Key

Trigonometric Identities and Special Angle Formulas Review

Verify the following identities:

1. $\csc\theta - \cot\theta = \frac{\sin\theta}{1+\cos\theta}$

Conjugate

$$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \frac{1-\cos\theta}{\sin\theta}\cdot\frac{(1+\cos\theta)}{(1+\cos\theta)} = \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} = \boxed{\frac{\sin\theta}{1+\cos\theta}} \checkmark$$

Simplify the following expression:

2. $\frac{\sin\theta - \sin^3\theta}{\tan\theta}$

GCF

$$= \frac{\sin\theta(1-\sin^2\theta)}{\left(\frac{\sin\theta}{\cos\theta}\right)} = \sin\theta\cos^2\theta \cdot \frac{\cos\theta}{\sin\theta} = \boxed{\cos^3\theta}$$

3. $\frac{(\sin\theta)}{(\sin\theta)}\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta\sin\theta}$

$$\frac{\sin^2\theta}{\sin\theta\cos\theta} - \frac{1}{\sin\theta\cos\theta} = \frac{\sin^2\theta - 1}{\sin\theta\cos\theta} = \frac{-\cos^2\theta}{\sin\theta\cos\theta} = \frac{-\cos\theta}{\sin\theta} = \boxed{-\cot\theta}$$

4. Evaluate $\sin 285°$ using the sum and difference formulas.

$$\sin(285) = \sin(240+45) = \sin 240\cos 45 + \cos 240 \sin 45 = \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

5. Evaluate $\sin 120°$ using the double angle formula.

2θ

$$\sin(120°) = 2\sin(60)\cos(60) = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{2\sqrt{3}}{4} = \boxed{\frac{\sqrt{3}}{2}}$$

6. Find the value of $\tan -\frac{5\pi}{3}$ $= \tan(\pi/3)$ ← coterminal

$$\frac{\sin(\pi/3)}{\cos(\pi/3)} = \frac{(\sqrt{3}/2)}{(1/2)} = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \boxed{\sqrt{3}}$$

7. Solve the function: $3\sin^2 x - 3\sin x = 0$

GCF $3\sin(x)[\sin(x)-1] = 0$

$3\sin(x) = 0$ $\quad$ $\sin(x) - 1 = 0$

$\sin(x) = 0$ $\quad$ $\sin(x) = 1$

$\boxed{x = 0, \pi, 2\pi \quad x = \pi/2}$

8. Find all possible solutions each equation below: Give answers in **degrees and radians.**

Radians

Ref∡ $= 1.26^R$

Degrees

Ref∡ $= 72.54°$

a) $\cos x = -0.3$

$x = 107.46°$ or 1.88^R

$x = 252.54°$ or 4.4^R

b) $\sin x = -\frac{\sqrt{2}}{5}$

$x = 196.43°$ or 3.42^R

$x = 343.57°$ or 5.99^R

c) $\sin x = -\frac{1}{2}$

$x = 210°$ or $330°$

$x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

9. Factor $(25\sin^2\theta - 49\cos^2\theta)$

$(5\sin\theta - 7\cos\theta)(5\sin\theta + 7\cos\theta)$

10. Find all the solutions to the equation $2\sin^2 x - \sin^2 x - 1 = 0$

$(2\sin(x) + 1)(\sin(x) - 1) = 0$

$2\sin(x) + 1 = 0$ $\quad$ $\sin(x) - 1 = 0$

$\sin(x) = -1/2$ $\quad$ $\sin(x) = 1$

$\boxed{x = 7\pi/6 \text{ and } \frac{11\pi}{6}}$ $\quad$ $\boxed{x = \pi/2}$