

Sum and Difference Formulas

Find the exact value of each.

1) $\sin 105 = \sin(60 + 45)$
 $\sin(60)\cos(45) + \cos(60)\sin(45)$
 $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$\boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

2) $\cos 165 = \cos(135 + 30)$
 $\cos(135)\cos(30) - \sin(135)\sin(30)$
 $\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$$\frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$$

3) $\cos 105 = \cos(225 - 120)$
 $\cos(225)\cos(120) + \sin(225)\sin(120)$
 $\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

4) $\tan 105 = \tan(45 + 60)$
 $\frac{\tan(45) + \tan(60)}{1 - \tan(45)\tan(60)} = \frac{(1) + (\sqrt{3})}{1 - [(1)(\sqrt{3})]}$

$$\frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})} = \frac{1 + \sqrt{3} + \sqrt{3} + 3}{1 + \sqrt{3} - \sqrt{3} - 3} = \frac{4 + 2\sqrt{3}}{-2}$$

$$\boxed{-2 - \sqrt{3}}$$

$\frac{2}{3}''$
 5) $\tan 195 = \tan(150 + 45) = \frac{\tan(150) + \tan(45)}{1 - \tan(150)\tan(45)}$
 $\frac{\left(-\frac{\sqrt{3}}{3}\right) + (1)}{1 - \left[\left(-\frac{\sqrt{3}}{3}\right)(1)\right]} = \frac{\left(-\frac{\sqrt{3} + 3}{3}\right)}{1 - \left[-\frac{\sqrt{3}}{3}\right]} = \frac{\left(-\frac{\sqrt{3} + 3}{3}\right)}{\left(\frac{3 + \sqrt{3}}{3}\right)}$

$$\frac{-\sqrt{3} + 3}{3} \cdot \frac{3}{3 + \sqrt{3}} = \frac{-\sqrt{3} + 3}{3 + \sqrt{3}} \cdot \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})} = \frac{-3\sqrt{3} + 3 + 9 - 3\sqrt{3}}{9 - 3\sqrt{3} + 3\sqrt{3} - 3}$$

$$\frac{12 - 6\sqrt{3}}{6} = \boxed{2 - \sqrt{3}}$$

6) $\sin 195 = \sin(150 + 45)$
 $\sin(150)\cos(45) + \cos(150)\sin(45)$
 $\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

7) $\sin 285 = \sin(225 + 60)$
 $\sin(225)\cos(60) + \cos(225)\sin(60)$
 $\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$$\frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2} - \sqrt{6}}{4}}$$

8) $\tan 165 = \tan(30 + 135) = \frac{\tan(30) + \tan(135)}{1 - \tan(30)\tan(135)}$
 $\frac{\left(\frac{\sqrt{3}}{3}\right) + (-1)}{1 - \left[\left(\frac{\sqrt{3}}{3}\right)(-1)\right]} = \frac{\left(\frac{\sqrt{3} - 3}{3}\right)}{1 + \frac{\sqrt{3}}{3}} = \frac{\left(\frac{\sqrt{3} - 3}{3}\right)}{\left(\frac{3 + \sqrt{3}}{3}\right)}$

$$\frac{\sqrt{3} - 3}{3} \cdot \frac{3}{3 + \sqrt{3}} = \frac{\sqrt{3} - 3}{3 + \sqrt{3}} \cdot \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})}$$

$$\frac{3\sqrt{3} - 3 - 9 + 3\sqrt{3}}{9 - 3\sqrt{3} + 3\sqrt{3} - 3} = \frac{-12 + 6\sqrt{3}}{6} = \boxed{-2 + \sqrt{3}}$$

$\left(\frac{-\sqrt{6} - \sqrt{2}}{4}\right)$ would also be ok (depending on angles!)