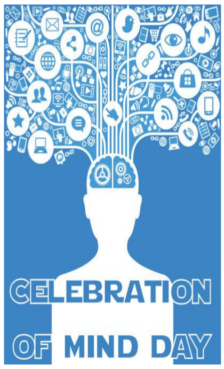
Happy Celebration of Mind Day!



- park your phones
- grab calculators
- grab warm-up sheet on circle table

Factoring & Double Angle side 1st.

Sum and Difference Formulas

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Warm-up – Sum and Difference Formulas

Use a sum formula to evaluate the following:

$\cos(465°) =$

$$\frac{-\sqrt{6}+\sqrt{2}}{4} \quad \text{or} \quad \frac{-\sqrt{2}-\sqrt{6}}{4}$$

Use a difference formula to evaluate the following:

$\sin\left(\frac{5\pi}{12}\right) =$

$$\frac{\sqrt{2}+\sqrt{6}}{4} \quad \text{or} \quad \frac{\sqrt{6}+\sqrt{2}}{4}$$

Use a sum formula to evaluate the following:

$\tan(75°) =$ $\boxed{2+\sqrt{3}}$

Warmup: Suppose that $\tan \alpha = -\frac{13}{5}$ **and** $cos(\beta) = -\frac{24}{25}$, **where** $\frac{\pi}{2} < \alpha < \pi$ **and** $\pi < \beta < \frac{3\pi}{2}$ **(set up a "cheat sheet!")**

(a) $\sin(\alpha - \beta)$

(b) $\cos(\alpha + \beta)$

(c) $\tan(\alpha - \beta)$

Honors Pre-Calculus Name___________________________

Factoring Practice

Factor each completely.

1) $\sin^2\theta + 6\sin\theta + 8$

$(\sin\theta + 4)(\sin\theta + 2)$

2) $\cos^2 x - 5\cos x + 6$

$(\cos x - 3)(\cos x - 2)$

3) $2\tan^2 x - 3\tan x + 1$

$(2\tan x - 1)(\tan x - 1)$

4) $5\sec^2\theta - 6\sec\theta - 8$

$(5\sec\theta + 4)(\sec\theta - 2)$

5) $16\csc^2\theta - 25$

$(4\csc\theta - 5)(4\csc\theta + 5)$

6) $4\cot^2 x - 9$

$(2\cot x - 3)(2\cot x + 3)$

7) $\sin^3\theta + 8\sin^2\theta + 15\sin\theta = 0$

$\sin\theta(\sin^2\theta + 8\sin\theta + 15) = 0$

$\sin\theta(\sin\theta + 3)(\sin\theta + 5) = 0$

8) $\cos^3\theta + 3\cos^2\theta + 2\cos\theta$

$\cos\theta(\cos\theta + 1)(\cos\theta + 2)$

show work using formula

Evaluate using Double Angle Formulas.

$\tan(\frac{4\pi}{3}) = \frac{2\tan(\frac{2\pi}{3})}{1-\tan^2(\frac{2\pi}{3})} = \frac{2(-\sqrt{3})}{1-(-\sqrt{3})^2} = \frac{-2\sqrt{3}}{1-3} = \frac{-2\sqrt{3}}{-2} = \boxed{\sqrt{3}}$

$\boxed{2\theta = \frac{4\pi}{3} \quad \theta = \frac{2\pi}{3} \quad \tan\frac{2\pi}{3} = -\sqrt{3}}$

$\cos(660°) = \cos^2(330) - \sin^2(330) = (\frac{\sqrt{3}}{2})^2 - (-\frac{1}{2})^2 = \frac{3}{4} - \frac{1}{4} = \boxed{\frac{1}{2}}$

$\boxed{2\theta = 660 \quad \theta = 330 \quad \begin{matrix}\cos & \sin\end{matrix} \; (\frac{\sqrt{3}}{2}, -\frac{1}{2})}$

Verify the following Trigonometric Identity.

double angle

$$\frac{3+\cos 2\theta}{\cos^3\theta+\cos\theta} = 2\sec\theta$$

$$= \frac{3+[2\cos^2\theta - 1]}{\cos^3\theta+\cos\theta}$$

$$= \frac{3+2\cos^2\theta - 1}{\cos^3\theta+\cos\theta}$$

$$= \frac{2\cos^2\theta + 2}{\cos\theta(\cos^2\theta+1)}$$

$$= \frac{2\cancel{(\cos^2\theta+1)}}{\cos\theta\cancel{(\cos^2\theta+1)}}$$

$$= \frac{2}{\cos\theta}$$

$$= 2\sec\theta \qquad = 2\sec\theta$$

$$= 2\frac{1}{\cos\theta} \qquad \frac{2}{\cos\theta}$$

Looking for 13 angles ⟹ Need to find ALL angles!

x=?

Trig Equations: A little more complex – Part 2

Solve the following trigonometric equations for $\boxed{0 \le x \le 360°}$ Degrees

1. $2cos^2x + 2cosx = 0$

zero prod. prop. $2\cos x(\cos x + 1) = 0$

$2x^2 + 2x = 0$

$2x(x+1) = 0$

$2\cos x = 0$

$\cos^{-1}$ $\cos x = 0$

$x = \cos^{-1}(0)$

$\boxed{x = 90° \checkmark \quad x = 270° \checkmark}$

$\cos x + 1 = 0$

$\cos^{-1}$ $\cos x = -1$

$x = \cos^{-1}(-1)$

$\boxed{x = 180°} \checkmark$

$2\cos^2(90) + 2\cos 90 \overset{?}{=} 0$

$2\cos^2(270) + 2\cos 270° = 0$

$2\cos^2(180) + 2\cos 180 = 0$

2. $3tan^2x - 1 = 0$

$3\tan^2 x = 1$

$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$

$\tan^{-1}$ $\tan x = \pm\frac{1}{\sqrt{3}}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

θ's $= \boxed{30°, 150°, 210°, 330°}$

$x = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$x = \tan^{-1}\left(-\sqrt{3}/3\right)$

Both pos & neg ratios

3. $2sin^2\theta - sin\theta - 3 = 0$

$(2\sin\theta - 3)(\sin\theta + 1) = 0$

$2\sin\theta - 3 = 0$

$2\sin\theta = 3$

$\sin^{-1}$ $\sin\theta = 3/2$

undefined

(Sine can't be bigger than 1)

$\sin\theta + 1 = 0$

$\sin^{-1}$ $\sin\theta = -1$

$\theta = \sin^{-1}(-1)$

$\boxed{\theta = 270°}$

Solving more Complex Trigonometric Equations - Homework

Degrees

$2\sin^2(x) - 5\sin(x) - 3 = 0$

210° & 330°

$2\sin^2(x) = -7\sin(x) + 4$

30° & 150°

$2\cos^2(x) + \cos(x) = 1$

60°, 180° & 300°

$4sin^2(x) - 3 = 0$

60°, 120°, 240°, 300°

$\cos^2(x) = 2\cos(x) - 1$

0° & 360°

$sin^2(\theta) + 2sin(\theta) = 0$

0°, 180°, 360°

Radians

Solving Trigonometric Equations Part 3

Notes/Example (angles)

$2\sin(5x) = -\sqrt{3}$

$\sin^{-1}\left[\sin(5x) = -\frac{\sqrt{3}}{2}\right]$

$5x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Q3 Q4

$\frac{4\pi}{3}$ $\frac{5\pi}{3}$

$\frac{1}{5} \cdot \frac{4\pi}{3} = 5x$ $\frac{1}{5} \cdot \frac{5\pi}{3} = 5x$

$\boxed{\frac{4\pi}{15} = x}$ $\boxed{x = \frac{\pi}{3}}$

Homework: Radians

$2\sin(3x) = 1$

$\frac{\pi}{18}$ & $\frac{5\pi}{18}$

$\sin(4t) = 1$

$\frac{\pi}{8}$

Homework: Degrees

$5\sin(2x) = 1$

$\sin^{-1}\left[\sin 2x = \frac{1}{5}\right]$

$2x = \sin^{-1}\left(\frac{1}{5}\right)$

ref angle 11.54

Q1 11.54° Q2 168.46°

$2x = 11.54$	$2x = 168.46$
$x = 5.77°$	$x = 84.23°$

$4\cos\left(\frac{x}{5}\right) = -3$

$x = 692.95°$

$x = 1107.05°$

$\cos A \neq \cos B$

A, B are angles

Sum and Difference Formulas Notes and Practice

Simplify each of the following by using the sum and difference formulas:

$\tan(A)\cos(B)[\cos(A) - \cos(A)\cot(A)\tan(B)]$

$\tan A \cos B \cos A - \tan A \cos B \cos A \cot A \tan B$

$\frac{\sin A}{\cancel{\cos A}} \cdot \cos B \cdot \cancel{\cos A} - \frac{\cancel{\sin A}}{\cancel{\cos A}} \cdot \cancel{\cos B} \cdot \cos A \cdot \frac{\cancel{\cos A}}{\cancel{\sin A}} \cdot \frac{\sin B}{\cancel{\cos B}}$

$= \sin A \cos B - \cos A \sin B$

$= \boxed{\sin(A - B)}$

$\sin A \cos B(\cot A + \tan B) =$

$\sin A \cos B \cot A + \sin A \cos B \tan B$

$\cancel{\sin A} \cos B \cdot \frac{\cos A}{\cancel{\sin A}} + \sin A \cdot \cancel{\cos B} \cdot \frac{\sin B}{\cancel{\cos B}}$

$\cos B \cdot \cos A + \sin A \sin B$

$\boxed{\cos(A - B)}$

Verify the following identity using a sum/difference formula:

sum form.

3) $\sec(A)[\cos(A + B)] = \cos B - \tan A \sin B$

$\sec A[\cos A \cos B - \sin A \sin B]$

$\sec A \cos A \cos B - \sec A \sin A \sin B$

$\frac{1}{\cancel{\cos A}} \cdot \cancel{\cos A} \cdot \cos B - \frac{1}{\cos A} \cdot \sin A \sin B$

$\cos B - \tan A \cdot \sin B \quad =$

✓ :)

4) $\cos(\theta + \pi) = -\cos(\theta)$

$\cos\theta \cos\pi - \sin\theta \sin\pi$

$\cos\theta \cdot (-1) - \sin\theta \cdot (0)$

$-\cos\theta \quad -0$

$-\cos\theta$ ✓ $= -\cos\theta$

5) $\csc\left(\theta - \frac{3\pi}{2}\right) = \sec(\theta) \rightarrow \frac{1}{\cos\theta}$ Goal "

$= \frac{1}{\sin\left(\theta - \frac{3\pi}{2}\right)}$

$= \frac{1}{\sin\theta \cos\frac{3\pi}{2} - \sin\frac{3\pi}{2}\cos\theta}$

$= \frac{1}{\sin\theta \cdot (0) - (-1)\cos\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$ ✓

Sum and Difference Formulas Notes and Practice

Verifying Practice **name ________________**

Prove each of the following by using the sum and difference formulas to verify the identities:

$$\sin(x-y)+\sin(x+y)=2\sin(x)\cos(y)$$

$$\cos(x-y)+\cos(x+y)=2\cos(x)\cos(y)$$

$$\frac{\cos(x-y)+\cos(x+y)}{\sin(x)\sin(y)}=2\cot(x)\cot(y)$$

$$\frac{\sin(x-y)+\sin(x+y)}{\cos(x)\cos(y)}=2\tan(x)$$

Verify each identity.

1. $\cot\left(\theta-\frac{\pi}{2}\right) = -\tan\theta$

$$\frac{\cos(\theta-\frac{\pi}{2})}{\sin(\theta-\frac{\pi}{2})} = \frac{\cos\theta\cos\frac{\pi}{2}+\sin\theta\sin\frac{\pi}{2}}{\dots}$$

2. $\cos\left(\theta-\frac{\pi}{2}\right) = \sin\theta$

3. $\sin\left(\theta-\frac{\pi}{2}\right) = -\cos\theta$

4. $\sec\left(\theta-\frac{\pi}{2}\right) = \csc\theta$

$$\frac{1}{\tan(\theta-\frac{\pi}{2})}$$

$\tan\frac{\pi}{2} = $ undef.