:  $(1, 4)$

Slant asymptote: \_\_\_\_\_

x-intercepts  $(-3, 0)$  y intercepts  $(0, 3)$

Graph:  $\frac{x^4-16}{x^2-4} = \frac{(x^2-4)(x^2+4)}{(x^2-4)} = x^2+4$  (a parabola w/ holes)

Vertical Asymptotes: \_\_\_\_\_

Horizontal Asymptote: \_\_\_\_\_

Holes:  $(2, 8)$   $(-2, 8)$

Slant asymptote: \_\_\_\_\_

x-intercepts \_\_\_\_\_ y intercepts  $(0, 4)$

Graph:  $\frac{x^3+x^2-2x}{x^2+2x-3} = \frac{x(x^2+x-2)}{(x+3)(x-1)} = \frac{x(x+2)}{x+3}$

Vertical Asymptotes:  $x = -3$

Horizontal Asymptote: \_\_\_\_\_

Holes:  $(1, 3/4)$

Slant asymptote:  $y = x - 1$

x-intercepts  $(0, 0)$   $(-2, 0)$  y intercepts  $(0, 0)$

$$\begin{array}{r} -3 \mid 1 \quad 2 \quad 0 \\ \downarrow -3 \quad 3 \\ 1 \quad -1 \quad 3 \end{array}$$

2 branches!