

Solving More Complex Trig Equations - Notes

Solve each equation for $0 \leq \theta < 2\pi$.

1)
$$\frac{-\sin \theta + \sqrt{2} \sin \theta \csc \theta}{-\sin \theta} = \sin \theta$$

$$-2 \sin \theta + \sqrt{2} \sin \theta \csc \theta = 0$$

$$\sin \theta [-2 + \sqrt{2} \csc \theta] = 0$$

$$\sin \theta = 0 \quad \left| \quad -2 + \sqrt{2} \csc \theta = 0$$

$$\theta = \sin^{-1}(0)$$

$$\csc \theta = \frac{2}{\sqrt{2}}$$

$$0, \pi, 2\pi$$

$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\frac{\pi}{4}, \frac{3\pi}{4}$$

$$\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, 2\pi$$

Only $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ work

3)
$$\cos^2 \theta - 2 - 3 \sin \theta = \sin^2 \theta$$

Pythag. Identity

$$\cos^2 \theta - \sin^2 \theta - 2 - 3 \sin \theta = 0$$

$$1 - \sin^2 \theta - \sin^2 \theta - 2 - 3 \sin \theta = 0$$

$$-1 - 2 \sin^2 \theta - 3 \sin \theta = 0$$

$$2 \sin^2 \theta + 3 \sin \theta + 1 = 0$$

$$(2 \sin \theta + 1)(\sin \theta + 1) = 0$$

$$\sin \theta = -\frac{1}{2}$$

$$\sin \theta = -1$$

$$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\sin^{-1}(-1) = 0$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\theta = \frac{3\pi}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

2)
$$-1 = 3 \cot^2 \theta - 2$$

$$1 = 3 \cot^2 \theta$$

$$\frac{1}{3} = \cot^2 \theta$$

$$3 = \tan^2 \theta$$

$$\pm \sqrt{3} = \tan \theta$$

$$\tan^{-1}(\sqrt{3}) = \theta \quad \text{and} \quad \tan^{-1}(-\sqrt{3}) = \theta$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

Check answers

4)
$$-\sqrt{3} \cos \theta - 2 \cos \theta = -\sin 2\theta - 2 \cos \theta$$

~~$$-\sqrt{3} \cos \theta = -2 \sin \theta \cos \theta$$~~

~~$$\sqrt{3} \cos \theta = 2 \sin \theta \cos \theta$$~~

~~$$\sqrt{3} = 2 \sin \theta$$~~

~~$$\frac{\sqrt{3}}{2} = \sin \theta$$~~

$$-\sqrt{3} \cos \theta - 2 \cos \theta = -\sin 2\theta - 2 \cos \theta$$

$$-\sqrt{3} \cos \theta = -2 \sin \theta \cos \theta$$

$$-\sqrt{3} \cos \theta + 2 \sin \theta \cos \theta = 0$$

$$(\cos \theta) [-\sqrt{3} + 2 \sin \theta] = 0$$

$$\cos \theta = 0$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$$