

Key

Factoring Trigonometric Expressions:

Factor each of the following expressions.

$$16\cos^2(\theta) - 81\sin^2(\theta)$$
$$(4\cos\theta - 9\sin\theta)(4\cos\theta + 9\sin\theta)$$

$$25\sin^2(\theta) - 49\cos^2(\theta)$$
$$(5\sin\theta - 7\cos\theta)(5\sin\theta + 7\cos\theta)$$

$$\sin^2 x - 10\sin x + 16$$
$$(\sin(x) - 8)(\sin(x) - 2)$$

$$2\cos^2 x - \cos x - 1$$
$$(2\cos(x) + 1)(\cos(x) - 1)$$

$$2\sin^2 x - \sin x - 3$$
$$(2\sin(x) - 3)(\sin(x) + 1)$$

$$4\sin^2(\theta) - 8\sin(\theta)$$
$$4\sin\theta(\sin\theta - 2)$$

$$6\cos^2 x - 7\cos x + 2$$
$$(3\cos(x) - 2)(2\cos(x) - 1)$$

$$\sin^3 x - 9\sin x$$
$$\sin(x)(\sin^2(x) - 9)$$
$$\sin(x)(\sin(x) - 3)(\sin(x) + 3)$$

$$\sin^4 x - 5\sin^2 x - 6$$
$$(\sin^2 x - 6)(\sin^2 x + 1)$$

$$\cos^4 \theta - 8\cos^2 \theta + 7$$
$$(\cos^2 \theta - 7)(\cos^2 \theta - 1)$$
$$(\cos^2 \theta - 7)(\cos \theta - 1)(\cos \theta + 1)$$

$$\cos^4 \theta - 16$$
$$(\cos^2 \theta - 4)(\cos^2 \theta + 4)$$
$$(\cos \theta + 2)(\cos \theta - 2)(\cos^2 \theta + 4)$$

$$9\sin^4 \theta - 25$$
$$(3\sin^2 \theta - 5)(3\sin^2 \theta + 5)$$

Key

Solving Trigonometric Equations (no calculator)

For the following examples, give answers in radians

$$(0 \leq x \leq 2\pi)$$

$$\cos(x) = -\frac{1}{2}$$

$$X = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$X = \frac{2\pi}{3} \text{ and } \frac{4\pi}{3}$$

$$\sin(x) = \frac{\sqrt{3}}{2}$$

$$X = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$X = \frac{\pi}{3} \text{ and } \frac{2\pi}{3}$$

$$\tan(x) = -1$$

$$X = \tan^{-1}(-1)$$

$$X = \frac{3\pi}{4} \text{ and } \frac{7\pi}{4}$$

$$\sec(x) = -\frac{2\sqrt{3}}{3}$$

$$\cos(x) = -\frac{3}{2\sqrt{3}} \quad \frac{\sqrt{3}}{\sqrt{3}} \quad -\frac{3\sqrt{3}}{6}$$

$$\cos(x) = -\frac{\sqrt{3}}{2}$$

$$X = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$X = \frac{5\pi}{6}$$
$$X = \frac{7\pi}{6}$$

$$\csc(x) = -2$$

$$\sin(x) = -\frac{1}{2}$$

$$X = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$X = \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}$$

$$2\cos(x) = \sqrt{2}$$

$$\cos(x) = \frac{\sqrt{2}}{2}$$

$$X = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$X = \frac{\pi}{4} \text{ and } \frac{7\pi}{4}$$

Solving Trig Equations without the Unit Circle

Solve each trigonometric equation. Give answers in degrees.

Example: Find all solutions for x .

$$\sin(x) = -\frac{4}{5}$$

$$\text{Reference Angle} = 53.13^\circ$$

$$\text{Q III} = 233.13^\circ$$

$$\text{Q IV} = 306.87^\circ$$

1. $5\cos(\theta) = -2$

$$\text{Reference Angle} = 66.42^\circ$$

$$\text{Q II} = 113.57^\circ$$

$$\text{Q III} = 246.42$$

2. $\cos(\theta) = -0.3$

$$\text{Reference Angle} = 72.54$$

$$\text{Q II} = 107.46^\circ$$

$$\text{Q III} = 252.54^\circ$$

3. $\tan(\theta) = -\frac{7}{24}$

$$\text{Reference Angle} = 16.26^\circ$$

$$\text{Q II} = 163.74^\circ$$

$$\text{Q IV} = 343.74^\circ$$

4. $\sec(\theta) = -\frac{25}{7}$

$$\cos(\theta) = -\frac{7}{25}$$

$$\text{Reference Angle} = 73.74^\circ$$

$$\text{Q II} = 106.26^\circ$$

$$\text{Q III} = 253.74$$

5. $3\sin(\theta) = 2$

$$\sin \theta = \frac{2}{3}$$

$$\text{Reference Angle} = 41.81^\circ$$

$$\text{Q I} = 41.81^\circ$$

$$\text{Q II} = 138.19^\circ$$

Solve each trigonometric equation:

$$\sin x - \sqrt{2} = -\sin x$$

$$2 \sin(x) = \sqrt{2}$$

$$\sin(x) = \frac{\sqrt{2}}{2}$$

$$X = 45^\circ \text{ or } \pi/4$$

$$X = 135^\circ \text{ or } 3\pi/4$$

$$2\cos^2 x + 2\cos x = 0$$

$$2\cos(x) [\cos(x) + 1] = 0$$

$$2\cos(x) = 0 \quad \cos(x) + 1 = 0$$

$$\cos(x) = 0 \quad \cos(x) = -1$$

$$X = 90^\circ \text{ or } \frac{\pi}{2}$$

$$X = 270^\circ \text{ or } \frac{3\pi}{2}$$

$$X = 180^\circ \text{ or } \pi$$

$$3\tan^2 x - 1 = 0$$

$$\tan^2(x) = \frac{1}{3}$$

$$\tan(x) = \pm \sqrt{\frac{1}{3}}$$

$$\tan(x) = \pm \frac{\sqrt{3}}{3}$$

$$X = 30^\circ \text{ or } \frac{\pi}{6}$$

$$X = 150^\circ \text{ or } 5\pi/6$$

$$X = 210^\circ \text{ or } 7\pi/6$$

$$X = 330^\circ \text{ or } 11\pi/6$$

$$2\cos x - 3 = 0$$

$$\cos(x) = \frac{3}{2}$$

No solution

cosine must be between
[-1, 1]

$$3\cos x - 2 = 0$$

$$\cos(x) = 2/3$$

$$\cos^{-1}\left(\frac{2}{3}\right)$$

$$= 48.18^\circ$$

$$X = 48.18^\circ$$

$$X = 311.81^\circ$$

$$4\sin x - 1 = 0$$

$$\sin(x) = \frac{1}{4}$$

$$\sin^{-1}\left(\frac{1}{4}\right)$$

$$= 14.48^\circ$$

$$X = 14.48^\circ$$

$$X = 165.52^\circ$$

Solving Trigonometric Equations

Find all solutions for θ : $2\sin^2\theta - \sin\theta - 3 = 0$

$$(2\sin\theta - 3)(\sin\theta + 1) = 0$$

$$2\sin^2(x) - 5\sin(x) - 3 = 0$$
$$(2\sin(x) + 1)(\sin(x) - 3) = 0$$

$$\sin(x) = -\frac{1}{2}$$

$$\sin(x) = 3$$

No additional solutions

$$X = 210^\circ \text{ or } \frac{7\pi}{6}$$

$$X = 330^\circ \text{ or } \frac{11\pi}{6}$$

$$2\cos^2(x) + \cos(x) = 1$$
$$(2\cos(x) - 1)(\cos(x) + 1) = 0$$

$$\cos(x) = \frac{1}{2}$$

$$\cos(x) = -1$$

$$X = 60^\circ \text{ or } \frac{\pi}{3}$$

$$X = 300^\circ \text{ or } \frac{5\pi}{3}$$

$$X = 180^\circ \text{ or } \pi$$

$$\cos^2(x) = 2\cos(x) - 1$$
$$\cos^2(x) - 2\cos(x) + 1 = 0$$
$$(\cos(x) - 1)(\cos(x) - 1) = 0$$

$$\cos(x) = 1$$

$$X = 0^\circ \text{ or } 0$$

$$X = 360^\circ \text{ or } 2\pi$$

$$2\sin^2(x) = -7\sin(x) + 4$$

$$2\sin^2(x) + 7\sin(x) - 4 = 0$$

$$(2\sin(x) - 1)(\sin(x) + 4) = 0$$

$$\sin(x) = \frac{1}{2}$$

$$\sin(x) = -4$$

No additional solutions

$$X = 30^\circ \text{ or } \frac{\pi}{6}$$

$$X = 150^\circ \text{ or } \frac{5\pi}{6}$$

$$4\sin^2(x) - 3 = 0$$

$$\sin^2(x) = \frac{3}{4} \quad \sin(x) = \pm \frac{\sqrt{3}}{2}$$

$$X = 60^\circ \text{ or } \frac{\pi}{3}$$

$$X = 240^\circ \text{ or } \frac{4\pi}{3}$$

$$X = 120^\circ \text{ or } \frac{2\pi}{3}$$

$$X = 300^\circ \text{ or } \frac{5\pi}{3}$$

$$\sin^2(\theta) + 2\sin(\theta) = 0$$

$$\sin(\theta)(\sin(\theta) + 2) = 0$$

$$\sin(\theta) = 0$$

$$\sin\theta = -2$$

No additional solutions

$$\theta = 0 \text{ or } 0$$

$$\theta = 180^\circ \text{ or } \pi$$

$$\theta = 360^\circ \text{ or } 2\pi$$

Does the given x -value represent a solution to the given trigonometric equation? Answer yes or no.

1. $x = \frac{\pi}{6}$; $2\cos^2(x) - 7\cos(x) = 4$

$$2\cos^2\left(\frac{\pi}{6}\right) - 7\cos\left(\frac{\pi}{6}\right)$$

$$2\left(\frac{\sqrt{3}}{2}\right)^2 - 7\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{2}{1}\left(\frac{3}{4}\right) - \frac{7\sqrt{3}}{2} = \frac{6}{4} - \frac{7\sqrt{3}}{2} \neq 4$$

No, $\frac{\pi}{6}$ is not a solution

2. $x = \frac{5\pi}{4}$; $3\tan^2(x) = 8\tan(x) - 5$

$$3\tan^2\left(\frac{5\pi}{4}\right) = 8\tan\left(\frac{5\pi}{4}\right) - 5$$

$$3(1)^2 \stackrel{?}{=} 8(1) - 5$$

$$3 = 3 \checkmark$$

Yes, $\frac{5\pi}{4}$ is a solution

3. $x = \frac{3\pi}{2}$; $\sin^3(x) - 4\sin^2(x) - 2\sin(x) = -3$

$$\sin^3\left(\frac{3\pi}{2}\right) - 4\sin^2\left(\frac{3\pi}{2}\right) - 2\sin\left(\frac{3\pi}{2}\right)$$

$$(-1)^3 - 4(-1)^2 - 2(-1)$$

$$-1 - 4 + 2 = -3 \checkmark$$

Yes, $\frac{3\pi}{2}$ is a solution

4. $x = -\frac{\pi}{3}$; $6\cos^2(x) - 7\cos(x) - 2 = 0$

$$6\cos^2\left(-\frac{\pi}{3}\right) - 7\left(\cos\left(-\frac{\pi}{3}\right)\right) - 2$$

$$6\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) - 2$$

$$\frac{6}{1} \cdot \frac{1}{4} - \frac{7}{2} - \frac{2}{1}$$

$$\frac{6}{4} - \frac{14}{4} - \frac{8}{4} = \frac{-8}{4} - \frac{8}{4} = \frac{-16}{4} \neq 0$$

No, $-\frac{\pi}{3}$ is not a solution

Solving Trigonometric Equations:

$$2 \sin(5x) = -\sqrt{3}$$

$$\sin(5x) = -\frac{\sqrt{3}}{2}$$

$$5x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$5x = \frac{4\pi}{3} \quad \text{and} \quad 5x = \frac{5\pi}{3}$$

$$x = \frac{4\pi}{15} \quad \text{and} \quad \frac{5\pi}{15}$$

$$2 \sin(3x) = 1 \quad \text{on} \quad [-\pi, \pi]$$

$$\sin(3x) = \frac{1}{2}$$

$$3x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$3x = \frac{5\pi}{6} \quad \text{and} \quad 3x = \frac{\pi}{6}$$

$$x = \frac{5\pi}{18} \quad \text{and} \quad x = \frac{\pi}{18}$$

$$\sin(4t) = 1 \quad \text{on} \quad [0, 2\pi]$$

$$4t = \sin^{-1}(1)$$

$$4t = \frac{\pi}{2}$$

$$t = \frac{\pi}{8}$$

$$5 \sin(2x) = 1$$

$$\sin(2x) = \frac{1}{5}$$

$$2x = \sin^{-1}\left(\frac{1}{5}\right)$$

Q1

$$2x = 11.54^\circ \quad \text{and} \quad 2x = 168.46^\circ$$

$$x = 5.77^\circ$$

Q2

$$x = 84.23^\circ$$

Q2

$$\frac{x}{5} = 138.59^\circ$$

$$x = 692.95^\circ$$

Q3

$$\frac{x}{5} = 221.41^\circ$$

$$x = 1107.05^\circ$$

Degrees

$$4 \cos\left(\frac{x}{5}\right) = -3$$

$$\cos\left(\frac{x}{5}\right) = -\frac{3}{4}$$

$$\frac{x}{5} = \cos^{-1}\left(-\frac{3}{4}\right)$$

Degrees