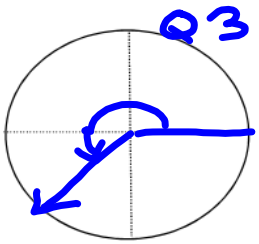
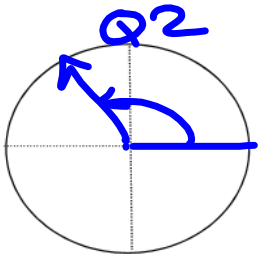
Warm-up: Radians

1) Sketch the angle in standard position and name the quadrant:

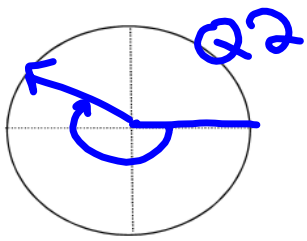
a) $\frac{5\pi}{4}$



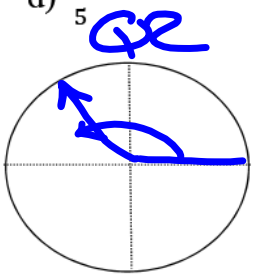
b) $\frac{2\pi}{3}$



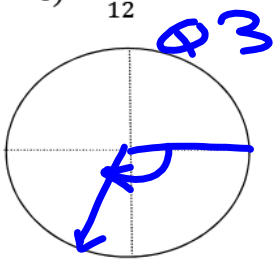
c) $-\frac{7\pi}{6}$



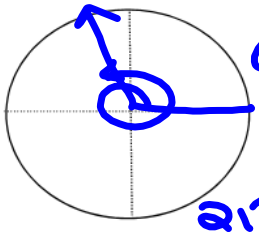
d) $\frac{3\pi}{5}$



e) $-\frac{7\pi}{12}$



f) $\frac{21\pi}{8}$ ← bigger than 2π



$\frac{21\pi}{8} - \frac{16\pi}{8}$

2) Unit Circle Mental Math: Convert without the calculator

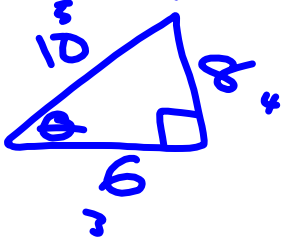
$\frac{5\pi}{6}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{5\pi}{3}$
210°	240°	120°	315°

3) If $\sec(B) = \frac{10}{6}$, find the value of the other five trig ratios:

← + ← A

reciprocal

$\cos\theta = \frac{A}{H}$



$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

$\csc\theta = \frac{5}{4}$

$\cot\theta = \frac{3}{4}$

Coterminal angle = angle with same terminal side +/- 360° or 2π

"Fastest way to x-axis"

Reference Angles – Unit Circle

Reference Angles: the reference angle is the acute angle formed by the terminal side of θ and the x-axis

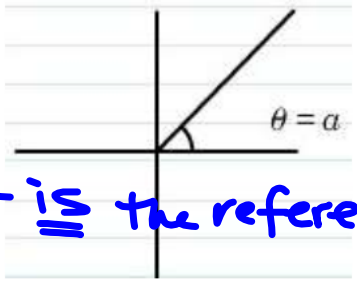
less than 90° / π/2

Basically, we need to know what angle measurement would get us back to the x-axis.

θ = theta = angle

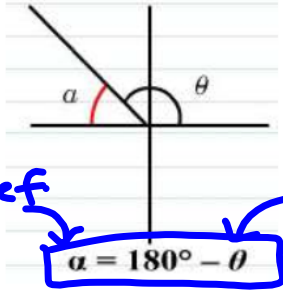
α = alpha = reference

Quadrant I



θ is the reference

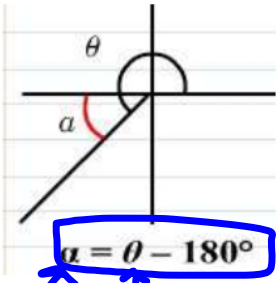
Quadrant II



ref angle

$\alpha = 180° - \theta$

Quadrant III

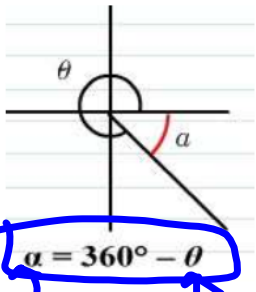


$\alpha = \theta - 180°$

ref angle

angle

Quadrant IV



$\alpha = 360° - \theta$

ref angle

angle

Quadrantals ⇒ no reference angles

Reference Angles – Unit Circle

If necessary, find a co-terminal angle. Tell what quadrant the angle lies and find the related reference angle for each of the following angles:

1) 210° Quadrant: Q3 Reference: 30°

2) 315° Quadrant: Q4 Reference: 45°

3) - 150° Co-terminal: 210° Quadrant: Q3 Reference: 30°

4) - 240° Co-terminal: 120° Quadrant: Q2 Reference: 60°

5) 480° Co-terminal: 120° Quadrant: Q2 Reference: 60°

6) -420° Co-terminal: 300° Quadrant: Q4 Reference: 60°

7) 195° Quadrant: Q3 Reference: 15°

8) 342° Quadrant: Q4 Reference: 18°

9) 54° Quadrant: Q1 Reference: 54°

10) 126° Quadrant: Q2 Reference: 54°

11) 905° Co-terminal: 185° Quadrant: Q3 Reference: 5°

Reference Angles – Unit Circle

12) $\frac{5\pi}{4}$ Quadrant: Q3 Reference: $\frac{\pi}{4}$

13) $\frac{4\pi}{3}$ Quadrant: Q3 Reference: $\frac{\pi}{3}$

14) $-\frac{13\pi}{6}$ Co-terminal: $\frac{11\pi}{6}$ Quadrant: Q4 Reference: $\frac{\pi}{6}$

$-\frac{13\pi}{6}+\frac{12\pi}{6}=-\frac{\pi}{6}+\frac{12\pi}{6}$

15) $-\frac{13\pi}{3}$ Co-terminal: $\frac{5\pi}{3}$ Quadrant: Q4 Reference: $\pi/3$

$-\frac{13\pi}{3}+\frac{6\pi}{3}+\frac{6\pi}{3}+\frac{6\pi}{3}$

$-\frac{13\pi}{3}+\frac{18\pi}{3}$

16) $\frac{5\pi}{8}$ Quadrant: Q2 Reference: $\frac{3\pi}{8}$

$\frac{8\pi}{8}-\frac{5\pi}{8}$

17) $\frac{10\pi}{7}$ Quadrant: Q3 Reference: $\frac{3\pi}{7}$

18) $\frac{13\pi}{7}$ Quadrant: Q4 Reference: $\frac{\pi}{7}$

19) $\frac{12\pi}{5}$ Co-terminal: $\frac{2\pi}{5}$ Quadrant: Q Reference: $\frac{2\pi}{5}$

$\frac{12\pi}{5}-\frac{10\pi}{5}=\frac{2\pi}{5}$

Reference Angles – Unit Circle

Find the reference angle for each of the given angles:

1. $\theta = 30°$ 2. $\theta = 225°$ 3. $\theta = 135°$ 4. $\theta = 315°$ 5. $\theta = 60°$ 6. $\theta = 120°$

7. $\theta = 150°$ 8. $\theta = 210°$ 9. $\theta = 300°$ 10. $\theta = 240°$ 11. $\theta = 45°$ 12. $\theta = 330°$

13. $\theta = 142°$ 14. $\theta = 85°$ 15. $\theta = 202°$ 16. $\theta = 341°$ 17. $\theta = 312°$ 18. $\theta = 195°$

19. $\theta = 228°$ 20. $\theta = 15°$ 21. $\theta = 117°$ 22. $\theta = 298°$ 23. $\theta = 167°$ 24. $\theta = 32°$

Determine the quadrant where the terminal side lies: Then find the reference angle.

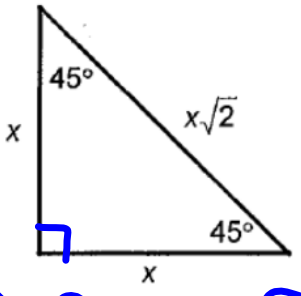
25. $\frac{12\pi}{7}$ **26.** $\frac{15\pi}{6}$ **27.** $\frac{6\pi}{5}$ **28.** $\frac{24\pi}{9}$

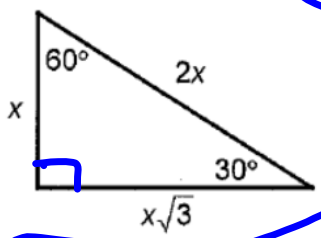
29. $\frac{24\pi}{8}$ **30.** $\frac{7\pi}{8}$ **31.** $\frac{5\pi}{4}$

Deriving the Unit Circle: 30, 45, and 60!

Special Right Triangles:

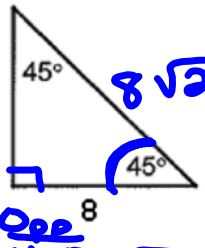
Quick



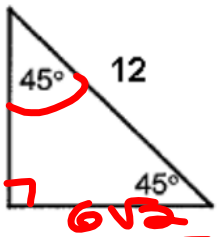


Practice: Fill in the missing side measurements, then evaluate sine and cosine.

ratio of side lengths

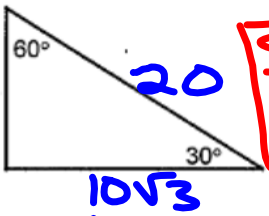


$\sin(45°) =$ $\cos(45°) =$

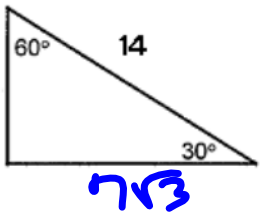


$\sin(45°) =$ $\cos(45°) =$

Similar △s



$\sin(30°) =$ $\cos(30°) =$



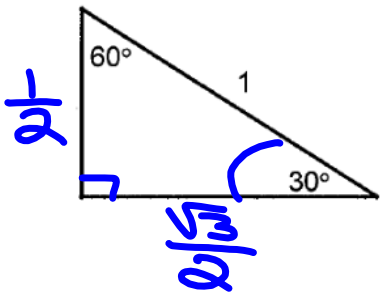
$\sin(30°) =$ $\cos(30°) =$

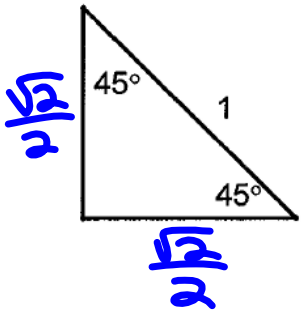
$\sin(60°) =$ $\cos(60°) =$

$\sin(60°) =$ $\cos(60°) =$

$(x, y) \Rightarrow (\cos\theta, \sin\theta)$

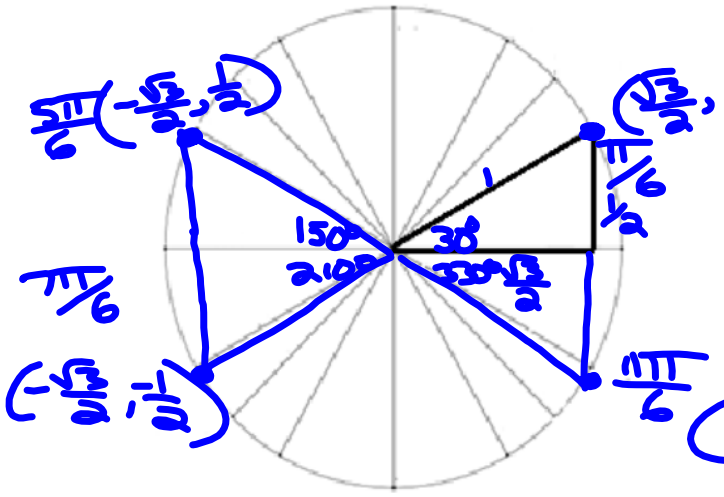
Now, on the Unit Circle, our radius is 1:

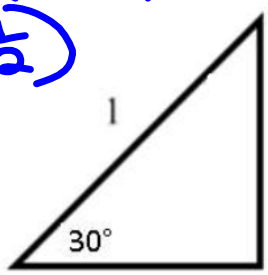




	cos	sin
30		
45		
60		

First, let's just look at 30°





Now, we can do the rest: