Precalculus

Name _____Key_____

Solving Trig Equations - more complex

Solve each equation for $0 \le \theta < 2\pi$.

1) $2\sin^2 \theta - 3\sin \theta - 4 = -5$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}(\frac{1}{2})$ $\theta = \sin^{-1}(1)$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

2) $4\sin^2 \theta + \sin \theta = 1 + 2\sin^2 \theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin^2\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}(\frac{1}{2})$ $\theta = \sin^{-1}(-1)$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

3) $-1 = -\cot^2 \theta$

$1 = \cot^2\theta$

$\pm 1 = \cot(\theta)$

$\theta = \cot^{-1}(1)$ $\theta = \cot^{-1}(-1)$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

4) $3\csc \theta\tan \theta + \sqrt{3}\csc \theta = 0$

$\csc\theta[3\tan\theta + \sqrt{3}] = 0$

$\csc\theta = 0$ $\tan\theta = \frac{-\sqrt{3}}{3}$

$\csc^{-1}(0) = \theta$ $\theta = \tan^{-1}(\frac{-\sqrt{3}}{3})$

$\sin^{-1}(\frac{1}{0}) = \theta$ $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

undefined

5) $-3 - \cos^2 \theta = -5\cos^2 \theta$

$-3 = -4\cos^2\theta$

$\frac{3}{4} = \cos^2\theta$

$\pm\frac{\sqrt{3}}{2} = \cos\theta$

$\theta = \cos^{-1}(\frac{\pm\sqrt{3}}{2})$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

6) $0 = \sec^2 \theta + 2\tan \theta$

$0 = \tan^2\theta + 1 + 2\tan\theta$

$0 = \tan^2\theta + 2\tan\theta + 1$

$0 = (\tan\theta + 1)(\tan\theta + 1)$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$ $\boxed{\theta = \frac{3\pi}{4}, \frac{7\pi}{4}}$

7) $2\sec^2 \theta = 4 - \sec^2 \theta$

$$3\sec^2\theta = 4$$

$$\sec^2\theta = \frac{4}{3}$$

$$\sec\theta = \pm\frac{2}{\sqrt{3}}$$

$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\pm\sqrt{3}/2\right)$$

$$\boxed{\theta = \pi/6,\ 5\pi/6,\ 7\pi/6,\ \frac{11\pi}{6}}$$

8) $-2\sin^2 \theta = -4\sin^2 \theta + 1$

$$2\sin^2\theta = 1$$

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \pm\frac{\sqrt{2}}{2}$$

$$\theta = \sin^{-1}\left(\pm\frac{\sqrt{2}}{2}\right)$$

$$\boxed{\theta = \frac{\pi}{4}\quad \frac{3\pi}{4}\quad \frac{5\pi}{4}\quad \frac{7\pi}{4}}$$

9) $3\sin 2\theta = 2\cos \theta + 4\sin 2\theta$

$$3\left[2\sin\theta\cos\theta\right] = 2\cos\theta + 4\left[2\sin\theta\cos\theta\right]$$

$$6\sin\theta\cos\theta = 2\cos\theta + 8\sin\theta\cos\theta$$

$$-2\sin\theta\cos\theta - 2\cos\theta = 0$$

$$-2\cos\theta\left[\sin\theta + 1\right] = 0$$

$$-2\cos\theta = 0 \qquad \sin\theta = -1$$
$$\cos\theta = 0 \qquad\quad \theta = \sin^{-1}(-1)$$
$$\theta = \cos^{-1}(0) = \boxed{\frac{\pi}{2},\ \frac{3\pi}{2}}$$

10) $-\cos 2\theta = 2\sin \theta + 3\sin^2 \theta$

$$-\left[1 - 2\sin^2\theta\right] = 2\sin\theta + 3\sin^2\theta$$

$$-1 + 2\sin^2\theta = 2\sin\theta + 3\sin^2\theta$$

$$0 = 1 + 2\sin\theta + \sin^2\theta$$

$$0 = (\sin\theta + 1)(\sin\theta + 1)$$

$$\sin\theta = -1$$

$$\theta = \sin^{-1}(-1) \qquad \boxed{\theta = \frac{3\pi}{2}}$$

11) $-6\sin^2 \theta - \cos 2\theta = -2$

$$-6\sin^2\theta - \left[1 - 2\sin^2\theta\right] = -2$$

$$-4\sin^2\theta - 1 = -2$$

$$-4\sin^2\theta = -1$$

$$\sin^2\theta = \frac{1}{4} \rightarrow \sin\theta = \pm\frac{1}{2}$$

$$\theta = \sin^{-1}\left(\pm\frac{1}{2}\right)$$

$$\boxed{\theta = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{11\pi}{6}}$$

12) $0 = \sqrt{3}\cos \theta + \sin 2\theta$

$$0 = \sqrt{3}\cos\theta + 2\sin\theta\cos\theta$$

$$0 = \cos\theta\left[\sqrt{3} + 2\sin\theta\right]$$

$$\cos\theta = 0 \qquad\qquad 2\sin\theta = -\sqrt{3}$$
$$\theta = \cos^{-1}(0) \qquad \sin\theta = -\frac{\sqrt{3}}{2}$$
$$\boxed{\theta = \frac{\pi}{2},\ \frac{3\pi}{2}} \qquad \theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$
$$\boxed{\theta = \frac{4\pi}{3},\ \frac{5\pi}{3}}$$