

Solving Exponential Equations Using Logarithms

Use the properties of logarithms to solve the exponential equations.

Key

$$1) 2^x = 45$$

$$x \cdot \log 2 = \log 45$$

$$x = \frac{\log 45}{\log 2} = \boxed{5.49}$$

$$2) 3^x = 3.6$$

$$x \cdot \log 3 = \log 3.6$$

$$x = \frac{\log 3.6}{\log 3} = \boxed{1.17}$$

$$3) 10^{2y} = 52$$

$$2y \cdot \log 10 = \log 52$$

$$2y = 1.716$$

$$y = \boxed{.858}$$

$$4) 7^{3y} = 126$$

$$3y \cdot \log 7 = \log 126$$

$$3y = 2.485357$$

$$y = \boxed{.8284}$$

$$5) 3^{x+4} = 6$$

$$(x+4) \cdot \log 3 = \log 6$$

$$x+4 = 1.6309$$

$$x = \boxed{-2.369}$$

$$6) 10^{x+6} = 250$$

$$(x+6) \log 10 = \log 250$$

$$x+6 = 2.3979$$

$$x = \boxed{-3.6}$$

$$7) 3e^x = 42$$

$$e^x = 14$$

$$x = \ln(14)$$

$$x = \boxed{2.639}$$

$$8) \frac{1}{4}e^x = 5$$

$$e^x = 20$$

$$x = \ln(20)$$

$$x = \boxed{2.996}$$

$$9) \frac{1}{2}e^{3x} = 20$$

$$e^{3x} = 40$$

$$3x = \ln(40)$$

$$3x = 3.688$$

$$x = \boxed{1.229}$$

$$10) 250(1.04)^x = 1000$$

$$1.04^x = 4$$

$$x \cdot \log(1.04) = \log(4)$$

$$x = \frac{\log(4)}{\log(1.04)} = \boxed{35.35}$$

$$11) 300e^{\frac{x}{2}} = 9000$$

$$e^{\frac{x}{2}} = 30$$

$$\frac{x}{2} = \ln(30)$$

$$\frac{x}{2} = 3.401$$

$$x = \boxed{6.802}$$

$$12) 1000^{0.12x} = 25000$$

$$.12x \cdot \log 1000 = \log 25000$$

$$.12x = 1.4659$$

$$x = \boxed{12.217}$$

$$13) \frac{1}{5}(4^{x+2}) = 300$$

$$4^{x+2} = 1500$$

$$(x+2) \cdot \log 4 = \log 1500$$

$$x+2 = 5.275$$

$$x = \boxed{3.275}$$

$$14) 6 + 2^{x-1} = 1$$

$$2^{x-1} = -5$$

$$(x-1) \cdot \log 2 = \log(-5)$$

No solution

$$15) 7 + e^{2-x} = 28$$

$$e^{2-x} = 21$$

$$2-x = \ln(21)$$

$$-x = 1.0445$$

$$x = \boxed{-1.0445}$$

$$16) 8 - 12e^{-x} = 7$$

$$-12e^{-x} = -1$$

$$12e^{-x} = 1$$

$$-x = \ln\left(\frac{1}{12}\right)$$

$$x = 2.485$$

$$17) 4 + e^{2x} = 10$$

$$e^{2x} = 6$$

$$2x = \ln(6)$$

$$x = .8958$$

$$18) 32 + e^{7x} = 46$$

$$e^{7x} = 14$$

$$7x = \ln(14)$$

$$x = 0.377$$

$$19) 23 - 5e^{x+1} = 3$$

$$-5e^{x+1} = -20$$

$$e^{x+1} = 4$$

$$x+1 = \ln(4)$$

$$x = .386$$

$$20) 4\left(1 + e^{\frac{x}{3}}\right) = 84$$

$$1 + e^{x/3} = 21$$

$$e^{x/3} = 20$$

$$\frac{x}{3} = \ln(20)$$

$$x = 8.987$$

23) The population  $P$  of a city is given by the function  $P = 2500e^{kt}$ , where  $t = 0$  represents the year 1990. In 1945 the population was 1350. Find the value of  $k$ , and use this value to predict the population in the year 2010.

45 years ago  
 $(t = -45) \quad 1350 = 2500e^{k(-45)}$   
 $.54 = e^{-45k}$

$$\ln(.54) = -45 \cdot k$$

$$.01369 = k$$

$$P = 2500e^{(0.01369 \cdot 20)}$$

$$P = 3287.58 \approx$$

$$3287$$

Solve each equation

$$24. 4^{1-2x} = 2$$

$$(1-2x) \cdot \log 4 = \log 2$$

$$1-2x = .5$$

$$-2x = -.5$$

$$x = .25$$

$$25. 5^x = 3^{x+2}$$

$$\frac{x \cdot \log 5}{\log 3} = \frac{(x+2) \log 3}{\log 3}$$

$$1.465x = x+2$$

$$.465x = 2$$

$$x = 4.301$$

$$26. 9^{2x} = 27^{3x-4}$$

$$2x \cdot \log 9 = \frac{(3x-4) \cdot \log 27}{\log 27}$$

$$1.33x = 3x-4$$

$$-1.66x = -4$$

$$x = 2.4$$

$$27. e^{1-x} = 5$$

$$1-x = \ln(5)$$

$$-x = .6094$$

$$x = -.6094$$

$$28. 2^{3x} = 3^{2x+1}$$

$$\frac{3x \cdot \log 2}{\log 3} = \frac{(2x+1) \cdot \log 3}{\log 3}$$

$$1.8927x = 2x+1$$

$$-.1072x = 1$$

$$x = -9.327$$