Happy National Bring Your Teddy Bear to School Day!

- Park your phones
- Start warm up on circle table

Warmup: Radical Skills

Simplify the following radical expressions:

1. $4 + 5\sqrt{3} + 7 - 2\sqrt{3} + 5 + 10\sqrt{3}$

$\boxed{16 + 13\sqrt{3}}$

2. $12 + 3\sqrt{2} - 5 + 6\sqrt{3} + 8\sqrt{2}$

$\boxed{7 + 11\sqrt{2} + 6\sqrt{3}}$

3. $(5 - 2\sqrt{3})(4 + 6\sqrt{3})$

$20 + 30\sqrt{3} - 8\sqrt{3} - 12(\sqrt{3})^2$

$20 + 22\sqrt{3} - 36$

$\boxed{-16 + 22\sqrt{3}}$

$(6-\sqrt{2})(6+\sqrt{2})$ conjugates

Rationalize the Denominator:

4. $\frac{5}{6\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\boxed{\frac{5\sqrt{2}}{12}}$

5. $\frac{5}{(6+\sqrt{2})} \cdot \left(\frac{6-\sqrt{2}}{6-\sqrt{2}}\right)$

$= \frac{30 - 5\sqrt{2}}{36 - 6\sqrt{2} + 6\sqrt{2} - 2}$

$= \boxed{\frac{30 - 5\sqrt{2}}{34}}$

6. $\frac{9}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{9\sqrt{3}}{6}$

$= \boxed{\frac{3\sqrt{3}}{2}}$

7. $\frac{(4-4\sqrt{5})}{(3+\sqrt{5})} \cdot \frac{(3-\sqrt{5})}{3-\sqrt{5}}$

$-4\sqrt{5} \cdot -\sqrt{5}$
$4(\sqrt{5})^2$
$4 \cdot 5$

$= \frac{12 - 4\sqrt{5} - 12\sqrt{5} + 20}{9 - 3\sqrt{5} + 3\sqrt{5} - (\sqrt{5})^2}$

$= \frac{32 - 16\sqrt{5}}{4}$

$= \frac{32}{4} - \frac{16\sqrt{5}}{4}$

$= \boxed{8 - 4\sqrt{5}}$

Warm-up: Trig Functions, Inverses and Triangles

1. $cos\left(\frac{5\pi}{3}\right) =$

2. $csc\left(\frac{5\pi}{3}\right) =$

3. If the $\boldsymbol{sec(\theta) = -\frac{\sqrt{29}}{3}}$ and $\boldsymbol{180° \leq \theta \leq 270°}$

a) Find $sin(\theta)$

b) Find $tan(\theta)$

Give answers in degrees AND radians:

4. $\boldsymbol{sin^{-1}\left(-\frac{1}{2}\right)}$

5. $\boldsymbol{tan^{-1}(-\sqrt{3})}$

6. $\boldsymbol{csc^{-1}\left(\frac{2\sqrt{3}}{3}\right)}$

7. $\boldsymbol{sec^{-1}\left(-\frac{1}{2}\right)}$

Angles
α = alpha
β = beta

DO NOT NEED TO MEMORIZE

Sum and Difference Formulas

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Question: Why do even need these formulas?

Is sin (30° + 60°) equal to sin 30° + sin 60°?

$= \sin(90°) \overset{?}{=} \sin(30) + \sin(60)$
$= 1 \neq \frac{1}{2} + \frac{\sqrt{3}}{2}$

{Use special Angles 30's, 45's, 60's quadrantals}

Exact Answer

Use a sum formula to evaluate the following without a calculator:

Options: $150° + 45° = 195°$ $240° - 45° = 195°$
$135° + 60° = 195°$ $225° - 30° = 195$

$\alpha = 150°$
$\beta = 45°$

$$\sin(195°) = \sin(150+45)$$
$$= \sin(150)\cos(45) + \cos(150)\sin(45)$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$
$$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}} \leftarrow \text{Exact answer}$$

$$\frac{-\sqrt{6}+\sqrt{2}}{4}$$

$300^\circ + 45^\circ$

$315^\circ + 30^\circ$

Use a ~~difference~~ sum formula to evaluate the following without a calculator:

$\tan(345^\circ) = \tan(300^\circ + 45^\circ)$

$\alpha = 300^\circ$

$\beta = 45^\circ$

$$= \frac{\tan(300) + \tan(45)}{1 - \tan(300)\cdot\tan(45)}$$

$$= \frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)} = \frac{-\sqrt{3}+1}{1+\sqrt{3}}$$ ← binomial × by conjugate

$$= \frac{(-\sqrt{3}+1)}{(1+\sqrt{3})}\left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right)$$

$$= \frac{-\sqrt{3} + (\sqrt{3})^2 + 1 - \sqrt{3}}{1 \cancel{-\sqrt{3}} \cancel{+\sqrt{3}} - (\sqrt{3})^2}$$

$$= \frac{-2\sqrt{3} + 4}{-2}$$

$$= \frac{-2\sqrt{3}}{-2} + \frac{4}{-2}$$

$$= \boxed{\sqrt{3} - 2}$$

Honors Pre-Calculus Name________________________________

Sum and Difference Formulas

Find the exact value of each.

1) sin 105

2) cos 165

3) cos 105

4) tan 105

$\tan(135 - 30)$

$$= \frac{\tan(135) - \tan(30)}{1 + \tan(135)\tan(30)}$$

$$= \frac{-1 - \frac{\sqrt{3}}{3}}{1 - (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{-1 - \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}}$$

$$= \frac{-\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}}$$

$$= \frac{\frac{-3-\sqrt{3}}{\cancel{3}}}{\frac{3-\sqrt{3}}{\cancel{3}}} = \frac{-3-\sqrt{3}}{3-\sqrt{3}}\left(\frac{3+\sqrt{3}}{3+\sqrt{3}}\right)$$

5) tan 195

6) sin 195

7) sin 285

8) tan 165

$30^\circ = \frac{\pi}{6}$ $45^\circ = \frac{\pi}{4}$ $60^\circ = \frac{\pi}{3}$

α

Fractions NEED a common denominator

L.C.D = 12

Sum and Difference Formulas

Sum and Difference Formulas (in radians)

Rewriting Radian Measures:

Rewrite each special angle below so that it has a denominator of 12.

(a) $\frac{\pi}{6} = \frac{2\pi}{12}$ (b) $\frac{\pi}{4} = \frac{3\pi}{12}$ (c) $\frac{\pi}{3} = \frac{4\pi}{12}$

(d) $\frac{2\pi}{3} = \frac{8\pi}{12}$ (e) $\frac{3\pi}{4} = \frac{9\pi}{12}$ (f) $\frac{5\pi}{6} = \frac{10\pi}{12}$

Rewrite each of the angles below as a *sum* of two special angles:

$\frac{2\pi}{12} + \frac{3\pi}{12}$ (a) $\frac{5\pi}{12}$ (b) $\frac{7\pi}{12}$ (c) $\frac{13\pi}{12}$ (d) $\frac{11\pi}{12}$

$\boxed{\frac{\pi}{6} + \frac{\pi}{4}}$ $\boxed{\frac{\pi}{4} + \frac{\pi}{3}}$ $\boxed{\frac{\pi}{3} + \frac{3\pi}{4}}$ $\boxed{\frac{3\pi}{4} + \frac{\pi}{6}}$

$\boxed{\frac{\pi}{4} + \frac{5\pi}{6}}$ $\boxed{\frac{2\pi}{3} + \frac{\pi}{4}}$

Rewrite each of the angles below as a *difference* of two special angles:

(a) $\frac{5\pi}{12}$ (b) $\frac{7\pi}{12}$ (c) $-\frac{7\pi}{12}$ (d) $-\frac{5\pi}{12}$

$\frac{3\pi}{4} - \frac{\pi}{3}$ $\frac{3\pi}{4} - \frac{\pi}{6}$ $\frac{\pi}{6} - \frac{3\pi}{4}$ $\frac{\pi}{3} - \frac{3\pi}{4}$

Example 1: **Evaluate** $\cos\frac{7\pi}{12} = \cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$

$\alpha = \frac{3\pi}{4}$

$\beta = \frac{\pi}{6}$

$= \cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

Sum and Difference Formulas

Find the exact value of each of the following by using a sum or difference formula:

$$\sin\left(\frac{43\pi}{12}\right)$$

$$\sin\left(\frac{7\pi}{12}\right)$$

$$\cos\left(-\frac{17\pi}{12}\right)$$

$$\cos\left(\frac{5\pi}{12}\right)$$

$$\sin\left(\frac{11\pi}{12}\right)$$

$$\tan\left(-\frac{13\pi}{12}\right)$$

$$\tan\left(\frac{7\pi}{12}\right)$$

$$\cos\left(\frac{31\pi}{12}\right)$$