Pre-Calculus Released Exam Practice

Key

1 A board is made up of 9 squares. A certain number of pennies is placed in each square, following a geometric sequence. The first square has 1 penny, the second has 2 pennies, the third has 4 pennies, etc. When every square is filled, how many pennies will be used in total?

$1, 2, 4, 8, \ldots$ Geometric $n = 9$ $r = 2$

$\frac{1(1-2^9)}{(1-2)}$

A 512

B 511

C 256

D 81

2 Let $f(x) = 14x^3 + 28x^2 - 46x$ and $g(x) = 2x + 7$. Which is the solution set to the equation $\frac{1}{12}f(x) = g(x)$?

$\frac{1}{12}(14x^3 + 28x^2 - 46x) = 2x + 7$ (Graph it!)

We need the intersections!

A $\{-3, 0, 1\}$

B $\{-3, -1, 2\}$

C $\{-2, 1, 3\}$

D $\{1, 5, 11\}$

3 The equation $2x^2 - 5x = -12$ is rewritten in the form of $2(x - p)^2 + q = 0$. What is the value of q?

$2(x^2 - \frac{5}{2}x + \_\_\_) = -12 + \_\_\_$

$2(\frac{25}{16})$ from left side

$(\frac{5}{4})^2$ $2(x^2 - \frac{5}{2}x + \frac{25}{16}) = -12 + \frac{25}{8}$

$2(x - \frac{5}{4})^2 = -\frac{71}{8}$

$2(x - 5/4)^2 + \frac{71}{8} = 0$

A $\frac{167}{16}$

B $\frac{71}{8}$

C $\frac{25}{8}$

D $\frac{25}{16}$

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4 A box with an open top will be constructed from a rectangular piece of cardboard.

- The piece of cardboard is 8 inches wide and 12 inches long.
- The box will be constructed by cutting out equal squares of side x at each corner and then folding up the sides.

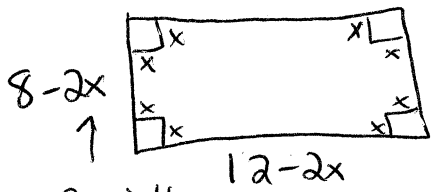
What is the entire domain for the function $V(x)$ that gives the volume of the box as a function of x?

(A) $0 < x < 4$

B $0 < x < 6$

C $0 < x < 8$

D $0 < x < 12$



If $x > 4$, we would run out of paper!

5 A function is shown below.

$$f(x) = \begin{cases} -x^2 + 2x & \text{for } x \le -3 \\ 2\left(\frac{1}{3}\right)^{2x} & \text{for } -3 < x < 4 \\ \frac{2x-5}{x-7} & \text{for } x \ge 4 \end{cases}$$

What is the value of the expression $f(-3) + 2f(-1) - f(4)$?

$$\left[-(-3)^2+2(-3)\right] + 2\left(2\left(\tfrac{1}{3}\right)^{2(-1)}\right) - \left[\frac{2(4)-5}{4-7}\right]$$

$$[-9-6] + [36] - \left[\tfrac{3}{-3}\right]$$

$$-15 + 36 + 1$$

A $\frac{101}{36}$

B $\frac{32}{9}$

C 4

(D) 22

6 Which function goes to positive ∞ most quickly as x increases?

A $y = \log(x) + 100$

(B) $y = e^{x-9} - 3$

C $y = x^2 + 5x + 6$

D $y = 3x^5 + 4x^3 - 11x - 6$

Graph!

Check the Table!

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7 Which expression is equivalent to $\frac{\sin^4(\theta) - \cos^4(\theta)}{\sin^2(\theta) - \cos^2(\theta)}$, where $\sin^2(\theta) \neq \cos^2(\theta)$?

A $\sin^2(\theta) - \cos^2(\theta)$

B $\cos^2(\theta) - \sin^2(\theta)$

C 2

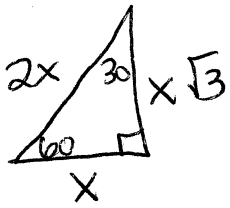
(D) 1

$= \frac{(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)}{(\sin^2\theta - \cos^2\theta)}$

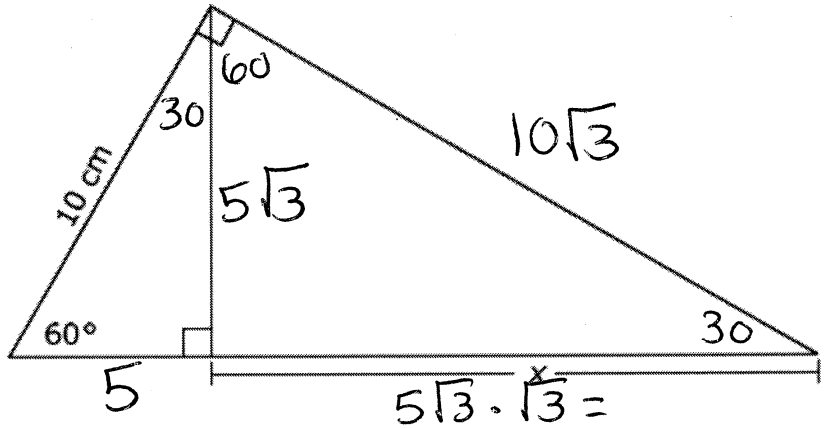
$= \sin^2\theta + \cos^2\theta$

$= 1$

8 What is the value of x in the triangle below?



30-60-90 Rules!



A $\frac{5\sqrt{3}}{2}$ cm

B $5\sqrt{3}$ cm

C 10 cm

(D) 15 cm

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9 Which expression is equivalent to $(x + 3)^3 - 9x(x + 3)$?

(A) $x^3 + 27$

B $x^3 - 27$

C $x^3 - 9x^2 - 27x + 27$

D $x^3 - 9x^2 + 27x + 27$

$(x+3)[(x+3)^2 - 9x]$

$(x+3)[x^2+6x+9-9x]$

$(x+3)[x^2-3x+9]$

$x^3-3x^2+9x+3x^2-9x+27$

10 Suppose $p(x) = x^3 - 2x^2 + 13x + k$. The remainder of the division of $p(x)$ by $(x + 1)$ is $^-8$. What is the remainder of the division of $p(x)$ by $(x - 1)$?

A $^-8$

B 8

C 16

(D) 20

$-1 \mid 1 \quad -2 \quad 13 \quad K$

$\downarrow \quad -1 \quad 3 \quad -16$

$1 \quad -3 \quad 16$

$K - 16 = -8$

$K = 8$

$1 \mid 1 \quad -2 \quad 13 \quad 8$

$\downarrow \quad 1 \quad -1 \quad 12$

$1 \quad -1 \quad 12 \quad 20$

11 What is the ***approximate*** solution to the equation $3^{x-1} = 4^{2x+5}$?

A 3.875

B 1.262

C $^-2.354$

(D) $^-4.797$

$(x-1)\cdot\log(3) = (2x+5)\log(4)$

$x - 1 = 1.2619(2x+5)$

$x - 1 = 2.5237x + 6.309$

$-7.309 = 1.5237x$

$x = -4.797$

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12 Samantha invested $10,000 in each of two different financial plans in 2013. The predicted value of each plan is modeled below.

- Plan M: a rate of 7.5%, compounded continuously.
- Plan N: The value is determined by the function $y = 5x^3 - 50x^2 + 4x + 10{,}000$, where x is the number of years after 2013.

Plan N has a greater predicted value than Plan M during which years?

A from 2014 to 2041

B from 2028 to 2055

(C) from 2042 to 2073

D Plan N never has a greater value than Plan M.

$y = 10000e^{.075t}$

Graph it!

Check the Table!

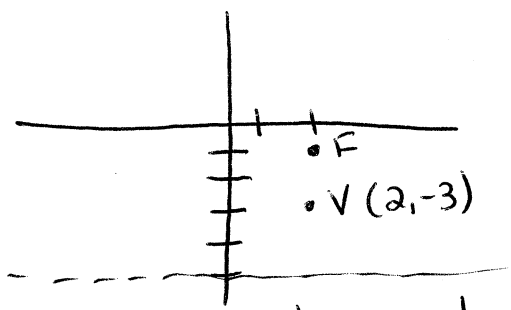
13 Which is an equation of a parabola that has a directrix of $y = -5$ and a focus at $(2, -1)$?

A $y = \frac{1}{2}(x + 2)^2 + 2$

B $y = \frac{1}{8}(x + 2)^2 + 3$

(C) $y = \frac{1}{8}(x - 2)^2 - 3$

D $y = \frac{1}{2}(x - 2)^2 - 2$



Focal length = 2

$a = \frac{1}{4c}$ $a = \frac{1}{4(2)}$

14 Which choice shows the solutions to the equation $8x^2 + 3x = -7$?

(A) $\frac{-3 \pm i\sqrt{215}}{16}$

B $\frac{3 \pm i\sqrt{215}}{16}$

C $\frac{-3 \pm \sqrt{233}}{16}$

D $\frac{3 \pm \sqrt{233}}{16}$

$8x^2 + 3x + 7 = 0$

$\frac{-3 \pm \sqrt{9 - 4(8)(7)}}{2(8)}$

$\frac{-3 \pm \sqrt{-215}}{16}$

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15 Fred drives an average of 15,000 miles per year, and his car gets 20 miles per gallon of gasoline.

- The average cost of gasoline is $3.25 per gallon.
- He buys a new car.
- In his new car, Fred continues to average 15,000 miles per year, and the average cost of gasoline remains the same.

Approximately how many more miles per gallon does the new car get if Fred has a savings of $650 per year on gasoline?

A 5.8 mpg

(B) 7.3 mpg

C 8.8 mpg

D 10.3 mpg

$\frac{15000 \text{ miles}}{550 \text{ gallons}} = 27.27 \text{mpg}$ (See below)

Old car 750 gallons in one year = $2437.50 per year

$2437.50 - 650 = \$1787.5$ per year (new amount)

$\frac{\$1787.5}{\$3.25} = 550$ gallons per year

27.27mpg (new) − 20mpg (old)

16 What is the ***approximate*** value of the sum:

$$8 - \frac{8}{7} + \frac{8}{49} - \ldots + 8 \cdot \left(\frac{-1}{7}\right)^{2,500} ?$$

A 1

(B) 7

C 8

D 9

Geometric $a_1 = 8$

$r = -1/7$

$n = 2500$ terms

$$S = \frac{8(1-(-1/7)^{2500})}{1-(-1/7)} = \frac{8}{(\frac{8}{7})} = \frac{8}{1} \cdot \frac{7}{8}$$

$= \boxed{7}$