

Sum and Difference Formulas

Sum and Difference Formulas (in radians)

Rewriting Radian Measures:

Rewrite each special angle below so that it has a denominator of 12.

(a) $\frac{\pi}{6}$ $\frac{2\pi}{12}$ (b) $\frac{\pi}{4}$ $\frac{3\pi}{12}$ (c) $\frac{\pi}{3}$ $\frac{4\pi}{12}$

(d) $\frac{2\pi}{3}$ $\frac{8\pi}{12}$ (e) $\frac{3\pi}{4}$ $\frac{9\pi}{12}$ (f) $\frac{5\pi}{6}$ $\frac{10\pi}{12}$

Rewrite each of the angles below as a *sum* of two special angles:

(a) $\frac{5\pi}{12}$ (b) $\frac{7\pi}{12}$ (c) $\frac{13\pi}{12}$ (d) $\frac{11\pi}{12}$

$\frac{\pi}{6} + \frac{\pi}{4}$ $\frac{\pi}{4} + \frac{\pi}{3}$ $\frac{3\pi}{4} + \frac{\pi}{3}$ $\frac{2\pi}{3} + \frac{\pi}{4}$

$\frac{\pi}{4} + \frac{5\pi}{6}$ $\frac{3\pi}{4} + \frac{\pi}{6}$

Rewrite each of the angles below as a *difference* of two special angles:

(a) $\frac{5\pi}{12}$ (b) $\frac{7\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{5\pi}{12}$

$\frac{2\pi}{3} - \frac{\pi}{4}$ $\frac{5\pi}{6} - \frac{\pi}{4}$ $\frac{\pi}{4} - \frac{5\pi}{6}$ $\frac{\pi}{3} - \frac{3\pi}{4}$

$\frac{3\pi}{4} - \pi/3$ $\frac{3\pi}{4} - \pi/6$

Example 1: Evaluate $\cos \frac{7\pi}{12}$

$$\begin{aligned} \cos\left(\frac{7\pi}{12}\right) &= \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right) \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \end{aligned}$$

$$\cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$$

Find the exact value of each of the following by using a sum or difference formula:

$$* \sin\left(\frac{7\pi}{12}\right) \quad \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$

$$\sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$\cos\left(\frac{5\pi}{12}\right) \quad \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$$\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$* \sin\left(\frac{11\pi}{12}\right) \quad \boxed{\frac{-\sqrt{2} + \sqrt{6}}{4}}$$

$$\sin\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)$$

$$\cos\left(\frac{31\pi}{12}\right) \quad \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

$$\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$\frac{31\pi}{12} = \frac{7\pi}{12}$$

(coterminal!)

coterminal!

$$\sin\left(\frac{43\pi}{12}\right) = \sin\left(\frac{19\pi}{12}\right)$$

$$= \sin\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)$$

$$\boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$$

$$* \cos\left(-\frac{17\pi}{12}\right) = \cos\left(-\frac{2\pi}{3} - \frac{3\pi}{4}\right)$$

$$\boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

$$\tan\left(-\frac{13\pi}{12}\right) = \tan\left(-\frac{\pi}{3} - \frac{3\pi}{4}\right)$$

$$\boxed{-2 + \sqrt{3}}$$

$$* \tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$\boxed{-2 - \sqrt{3}}$$