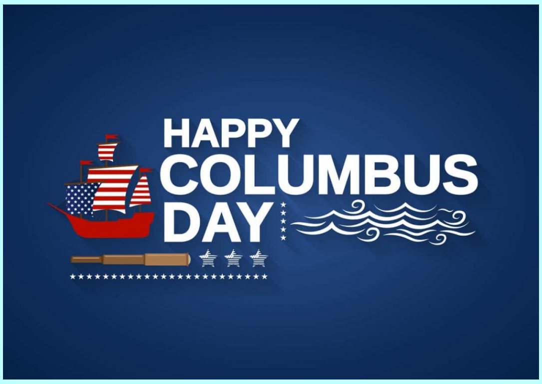


- Park your phones
- Grab your laptops

SUM AND DIFFERENCE RIGHT TRIANGLE COORDINATE PLANE WARM-UP

Use the Sum/Difference Formulas to evaluate

Suppose that $\tan \alpha = -\frac{12}{5}$ **and** $cos(\beta) = -\frac{24}{25}$**, where**

$\frac{\pi}{2} < \alpha < \pi$ **and** $\pi < \beta < \frac{3\pi}{2}$ **(set up a "cheat sheet!")**

Find $cos(\alpha - \beta)$**:**

Find $tan(\alpha + \beta)$**:**

Goal:

$\sec\theta - \sin\theta \tan\theta = \cos\theta$

Start by building our toolbox! (")

Simplifying Trig Expressions NOTES Name: _______________________

Trigonometric Identities

Pythagorean Theorem

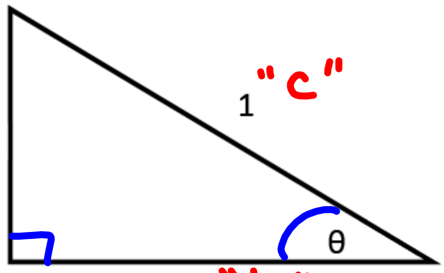
$a^2+b^2=c^2$

$\sin^2\theta + \cos^2\theta =$ "a"

* $\sin^2\theta = \sin\theta \cdot \sin\theta$

$\cos^2\theta = \cos\theta \cdot \cos\theta$

. . . .



Using the triangle above, define each of the following:

tan θ = $\frac{\sin\theta}{\cos\theta}$ cot θ = $\frac{\cos\theta}{\sin\theta}$ csc θ = $\frac{1}{\sin\theta}$ sec θ = $\frac{1}{\cos\theta}$

in terms of sine and cosine

*substitution

Strategy 1: Rewriting expressions in terms of sine or cosine

"Hammer"

1) $\frac{\sin^2\theta}{\sin\theta\cos\theta} = \frac{\cancel{\sin\theta}\cdot\sin\theta}{\cancel{\sin\theta}\cos\theta} = \frac{\sin\theta}{\cos\theta} = \boxed{\tan\theta}$

2) $\frac{\cot\theta}{\cos\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\cos\theta} = \frac{\cancel{\cos\theta}}{\sin\theta}\cdot\frac{1}{\cancel{\cos\theta}} = \frac{1}{\sin\theta} = \boxed{\csc\theta}$

3) $\frac{\tan\theta}{\sec\theta} = \frac{\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{1}{\cancel{\cos\theta}}} = \boxed{\sin\theta}$

4) $\csc\theta\cos\theta$

$= \frac{1}{\sin\theta}\cdot\frac{\cos\theta}{1}$

$= \frac{\cos\theta}{\sin\theta}$

$= \boxed{\cot\theta}$

5) $\csc^2\theta\tan^2\theta$

$= \frac{1}{\cancel{\sin\theta}}\cdot\frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{\cos\theta}\cdot\frac{\cancel{\sin\theta}}{\cos\theta}$

$= \frac{1}{\cos^2\theta} = \boxed{\sec^2\theta}$

6) $\frac{\sin\theta}{\cos^2\theta} = \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$

$= \boxed{\tan\theta\cdot\sec\theta}$

7) $\frac{\sin^2\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{1}$

$= \boxed{\tan\theta\cdot\sin\theta}$

Simplifying Trig Expressions NOTES Name: ______________________

Strategy 2! Using the Pythagorean Identity

"Screwdriver"

1) Using the triangle, apply the Pythagorean Theorem to obtain the Pythagorean Identity:

V1. $\sin^2\theta + \cos^2\theta = 1$

V2. $1 + \cot^2\theta = \csc^2\theta$

V3. $\tan^2\theta + 1 = \sec^2\theta$

$\sin^2(5) + \cos^2(5) = 1$ ✓

2) Divide the Pythagorean Identity by $sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

3) Divide the Pythagorean Identity by $cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

"Drill" Strategy 3: Look for a GCF to help you factor, then use the Pythagorean Identities

1. $\sin^2\theta\tan^2\theta + sin^2(\theta)\cdot 1$ $= \sin^2\theta(\tan^2\theta + 1)$
$= \sin^2\theta \cdot \sec^2\theta$
$= \frac{\sin^2\theta}{1} \cdot \frac{1}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \boxed{\tan^2\theta}$

2. $\sin\theta - \sin^3\theta$ $= \sin\theta(1 - \sin^2\theta)$
$= \boxed{\sin\theta \cdot \cos^2\theta}$

3. $\cot^3\theta + cot\theta$ $= \cot\theta(\cot^2\theta + 1)$
$= \boxed{\cot\theta \cdot \csc^2\theta}$

$\cancel{\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin^2\theta}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

Simplifying Trig Expressions NOTES Name: ________________________

Strategy 4: Getting a Common Denominator

1. $\left(\frac{\sin\theta}{\sin\theta}\right)\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right)$

$= \frac{\sin^2\theta}{\cos\theta\cdot\sin\theta} + \frac{\cos^2\theta}{\cos\theta\cdot\sin\theta}$

$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\cdot\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta} = \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$

$= \boxed{\sec\theta\csc\theta}$

2. $\left(\frac{\sin\theta}{\sin\theta}\right)\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta\sin\theta}$

$= \frac{\sin^2\theta}{\cos\theta\cdot\sin\theta} - \frac{1}{\cos\theta\cdot\sin\theta}$

$= \frac{\sin^2\theta - 1}{\cos\theta\cdot\sin\theta}$ ★

$= -\frac{\cancel{\cos^2\theta}}{\cancel{\cos\theta}\cdot\sin\theta}$

$= -\frac{\cos\theta}{\sin\theta}$

$= \boxed{-\cot\theta}$

3. $\sin(x) + \cos(x)\cot(x)$

$\left(\frac{\sin(x)}{\sin(x)}\right)\frac{\sin(x)}{1} + \frac{\cos(x)}{1}\cdot\frac{\cos(x)}{\sin(x)}$

$= \frac{\sin^2(x)}{\sin(x)} + \frac{\cos^2(x)}{\sin(x)}$

$= \frac{\sin^2(x)+\cos^2(x)}{\sin(x)}$

$= \frac{1}{\sin(x)}$

$= \boxed{\csc(x)}$

4. $\left(\frac{1+\cos\theta}{1+\cos\theta}\right)\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta}\left(\frac{1-\cos\theta}{1-\cos\theta}\right)$ — conjugates

$\frac{\sin\theta + \sin\theta\cos\theta}{\underbrace{1-\cos^2\theta}_{\sin^2\theta}} + \frac{\sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta}$

$= \frac{2\sin\theta}{\sin^2\theta}$

$= \frac{2}{\sin\theta} = 2\cdot\frac{1}{\sin\theta}$

$= \boxed{2\csc\theta}$

Simplifying Trig Expressions NOTES Name: ______________________

Strategy 5: Splitting one fraction into two fractions

11. $\frac{1+\cos\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$

$= \boxed{\sec\theta + 1}$

* NOT Pythagorean Identity

12. $\frac{1-\sin\theta}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\sin\theta}{\sin\theta}$

$= \boxed{\csc\theta - 1}$

13. $\frac{\cos\theta-\sin\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\sin\theta}$

$\boxed{\cot\theta - 1}$

Strategy 6: Simplify using Factoring (Difference of Squares)

Perfect Square   Perf. square   D.O.T.S

8. $\frac{\sin^2\theta-1}{\sin\theta+1} = \frac{\cancel{(\sin\theta+1)}(\sin\theta-1)}{\cancel{\sin\theta+1}}$

$= \boxed{\sin\theta - 1}$

9. $\frac{\cos^2\theta-1}{\cos\theta-1} = \frac{(\cos\theta+1)\cancel{(\cos\theta-1)}}{\cancel{\cos\theta-1}}$

$= \boxed{\cos\theta + 1}$

Put it all together – Simplify using the appropriate strategy

$\frac{\cos\theta-\cos^3\theta}{\cot\theta} = \frac{\cos\theta(1-\cos^2\theta)}{\cot\theta}$

$= \frac{\cos\theta \cdot \sin^2\theta}{\cot\theta}$

$= \frac{\cos\theta \cdot \sin^2\theta}{\frac{\cos\theta}{\sin\theta}}$

$= \frac{\cancel{\cos\theta} \cdot \sin^2\theta}{1} \cdot \frac{\sin\theta}{\cancel{\cos\theta}}$

$= \boxed{\sin^3\theta}$

Simplifying Trig Expressions PRACTICE Name: ____________________

$\cos(\theta)\tan(\theta)$

$\sin(\theta)\cot(\theta)$

$\csc^2(x)\tan^2(x)$

$\sin(x)\sec(x)\cot^2(x)$

$\cot(x)\csc(x)\tan^2(x)$

$\cos x(\csc x + \tan x) =$

$\cos x \cdot \csc x + \cos x \tan x$

$\cos x \cdot \frac{1}{\sin x} + \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}}$

$\frac{\cos x}{\sin x} + \sin x$

$\cot\theta \sec^2\theta - \cot\theta$

$\boxed{\cot x + \sin x}$

$\sin^2 t \cot^2 t\,(1 + \tan^2 t) = \cancel{\sin^2\theta} \cdot \frac{\cos^2\theta}{\cancel{\sin^2\theta}} \cdot \sec^2\theta$

$= \cancel{\cos^2\theta} \cdot \frac{1}{\cancel{\cos^2\theta}} = \boxed{1}$

$\frac{1-\sin^2(x)}{\cos^2(x)-1} = \frac{\cos^2\theta}{-\sin^2\theta}$

$= -\cot^2\theta$

$\cos(\theta)[\sec(\theta) - \cos(\theta)]$

$\frac{1}{\sin\theta}$

$\sin(\theta)[\csc(\theta) - \sin(\theta)]$

$\sin\theta \csc\theta - \sin^2\theta$

$\frac{\sin\theta}{\sin\theta} - \sin^2\theta$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

$\sin(x)[\cot(x) + \tan(x)]$

$\cos(x)[\tan(x) + \cot(x)]$

Simplifying Trig Expressions PRACTICE Name: ____________________

$$\left[\sec(x)-\tan(x)\right]\left[\sec(x)+\tan(x)\right]$$

$$\frac{\cot(x)+\tan(x)}{\csc^2(x)}$$

$$\left[\csc(x)-\cot(x)\right]\left[\csc(x)+\cot(x)\right]$$

$$\frac{\sec^2(x)}{\tan(x)+\cot(x)}$$

$$\left[1-\cos(x)\right]\left[\csc(x)+\cot(x)\right]$$

$$\frac{\cos(x)}{1-\sin(x)}-\frac{\cos(x)}{1+\sin(x)}$$

$$\left[\csc(x)-1\right]\left[\sec(x)+\tan(x)\right]$$

$$\frac{\cot(\theta)}{\csc(\theta)+1}+\frac{\cot(\theta)}{\csc(\theta)-1}$$