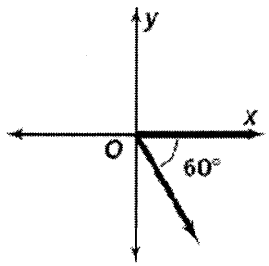
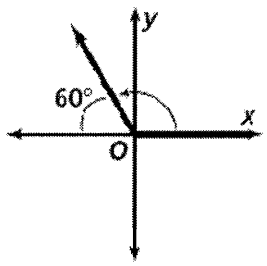
Reference Angles – Unit Circle

Key

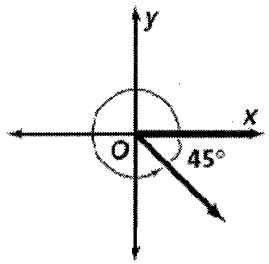
Find the measure of each angle in standard position.



300°



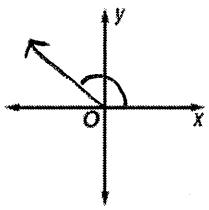
120°



315°

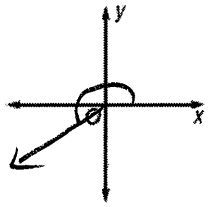
Sketch each angle. Then find its reference angle.

9. 135°



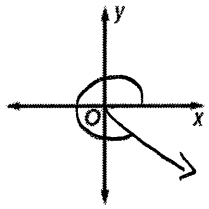
180
−135
45°

10. 200°



200
−180
20°

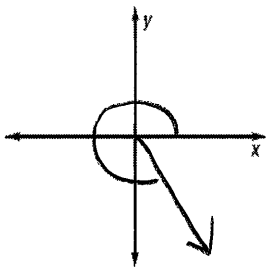
11. $\frac{5\pi}{3}$



2π
$-\frac{5}{3}\pi$
$\frac{\pi}{3}$

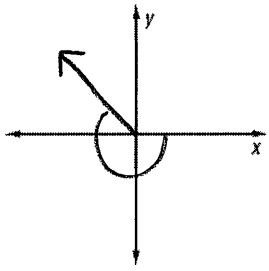
Sketch each angle. Then find its reference angle.

4. $\frac{13\pi}{8}$



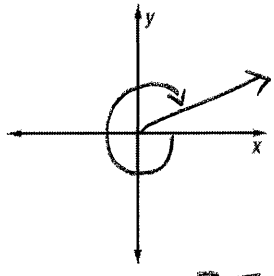
2π
$-13/8\pi$
$\frac{3\pi}{8}$

5. $-210°$



$-210° = 150°$
180
−150
30°

6. $-\frac{7\pi}{4}$



$-\frac{7\pi}{4} = \frac{\pi}{4}$

Reference Angles – Unit Circle

Find the related reference angle for each of the following angles:

1) 210° $30°$

2) 315° $45°$

3) - 150° $30°$

Coterminal: 210

4) - 240° $60°$

Coterminal 120

5) 480° $60°$

coterminal 120

6) -420° $60°$

Coterminal 300

7) $\frac{5\pi}{4}$ $\frac{\pi}{4}$

8) $\frac{4\pi}{3}$ $\frac{\pi}{3}$

9) $-\frac{13\pi}{6}$ $\frac{\pi}{6}$

10) $-\frac{13\pi}{3}$ $\frac{\pi}{3}$

11) 195° $15°$

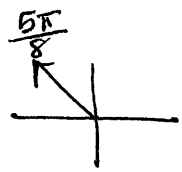
12) 342° $18°$

13) 54° $54°$

14) 126° $54°$

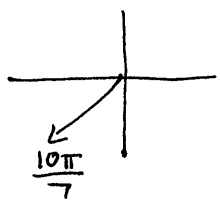
15) $\frac{5\pi}{8}$ $\frac{3\pi}{8}$

QII



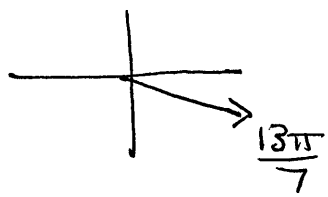
16) $\frac{10\pi}{7}$

QIII $\frac{3\pi}{7}$



17) $\frac{13\pi}{7}$ $\frac{\pi}{7}$

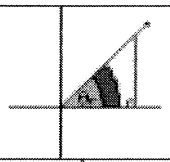
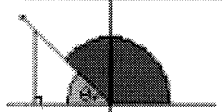
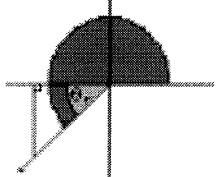
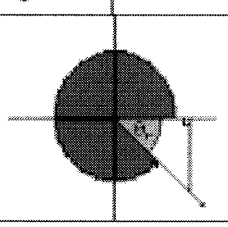
QIV



18) $\frac{12\pi}{5} - 2\pi$

$\frac{2\pi}{5}$ QI

Reference Angles – Unit Circle

If the given angle, θ, is in…	Then the related angle, θ_r, is found by…	Graphical Representation of how θ_r is found
Quadrant I $(0° < \theta < 90°)$	$\theta_r = \theta$	
Quadrant II $(90° < \theta < 180°)$	$\theta_r = 180° - \theta$	
Quadrant III $(180° < \theta < 270°)$	$\theta_r = \theta - 180°$	
Quadrant IV $(270° < \theta < 360°)$	$\theta_r = 360° - \theta$	

Find the related angle, θ_r, for each of the following given angles.

1. $\theta = 30°$ 2. $\theta = 225°$ 3. $\theta = 135°$ 4. $\theta = 315°$ 5. $\theta = 60°$ 6. $\theta = 120°$
7. $\theta = 150°$ 8. $\theta = 210°$ 9. $\theta = 300°$ 10. $\theta = 240°$ 11. $\theta = 45°$ 12. $\theta = 330°$

1. 30° 2. 45° 3. 45° 4. 45° 5. 60° 6. 60°
7. 30° 8. 30° 9. 60° 10. 60° 11. 45° 12. 30°

13. $\theta = 142°$ 14. $\theta = 85°$ 15. $\theta = 202°$ 16. $\theta = 341°$ 17. $\theta = 312°$ 18. $\theta = 195°$
19. $\theta = 228°$ 20. $\theta = 15°$ 21. $\theta = 117°$ 22. $\theta = 298°$ 23. $\theta = 167°$ 24. $\theta = 32°$

13. 38° 14. 85° 15. 22° 16. 19° 17. 48° 18. 15°
19. 48° 20. 15° 21. 63° 22. 62° 23. 13° 24. 32°

Determine the quadrant where the terminal side lies: Then find the reference angle.

25. $\frac{12\pi}{7}$ 26. $\frac{15\pi}{6}$ 27. $\frac{6\pi}{5}$ 28. $\frac{24\pi}{9} = \frac{8\pi}{3}$ 29. $\frac{24\pi}{8} = 3\pi$ 30. $\frac{7\pi}{8}$ 31. $\frac{5\pi}{4}$

QIV Quadrantal QIII QII Quadrantal QII QIII

$\frac{2\pi}{7}$ $\frac{\pi}{2}$ $\frac{\pi}{5}$ $\frac{\pi}{3}$ π $\frac{\pi}{8}$ $\frac{\pi}{4}$