Ch. 2 Review

Key

1. If $f(x) = x^2 - 5x - 6$ and $g(x) = \frac{x+3}{4}$....

a) Evaluate $g(f(4))$

$= g[4^2 - 5(4) - 6]$

$= g[-10]$

$= \frac{-10+3}{4} = \boxed{\frac{-7}{4}}$

b) Evaluate $(f+g)(9) = f(9) + g(9)$

$(9^2 - 5(9) - 6) + \left(\frac{9+3}{4}\right)$

$30 + 3$

$\boxed{33}$

c) Evaluate $f(g(x)) = f\left[\frac{x+3}{4}\right]$

$= \left(\frac{x+3}{4}\right)^2 - 5\left(\frac{x+3}{4}\right) - 6$

$= \frac{x^2+6x+9}{16} + \frac{-5x-15}{4} - 6$

$= \frac{x^2+6x+9-20x-60-96}{16}$

$= \boxed{\frac{x^2-14x-147}{16}}$

2. Write a polynomial whose roots are 5, and $3 + 2i$, $3-2i$

$(x-5)(x-3-2i)(x-3+2i)$

$(x-5)(x^2-3x+2ix-3x+9-6i-2ix+6i-4i^2)$

$(x-5)(x^2-6x+13) = x^3-6x^2+13x-5x^2+30x-65 = \boxed{x^3-11x^2+43x-65}$

3. Three consecutive integers have a sum of 261. Find the three integers. Show your work.

no work = no credit!

$x, x+1, x+2$

$x + x+1 + x+2 = 261$

$3x+3 = 261$

$3x = 258$

$x = 86$

$\boxed{86, 87, 88}$

4. Your salary was \$27,500 in 2003 and \$29,900 in 2006. If your salary follows a linear growth pattern, write an equation to model the value of your salary each year. Then, evaluate what your salary would be in 2018?

$\frac{29900-27500}{2006-2003} = 800$

$y = 800x + 27500$

$x = 15$ (15 years since 2003)

$y = \$39500$

\$39 500 in 2018

Find the domain.

4. $\frac{5x}{2x+6} \neq 0$

$x \neq -3$

$\boxed{(-\infty,-3)\cup(-3,\infty)}$

5. $\frac{7x^2}{x^2-10x-24} \neq 0$

$(x-12)(x+2) \neq 0$

$x \neq 12 \quad x \neq -2$

$\boxed{(-\infty,-2)\cup(-2,12)\cup(12,\infty)}$

6. $\sqrt{3x-9}$

$3x-9 \geq 0$

$x \geq 3$

$\boxed{[3,\infty)}$

7. $\frac{8}{\sqrt{8x+16}}$

no = b/c in den.

$8x+16 > 0$

$x > -2$

$\boxed{(-2,\infty)}$

Factor each polynomial and find the zeroes.

8. $3x^2 + 6x = 0$

$3x(x+2) = 0$

$\boxed{x=0 \quad x=-2}$

9. $x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$\boxed{x=5 \quad x=-3}$

10. $25x^2 - 121 = 0$

$(5x-11)(5x+11) = 0$

$\boxed{x = \pm\frac{11}{5}}$

11. $2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$\boxed{x=\frac{1}{2} \quad x=-3}$

12. $x^2 - 19x + 48$

$(x-16)(x-3) = 0$

$\boxed{x=16 \quad x=3}$

13. $3x^2 - 25x + 28 = 0$

$(3x-4)(x-7) = 0$

$\boxed{x=\frac{4}{3} \quad x=7}$

14. $3x^2 - 2x - 5 = 0$

$(3x-5)(x+1) = 0$

$\boxed{x=\frac{5}{3} \quad x=-1}$

15. $x^3 + x^2 - 16x - 16 = 0$

$(x^2-16)(x+1) = 0$

$(x-4)(x+4)(x+1) = 0$

$\boxed{x=4,\ x=-4\ x=-1}$

16. $x^2 - 100 = 0$

$(x-10)(x+10) = 0$

$\boxed{x = \pm 10}$

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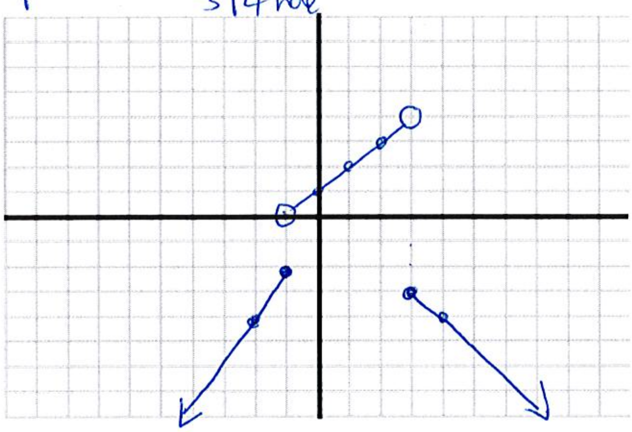
Graph each piecewise function: Create a T-chart for each function

17. $f(x)=\begin{cases} 2x & x\le -1 \quad (-\infty,-1] \\ x+1, & -1<x<3 \quad (-1,3) \\ -x, & x\ge 3 \quad [3,\infty) \end{cases}$

$2x$	
-1	-2
-2	-4

$x+1$	
-1	0 hole
0	1
1	2
2	3
3	4 hole

$-x$	
3	-3
4	-4

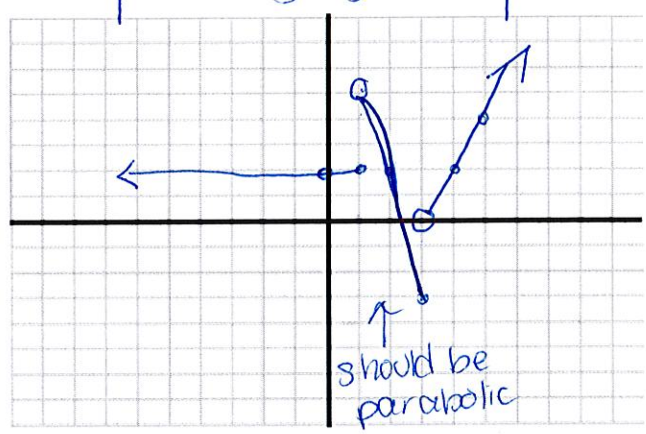


18. $f(x)=\begin{cases} 2, & x\le 1 \quad (-\infty,1] \\ 6-x^2, & 1<x\le 3 \quad (1,3] \\ 2x-6, & x>3 \quad (3,\infty) \end{cases}$

$y=2$	
1	2
0	2

$6-x^2$	
1	5 hole
2	2
3	-3

$2x-6$	
3	0 hole
4	2
5	4



19. Using the piecewise function f(x), **Evaluate** $f(-3), f(0), f(2), f(3), \textit{and } f(5)$

$f(x)=\begin{cases} ① \; 2, & x\le 1 \\ ② \; 6-x^2, & 1<x\le 3 \\ ③ \; 2x-6, & x>3 \end{cases}$

① $f(-3)=\boxed{2}$

② $f(0)=\boxed{2}$

② $f(2)=6-(2)^2=\boxed{2}$

② $f(3)=6-(3)^2=\boxed{-3}$

③ $f(5)=2(5)-6=\boxed{4}$

Find the zeroes: #20 and 21, use the quadratic formula. For #22, complete the square

20. $x^2-2x+3=0$

$a=1$
$b=-2$
$c=3$

$x=\dfrac{2\pm\sqrt{(-2)^2-4(1)(3)}}{2(1)}$

$x=\dfrac{2\pm\sqrt{-8}}{2}$

$x=\dfrac{2+2i\sqrt{2}}{2}$

$\boxed{x=1\pm i\sqrt{2}}$

21. $x^2+4x-21=0$

$a=1$
$b=4$
$c=-21$

$x=\dfrac{-4\pm\sqrt{(4)^2-4(1)(-21)}}{2(1)}$

$x=\dfrac{4\pm\sqrt{100}}{2}$

$x=\dfrac{4\pm 10}{2}$

$x=-2\pm 5$

$\boxed{x=-7 \quad x=3}$

22. $\dfrac{2x^2}{2}+\dfrac{5x}{2}-\dfrac{12}{2}=0$

$x^2+\frac{5}{2}x-6=0$ $\quad \left(\dfrac{\frac{5}{2}}{2}\right)^2$

$x^2+\frac{5}{2}x+\frac{25}{16}=6+\frac{25}{16}$ $\quad \left(\frac{5}{4}\right)^2$

$\left(x+\frac{5}{4}\right)^2=\frac{121}{16}$

$x+\frac{5}{4}=\pm\frac{11}{4}$

$x=-\frac{5}{4}\pm\frac{11}{4}$

$\boxed{x=\frac{3}{2} \quad x=-4}$

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$x + y = 84$

23. The sum of two numbers is 84. Find the maximum value of their product.

$x = 1\text{st}\ \#$
$84 - x = 2\text{nd}\ \#$

a) Create a polynomial that represents all possible products.

$x(84 - x) = \text{Max Product}$
$84x - x^2 = \text{Max Product}$

$\text{Max } (42, 1764)$

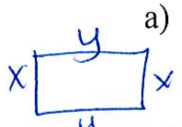
b) Find the maximum value of the product. 1764

c) What two numbers would you use to get this maximum value? 1st is 42
2nd is 42

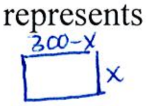
24. Suppose that the perimeter of a rectangle is 600 ft. If x represents the width of the rectangle (in feet), then express the area of the rectangle as a function of x.

$A = L \cdot W$

a) Create a polynomial that represents the area of the rectangle in term of x.



$2x + 2y = 60$
$y = 300 - x$



$x(300 - x) = \text{Area}$
$300x - x^2 = \text{Area}$

x is width
y is length

$\text{Max } (150, 22500)$

b) Find the maximum possible area of the box. $22{,}500\ ft^2$

c) What are the dimensions that yield a maximum area.

width is 150 ft
Length is 150 ft

25. The sum of two numbers is 29. Find the smallest possible sum of their squares.

$x + y = 29$ $x^2 + y^2 = \text{sum of squares}$

a) Create a polynomial that represents the sum of squares.

x is 1st #
29-x is 2nd #

$x^2 + (29 - x)^2 = \text{Sum of Squares}$
$2x^2 - 58x + 841 = \text{Sum of Squares}$

$\text{Min. } (14.5, 420.5)$

b) Find the smallest possible value of the sum of squares.

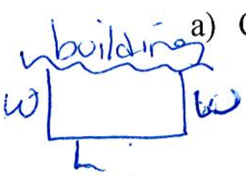
420.5

c) What two numbers would you use to get this minimum value?

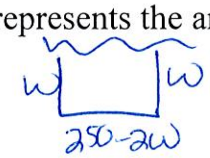
1st # is 14.5, 2nd # is 14.5

26. Suppose that you are building a fence, using the side of a building as one side. You have 250 feet of fencing to use for the other 3 sides.

a) Create a polynomial that represents the area of the rectangle, in terms of the width w.



$2w + L = 250$
$L = 250 - 2w$



$A = w(250 - 2w)$
$A = 250w - 2w^2$

$\text{Max } (62.5, 7812.5)$

b) Find the maximum possible area of the box. $7812.5\ ft^2$

c) What are the dimensions that yield a maximum area.

width is 62.5 ft Length is 125 ft

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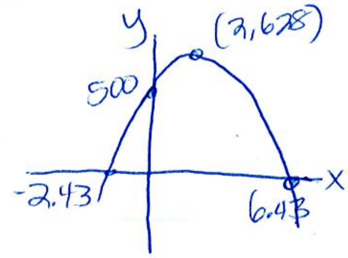
27. A projectile is fired from a cliff that is 500 feet above the water. The height, h (in feet), is given by the function $h(x) = -32x^2 + 128x + 500$ where x represents time in seconds.



a) Sketch a graph of the function and label x intercepts, y intercepts, and maximum.

$x = -2.43$ y-int 500 max
$x = 6.43$ (2, 628)

b) Identify the x intercept that is important in this problem: 6.43

c) What does this value represent in practical terms? (time) when the projectile hits the water

d) Identify the y-intercept: 500

e) What does this represent in practical terms? Projectile starts 500ft above water

f) Maximum height: 628ft How long does it take to reach this height? 2 sec.

28. Use a graphing calculator to find ONE real root. Then, find the rest by hand.

$x = 3$ from calc.

$P(x) = x^3 - 3x^2 + x - 3$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 1 & -3 \\ & \downarrow & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

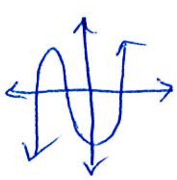
$x^2 + 1$

$x^2 + 1 = 0$
$x^2 = -1$
$x = \pm i$

3 roots
$\boxed{-i, i \text{ and } 3}$

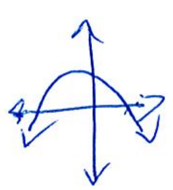
29. Using your calculator, graph $P(x) = x^3 + 5x^2 - x - 9$

Rel. Max (-3.43, 12.9)
Rel Min (.097, -9.05)



a) Identify any extrema as well as any points of inflection. Pt of Inflection $x \approx 1.7$
b) Describe the end behavior. $x \to -\infty, y \to -\infty$ and $x \to \infty, y \to \infty$
c) Find the zeroes.
$x = -1.45$ and $x = -4.82$

30. Using your calculator, graph $P(x) = -x^2 - 3x + 10$



a) Identify any extrema as well as any points of inflection. none!
b) Describe the end behavior. $x \to -\infty, y \to -\infty$
$x \to \infty, y \to -\infty$
c) Find the zeroes.
$x = -5$
$x = 2$

Abs. max
(-1.5, 12.25)

31. a) vertical stretch by factor of 2
b) horizontal compression by factor of 2
c) vertical compression by factor of 2
d) horizontal stretch by factor of 2