HA and Slant asymptote affect the end behavior.

Slant (Oblique) Asymptotes

Quick Review: Long Division

$\frac{3x^2-2x+2}{x-1}$

$\boxed{3x+1}$ rem 3

$x-1 \,\overline{)\, 3x^2-2x+2}$

$-\;3x^2-3x$

$x+2$

$-\;x-1$

3

$3x+1+\frac{3}{x-1}$

$\frac{2x^3+7x^2-4}{x^2+2x-3}$

$2x+3+\frac{5}{x^2+2x-3}$

$\boxed{2x+3}$ rem 5

$x^2+2x-3 \,\overline{)\, 2x^3+7x^2+0x-4}$

$-\;2x^3+4x^2-6x$

$3x^2+6x-4$

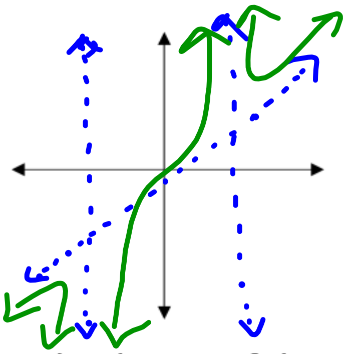
$-\;3x^2+6x-9$

5

What does a graph with a slant asymptote look like?

can cross near the origin

$y=mx+b$



slant

Slant asymptotes occur when the degree of the numerator is exactly one greater than that of the denominator.

Decide whether or not each function has a slant asymptote. If so, find the equation.

$f(x) = \frac{x^3-1}{x^2+3x+5}$

no HA, but yes! Slant!

$x-3$

$x^2+3x+5 \,\overline{)\, x^3+0x^2+0x-1}$

$-\;x^3+3x^2+5x$

$-3x^2-5x-1$

$-\;-3x^2-9x-15$

rem $4x+14$

$\boxed{y=x-3}$ slant

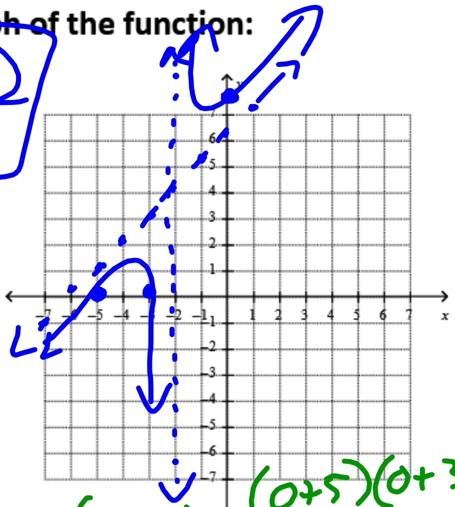
$f(x) = \frac{3x^3+2}{2x-5}$

no HA

no slant

Identify any asymptotes and the ~~sketch a graph of~~ the function:

Example: $f(x) = \frac{x^2+8x+15}{x+2} = \boxed{\frac{(x+5)(x+3)}{(x+2)}}$



den=0 $x+2=0$

VA: $x=-2$ holes: none HA: no

yes :) Slant Asym: $y = x+6$

$$\begin{array}{r} x+6 \\ x+2 \overline{\smash{)}\, x^2+8x+15} \\ \underline{x^2+2x} \\ 6x+15 \\ \underline{6x+12} \\ 3 \end{array}$$

$y=0$ x-intercepts: $(-5,0)(-3,0)$

$0 = \frac{x^2+8x+15}{x+2}$

$0 = x^2+8x+15$

$0 = (x+5)(x+3)$

$x=-5$ $x=-3$

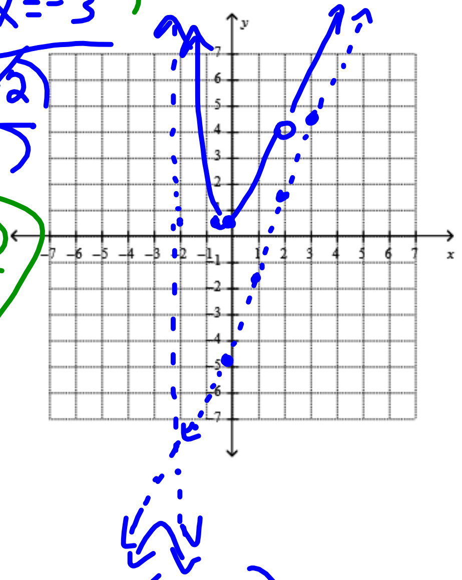
y-intercept: $x=0$

$f(0) = \frac{(0+5)(0+3)}{(0+2)}$

$= \frac{15}{2}$

deg num. 1

$f(x) = \frac{3x^3-5x^2-2x}{x^2-4} = \frac{x(3x^2-5x-2)}{(x-2)(x+2)} = \frac{x(3x+1)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$



den=0 Vertical Asymptotes: $\boxed{x=-2}$

cancel Holes: $\boxed{(2, \tfrac{7}{2})}$

$f(x) = \frac{x(3x+1)}{x+2}$

Horizontal Asymptote: none

yes Slant asymptote: $y=3x-5$

$$\begin{array}{r} 3x-5 \\ x^2+0x-4 \overline{\smash{)}\, 3x^3-5x^2-2x} \end{array}$$

$y=0$ x-intercepts: $(0,0), (-\frac{1}{3}, 0)$

$0 = \frac{x(3x+1)}{x+2}$

$0 = x(3x+1)$

$3x+1=0$

$x=0$ $x = -\frac{1}{3}$

$x=0$ y intercept: $(0,0)$

$y = \frac{0(3(0)+1)}{0+2}$

$= \frac{0}{2}$

$y=0$

8) $f(x) = -\frac{(x-4)(x-3)(x-2)(x+1)}{(x-4)(x-2)}$

VA: hole(s):

HA: Slant:

x-int: yint:

9) $f(x) = \frac{2x^3-7x^2-4x}{x^2-x-12}$

VA: hole(s):

HA: Slant:

x-int: yint:

10) ***may need a calculator to graph**

$$f(x) = \frac{x^4 - 10x^2 + 9}{x^3}$$

VA: hole(s):

HA: Slant:

x-int: yint:

11) $f(x) = \frac{(x-2)(x+3)}{(x-2)(x-4)}$

VA: hole(s):

HA: Slant:

x-int: yint:

12) $f(x) = \frac{(x+3)(6-x)}{(x-2)(x+3)}$

VA: hole(s):

HA: Slant:

x-int: yint:

13) $f(x) = \frac{x^2+x-20}{x-4}$

VA: hole(s):

HA: Slant:

x-int: yint:

Describe the conditions that would produce a rational function without a vertical asymptote.

Describe how you would determine whether a rational function has a horizontal or slant asymptote.