Happy Letter Writing Day!

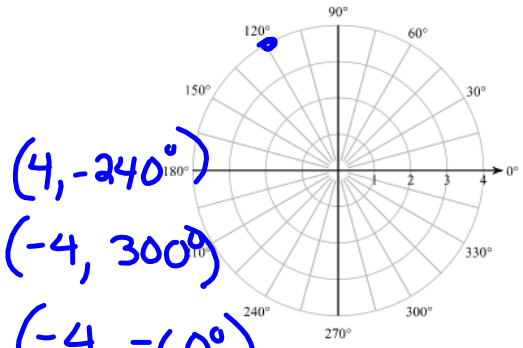
- Park your phones
- Grab your calculators
- Start the warm up (on circle table)

Pre-Calculus Name_____ ID: 1

Warmup - Polar Points and Polar Parabolas

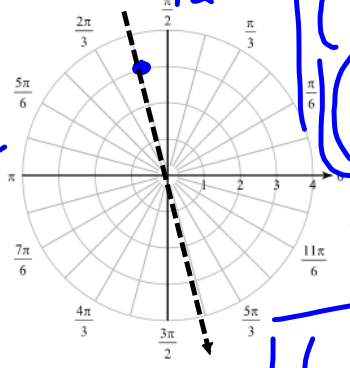
Find all pairs of polar coordinates that describe the same point as the provided polar coordinates.

1) (4, 120°)



(4, -240°)
(-4, 300°)
(-4, -60°)

2) $\left(3, \frac{7\pi}{12}\right)$ $\frac{6\pi}{12}$



12π

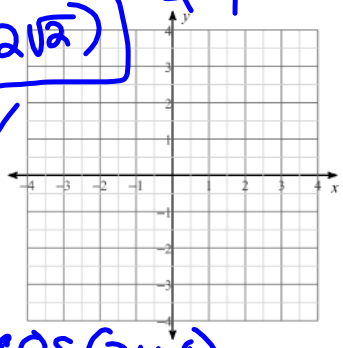
$\left(3, -\frac{17\pi}{12}\right)$
$\left(-3, \frac{19\pi}{12}\right)$

$\frac{24\pi}{12}$

$\left(-3, -\frac{5\pi}{12}\right)$

Convert each pair of polar coordinates to rectangular coordinates.

3) (4, 315°) Q4

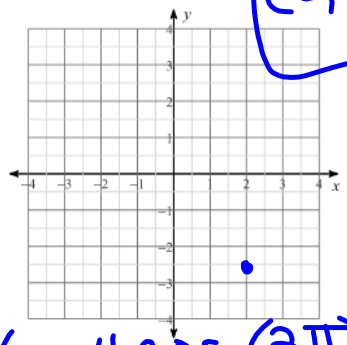


$(2\sqrt{2}, -2\sqrt{2})$
Q4 ✓

$x = 4\cos(315°)$
$x = 4\left(\frac{\sqrt{2}}{2}\right)$
$x = 2\sqrt{2}$

$y = 4\sin(315°)$
$y = 4\left(\frac{-\sqrt{2}}{2}\right)$
$y = -2\sqrt{2}$

4) $\left(-4, \frac{2\pi}{3}\right)$ Q4



$(2, -2\sqrt{3})$

$x = -4\cos\left(\frac{2\pi}{3}\right)$
$x = -4\left(-\frac{1}{2}\right)$
$x = 2$

$y = -4\sin\left(\frac{2\pi}{3}\right)$
$y = (-4)\left(\frac{\sqrt{3}}{2}\right)$
$y = -2\sqrt{3}$

positive radius

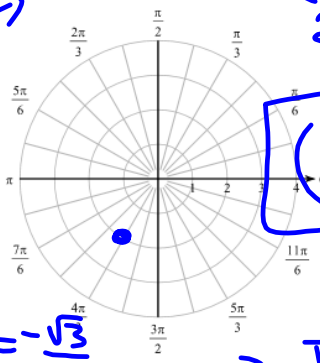
Convert each pair of rectangular coordinates to polar coordinates where $r > 0$ and $0 \le \theta < 2\pi$.

5) $(-1, -\sqrt{3})$

Quad 3

$(-1)^2 + (-\sqrt{3})^2 = r^2$

$2 = r$



$(2, \frac{4\pi}{3})$

$\tan\theta = \frac{-\sqrt{3}}{-1}$

$\theta = \tan^{-1}(\sqrt{3})$

$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

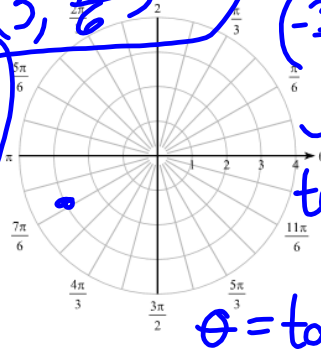
6) $\left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

$(3, \frac{7\pi}{6})$ Q3

positive

$\left(\frac{-3\sqrt{3}}{2}\right)^2 + \left(\frac{-3}{2}\right)^2 = r^2$

$3 = r$



$\tan\theta = \frac{\frac{-3}{2}}{\frac{-3\sqrt{3}}{2}}$

$\theta = \tan^{-1}\left(\frac{\frac{-3}{2}}{\frac{-3\sqrt{3}}{2}}\right)$

$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$

Each polar equation describes a conic section with a focus at the origin. Find the equation of the directrix associated with the focus at the origin, and classify the conic section.

7) $r = \frac{6}{1 + \cos\theta}$

$r(1+\cos\theta) = 6$

$r + r\cos\theta = 6$

$\sqrt{x^2+y^2} + x = 6$

$(\sqrt{x^2+y^2})^2 = (6-x)^2$

$x^2 + y^2 = 36 - 12x + x^2$ ← horizontal

$x =$

$\frac{y^2}{-12} - \frac{36}{-12} = \frac{-12x}{-12}$

$-\frac{1}{12}y^2 + 3 = x$

left horizontal

$\frac{1}{4c} = \frac{1}{12}$

$c = 3$

vertex $(3, 0)$

focus $(0, 0)$

directrix $x = 6$

8) $r = \frac{1}{1 - \sin\theta}$

$y = \frac{1}{2}x^2 - \frac{1}{2}$

vertex $(0, -\frac{1}{2})$

$r = \text{constant}$ Circle centered @ pole

$\theta = \text{angle}$ Line through the pole

$r = a\sin\theta$
$r = a\cos\theta$ circles, symmetry about x- or y-axis, a is diameter

$r = \frac{a}{b \pm \sin\theta}$ vertical
$r = \frac{a}{b \pm \cos\theta}$ horizontal parabolas

$x^2 + y^2 = r^2$ $\quad r = \sqrt{x^2 + y^2}$

$x = r\cos\theta$
$y = r\sin\theta$

$\tan\theta = \frac{y}{x}$

$Ax+By=C \Rightarrow$ Standard Form

$y=mx+b \Rightarrow$ slope-intercept form

Converting Linear Equations – Polar and Rectangular

Convert the following equation from its rectangular form to its polar form.

1. $2x+3y=7$

$2r\cos\theta + 3r\sin\theta = 7$

$r(2\cos\theta + 3\sin\theta) = 7$

$r = \frac{7}{(2\cos\theta+3\sin\theta)}$

2. $y=5x-10$

$r\sin\theta = 5r\cos\theta - 10$

$r\sin\theta - 5r\cos\theta = -10$

$r(\sin\theta - 5\cos\theta) = -10$

$r = \frac{-10}{(\sin\theta - 5\cos\theta)}$

3. $8x-4y=12$

$8r\cos\theta - 4r\sin\theta = 12$

$r(8\cos\theta - 4\sin\theta) = 12$

$r = \frac{3}{(2\cos\theta - \sin\theta)}$

reduced by 4

4. $y+6=3x-11$

$r = \frac{-17}{\sin\theta - 3\cos\theta}$

Convert the following equation from its polar form to its rectangular form.

(Use standard form)

5. $r = \frac{9}{2\cos(\theta)-3\sin(\theta)}$

$2r\cos\theta - 3r\sin\theta = 9$

$2x - 3y = 9$

(Use slope-intercept form)

6. $r = \frac{12}{8\cos(\theta)+5\sin(\theta)}$

$8r\cos\theta + 5r\sin\theta = 12$

$8x + 5y = 12$

$5y = -8x + 12$

$y = -\frac{8}{5}x + \frac{12}{5}$

Converting Linear Equations – Polar and Rectangular Homework

Convert the following equation from its rectangular form to its polar form.

1. $x - 6 = 2x - 14$

2. $-4x + y = -9$

3. $y + 5 = 6x + 12$

4. $8y + 3x = 13$

Convert the following equation from its polar form to its rectangular form. Show and label the standard form and then the slope-intercept form!

5. $r = \frac{3}{4cos(\theta)+7sin(\theta)}$

6. $r = \frac{15}{6sin(\theta)+8cos(\theta)}$

Graphing
Substitution
Elimination

Solving systems of polar equations

Answer is (x,y) a point(s)

Let's review some basic algebra first....

Solve the following system of equations:

$$\begin{cases} y = x + 6 \\ y = 5x - 2 \end{cases}$$

$5x-2 = x+6$

$4x = 8$

$x = 2$

plug it in: $y = x+6, x=2$

$y = 8$

$(2, 8)$

(1 1)

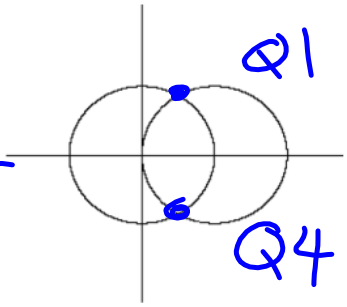
Now, let's try it with Polar!

What if we were to graph two polar curves simultaneously?

Below are the graphs of $r = 1$ and $r = 2\cos\theta$

circle centered at pole

circle symmetric +x-axis



First, solve for θ using substitution:

$1 = 2\cos\theta$

$\frac{1}{2} = \cos\theta$

$\cos^{-1}\left(\frac{1}{2}\right) = \theta$

$\frac{\pi}{3}$ and $\frac{5\pi}{3}$

(1 1)

Next, find r.

$\left(1, \frac{\pi}{3}\right)$ $\left(1, \frac{5\pi}{3}\right)$

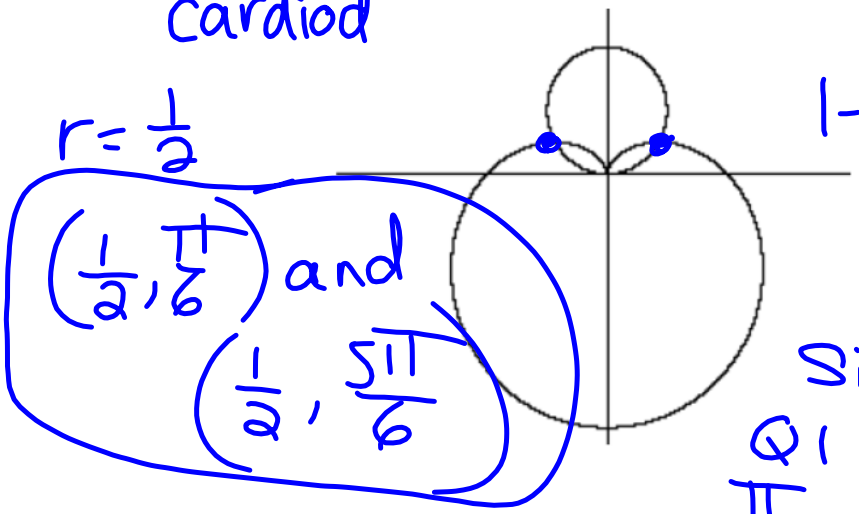
At what polar coordinates do they intersect each other?

Example2: Find the intersection points for $r = sin\theta$ and $r = 1 - sin\theta$

circle

cardiod

(0, 0) Pole



$1 - \sin\theta = \sin\theta$

$1 = 2\sin\theta$

$\frac{1}{2} = \sin\theta$

$\sin^{-1}\left(\frac{1}{2}\right) = \theta$

$r = \frac{1}{2}$

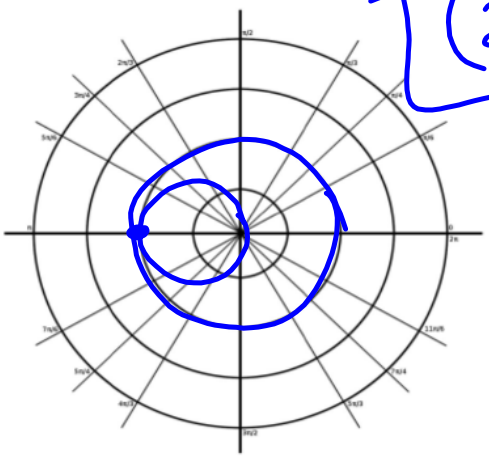
$\left(\frac{1}{2}, \frac{\pi}{6}\right)$ and $\left(\frac{1}{2}, \frac{5\pi}{6}\right)$

Q1 $\frac{\pi}{6}$ Q2 $\frac{5\pi}{6}$

circle centered at pole

circle symm: x-axis d=2

Example 3: Find the intersection of r = 2 and $r = -2cos\theta$



(2, π)

$2 = -2\cos\theta$

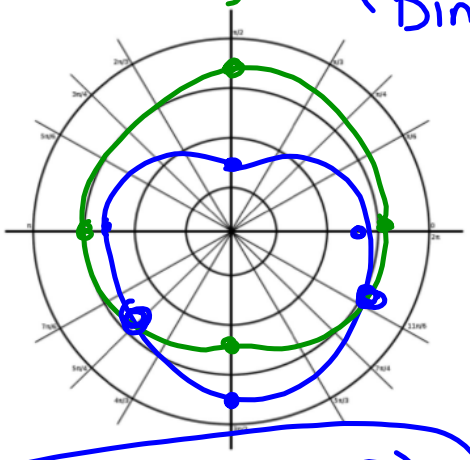
$-1 = \cos\theta$

$\cos^{-1}(-1) = \theta$

$\pi = \theta$

Example 4: $r = 5 - 2sin\theta$ and $r = 6 + sin\theta$

-y-axis ↑ Dimpled Limacon ↑ +y axis



$5 - 2\sin\theta = 6 + \sin\theta$

$-1 = 3\sin\theta$

$-\frac{1}{3} = \sin\theta$

$\sin^{-1}(-\frac{1}{3}) = \theta$

$-.34^R = \theta$

ref angle $.34^R$

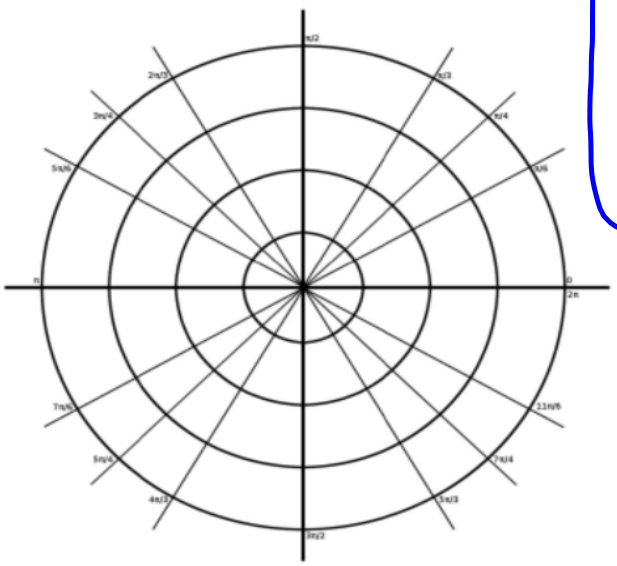
$6.28^R - .34^R = 5.94^R$

$(5.67, 5.94^R)$

$(5.67, 3.48^R)$

$3.14^R + .34^R = 3.48^R$

Example 5: $r = \sqrt{3}$ and $r = 2cos\theta$



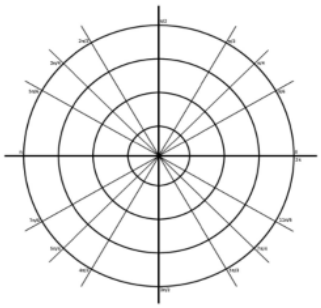
$(\sqrt{3}, \frac{11\pi}{6})$

$(\sqrt{3}, \frac{\pi}{6})$

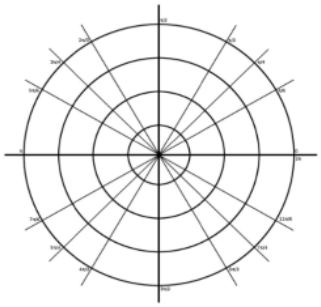
Systems of Polar

Find the intersection of each system of polar equations and sketch a graph of the system. Find the intersection points in degrees, then find them in radians.

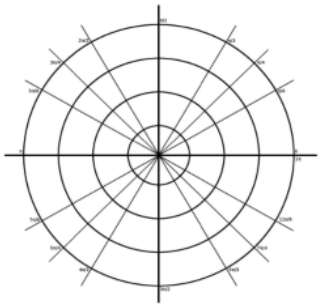
$r = 2 - 3cos\theta$ and $r = 2cos\theta$



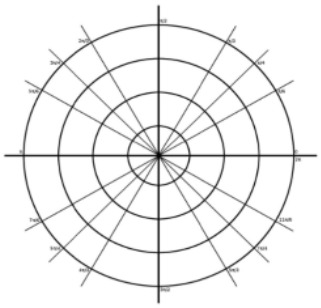
$r = 2 - \sin\theta$ and $r = \sin\theta + 1$



$r = 2 + 3\sin\theta$ and $r = \sin\theta$



$r = 2cos\theta$ and $r = \sqrt{3}$



Warm-up Quiz – Naming Polar Graphs name _____

For each graph below, name the shape and the axis it is symmetric (positive x-axis, negative y-axis, etc).

$r = 2 - 4\sin\theta$ _____ _____

$r = 3 + 3\sin\theta$ _____ _____

$r = 5 + 3\cos\theta$ _____ _____

For each rose curve, tell the number of petals and length of each petal.

$r = 3\sin 2\theta$ _____ _____

$r = 4\sin 3\theta$ _____ _____