

Exponent Rules

- Same base, multiply $\Rightarrow X^a \cdot X^b = X^{a+b}$
- Same base, divide $\Rightarrow \frac{X^a}{X^b} = X^{a-b}$
- Power to a power $\Rightarrow (X^a)^b = X^{a \cdot b}$

Properties of Logarithms

Let a be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, the following properties are true,

same base "a"

1. $\log_a uv = \log_a u + \log_a v$
2. $\log_a \frac{u}{v} = \log_a u - \log_a v$
3. $\log_a u^n = n \log_a u$

1. $\ln(uv) = \ln u + \ln v$
2. $\ln \frac{u}{v} = \ln u - \ln v$
3. $\ln u^n = n \ln u$

} base e

Left to right, put exponents back 1st

Condense the logarithmic functions

$\log_4 2 + \log_4 32$

$\log_4 (2 \cdot 32)$
 $\log_4 (64) = x$
 $4^x = 64 \Rightarrow x = 3$

$\frac{1}{3} \ln(8) + 4 \ln(x) - 3 \ln(y)$

$\ln 8^{1/3} + \ln x^4 - \ln y^3$
 $\ln 2 + \ln x^4 - \ln y^3$
 $\ln(2x^4) - \ln y^3$
 $\ln \left(\frac{2x^4}{y^3} \right)$

$5 \log(x) - 3 \log(x) + 8 \log(x)$

$\log x^5 - \log x^3 + \log x^8$
 $\log \left(\frac{x^5}{x^3} \right) + \log x^8$
 $\log(x^2) + \log x^8 \rightarrow \log(x^2 \cdot x^8)$
 $\log(x^{10})$

$2 \ln(x^3) - 9 \ln x$

$\ln(x^3)^2 - \ln x^9$
 $\ln x^6 - \ln x^9$
 $\ln \left(\frac{x^6}{x^9} \right) = \ln \left(\frac{1}{x^3} \right)$

Expand logarithmic functions

$\log_2 6x$

$\log_2 6 + \log_2 x$

$\ln \frac{x^2 y^7}{wz^3}$

$\ln x^2 y^7 - (\ln w z^3)$
 $\ln x^2 + \ln y^7 - (\ln w + \ln z^3)$
 $\ln x^2 + \ln y^7 - \ln w - \ln z^3$
 $2 \ln x + 7 \ln y - \ln w - 3 \ln z$
 $\ln \left(\frac{a^b}{\sqrt[3]{c}} \right)$

$\log_5 5x^3y$

$\log_5 5 + \log_5 x^3 + \log_5 y$
 $1 + 3 \log_5 x + \log_5 y$

$\ln ab - (\ln c^{1/3})$
 $\ln a + \ln b - \frac{1}{3} \ln c$

$\log_2(2y)$
 $\log_2 2 + \log_2 y$
 $1 + \log_2 y$

$\log_2(2y)^3$
 $\log_2 2^3 + \log_2 y^3$
 $3 + 3 \log_2 y$

Logarithmic Functions – Properties HW

Use the Laws of Logarithms to rewrite the expression in a form with no logarithm of a product, quotient, or power.

1. $\log_2(8x)^{\frac{1}{3}}$ 2. $\log_2 \frac{8x^3}{2y}$ 3. $\log_b \frac{xy}{z}$ 4. $\log_5 5^p$

Use properties of logarithms to simplify the expression.

5. (a) $\log x^2 - \log xy + 4 \log y$ 6. $\log_3 5 + \log_3 2$

7. Use the logarithm laws to simplify the following:

(a) $\log_2 xy - \log_2 x^2$ (b) $\log_2 \frac{8x^2}{y} + \log_2 2xy$

(c) $\log_3 9xy^2 - \log_3 27xy$ (d) $\log_4 (xy)^3 - \log_4 xy$

(e) $\log_3 9x^4 - \log_3 (3x)^2$

Evaluate the following using the laws of logarithms

8. $\log_2 12 - \log_2 3$ 9. $\log_6 9 + \log_6 8 - \log_6 2$ 10. $\log_2 2 + \log_2 3 - \log_2 6 - \log_2 8$

11. $\log_8 16 + \log_8 4$

12. $\log_2 24 - \log_2 3$

13. $\log_3 45 - \log_3 5 + \log_3 \sqrt[4]{3}$

14. $\log_2 y + \log_2 2x - \log_2 x^2 - \log_2 y$

15. $\frac{1}{2} \log_2 16 - \frac{1}{3} \log_2 8$

16. $\log_2 8^{25} + \log_2 16^5$

17. $2 \log_b 4 - 3 \log_b 2 + \log_b 2 \rightarrow \log_b x$

