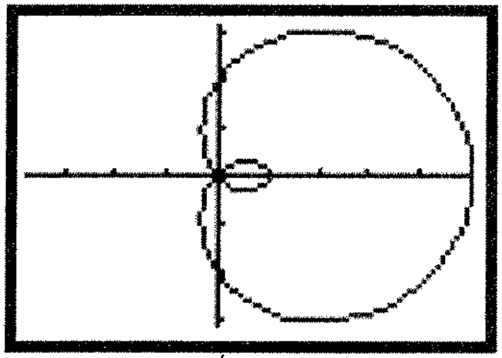
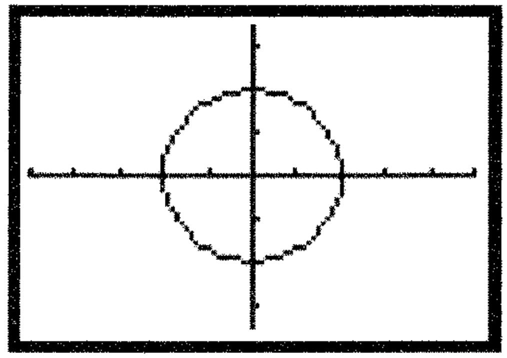
Polar Coordinates, Graphs, and Equations Review

1. Name the shape, its axis of symmetry, and match the appropriate equation:

$r = 2+3\cos\theta$

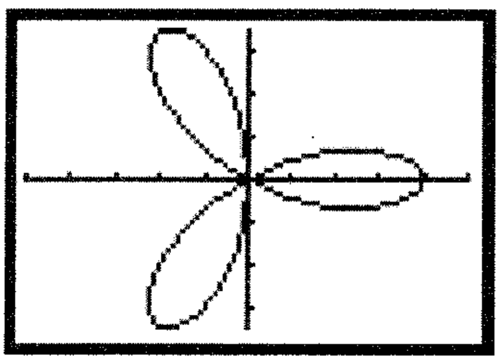


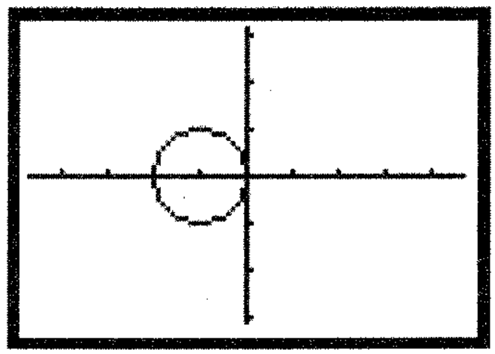
Limacon w/ inner loop
Symmetric w/ positive x axis



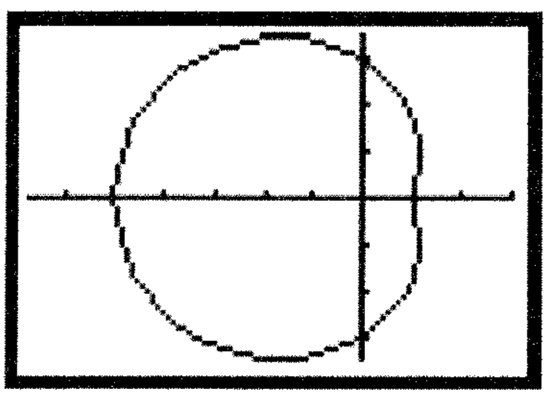
Circle
$r = 2$

Rose
3 petals
4 units long
$r = 4\cos(3\theta)$





Circle
$r = -2\cos\theta$



Dimpled
Limacon
$r = 3-2\cos\theta$

Equation Matching Bank:

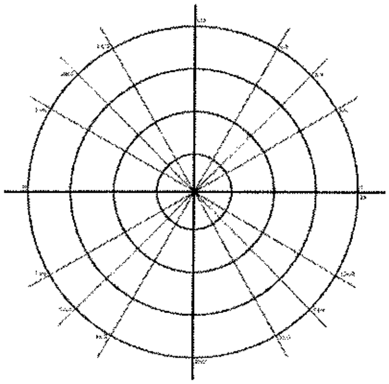
$\cancel{r = 4\cos(3\theta)}$ $\quad$ $\cancel{r = -2\cos\theta}$ $\quad$ $r = 4 - 3\cos\theta$ $\quad$ $\cancel{r = 2}$ $\quad$ $\cancel{r = 2 + 3\cos\theta}$

$\cancel{r = 3 - 2\cos\theta}$ $\quad$ $r = 2\sin\theta$ $\quad$ $r = 3\cos(4\theta)$ $\quad$ $r = 4$ $\quad$ $r = 3 - 4\cos\theta$

2. Sketch the following graphs on the grids given below: You may use your calculator.

$r = 3\cos(5\theta)$

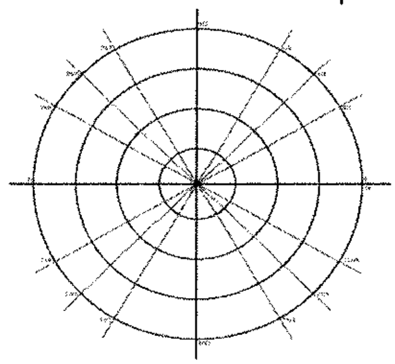
This graph is a rose



$r = 3 + 2cos\theta$

This graph is a Dimpled Limacon

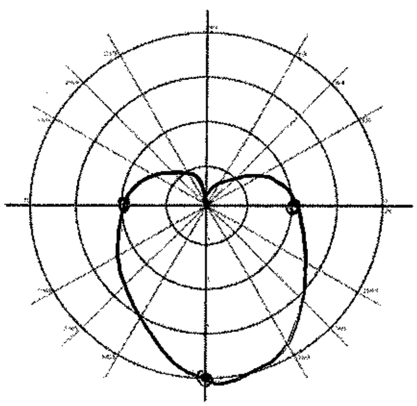
It is symmetric about the positive x axis



$r = 2 - 2sin\theta$

This graph is a Cardioid

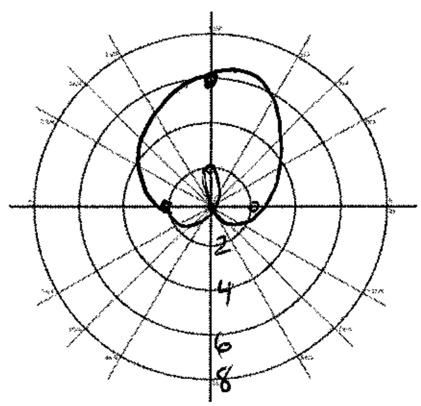
It is symmetric about the negative y axis



$\boldsymbol{r = 2 + 4sin\theta}$

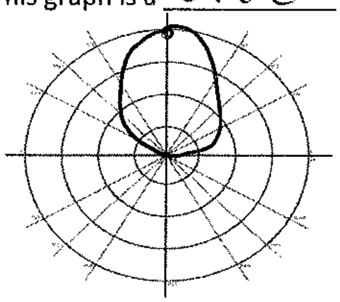
This graph is a limacon with an inner loop

It is symmetric about the positive y axis



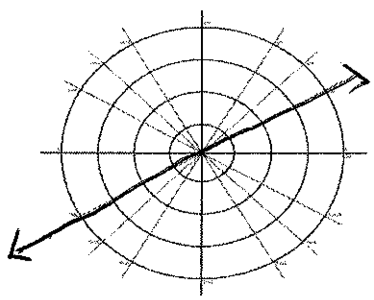
$r = 4sin\theta$

This graph is a circle



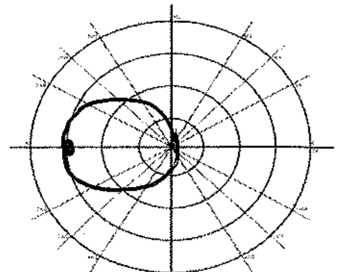
$\boldsymbol{\theta = -\frac{5\pi}{6}}$

This graph is a line



$\boldsymbol{r = -3cos\theta}$

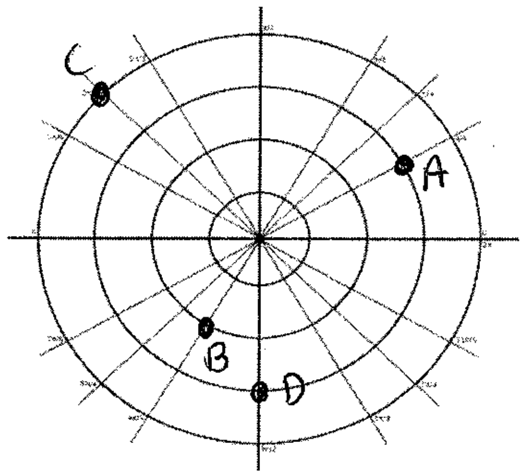
This graph is a circle



Polar Coordinates, Graphs, and Equations Review

3. Graph the following points:

$A\left(3, \frac{\pi}{6}\right)$ $B\left(-2, \frac{\pi}{3}\right)$ $C\left(-4, \frac{7\pi}{4}\right)$ $D\left(3, \frac{3\pi}{2}\right)$



4. Solve the following systems (find the intersection points and sketch):

Give your answers in degrees and radians.

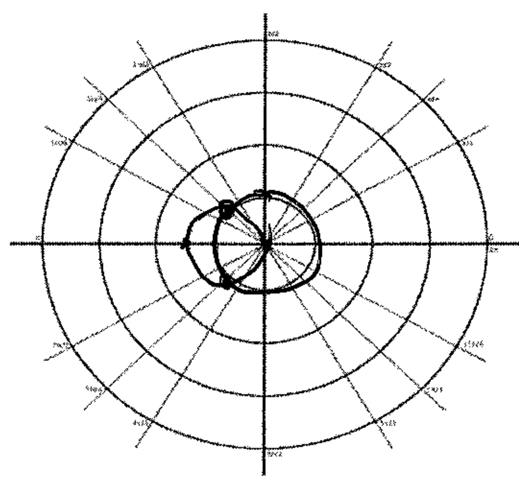
$r = 1$ and $r = -\sqrt{2}\cos\theta$

$1 = -\sqrt{2}\cos\theta$

$\frac{1}{-\sqrt{2}} = \cos\theta$

$-\frac{\sqrt{2}}{2} = \cos\theta \quad \theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\theta = 135°$ and $225°$ or $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$



Radians

$(1, \frac{3\pi}{4})$

$(1, \frac{5\pi}{4})$

Degrees

$(1, 135°)$

$(1, 225°)$

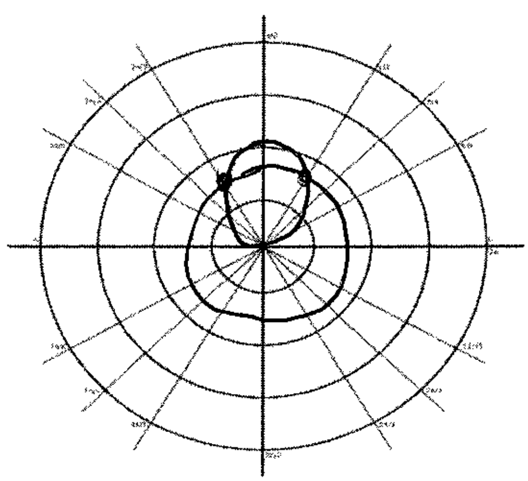
$r = 2\sin\theta$ and $r = \sqrt{3}$

$2\sin\theta = \sqrt{3}$

$\sin\theta = \sqrt{3}/2$

$\theta = \sin^{-1}(\sqrt{3}/2)$

$\theta = 60°$ and $120°$ or $\frac{\pi}{3}$ and $\frac{2\pi}{3}$



Radians

$(\sqrt{3}, \pi/3)$

$(\sqrt{3}, 2\pi/3)$

Degrees

$(\sqrt{3}, 60°)$

$(\sqrt{3}, 120°)$

Polar Coordinates, Graphs, and Equations Review

Convert each rectangular coordinate to a polar coordinate give answers in degrees and radians.

(#5 and 7 are calculator INACTIVE, while #6 and 8 are calculator ACTIVE)

5. (-4,4) Q2

$(-4)^2+(4)^2=r^2 \quad \tan\theta=\frac{4}{-4}$

$32=r^2 \quad \theta=135^\circ \text{ or } 315^\circ$

$4\sqrt{2}=r$

r θ

$\boxed{(4\sqrt{2},135^\circ)}$ or $\boxed{(4\sqrt{2},\frac{3\pi}{4})}$

6. (2, -5) Q4

$2^2+(-5)^2=r^2 \quad \tan\theta=\frac{-5}{2}$

$29=r^2 \quad \theta=-1.19^R$

$\sqrt{29}=r$

$Q4=2\pi-1.19$

$=5.09^R$

$\boxed{(\sqrt{29},5.09^R)\ (\sqrt{29},291.8^\circ)}$

7. $(3,-\sqrt{3})$ Q4

$(3)^2+(-\sqrt{3})^2=r^2 \quad \tan\theta=\frac{-\sqrt{3}}{3}$

$12=r^2 \quad \theta=330^\circ \text{ or } \frac{11\pi}{6}$

$2\sqrt{3}=r$

$\boxed{(2\sqrt{3},330^\circ) \text{ or } (2\sqrt{3},\frac{11\pi}{6})}$

8. (-5, -8) Q3

$(-5)^2+(-8)^2=r^2 \quad \tan\theta=\frac{-8}{-5} \quad \theta=58^\circ$ or 1.01^R

$89=r^2$

$\sqrt{89}=r$

r θ

$\boxed{(\sqrt{89},238^\circ) \text{ or } (\sqrt{89},4.15^R)}$

Convert each polar coordinate to a rectangular coordinate (do not use a calculator)

9. (6,150°)

$x=6\cos 150=6(-\sqrt{3}/2)$

$y=6\sin 150=6(1/2)$

$\boxed{(-3\sqrt{3},3)}$

x y

10. (-4,135°)

$x=-4\cos 135=-4(-\sqrt{2}/2)=2\sqrt{2}$

$y=-4\sin 135=-4(\sqrt{2}/2)=-2\sqrt{2}$

$\boxed{(2\sqrt{2},-2\sqrt{2})}$

x y

11. $\left(3,\frac{5\pi}{3}\right)$

$x=3\cos 5\pi/3=3(1/2)$

$y=3\sin 5\pi/3=3(-\sqrt{3}/2)$

x y

$\boxed{\left(\frac{3}{2},-\frac{3\sqrt{3}}{2}\right)}$

12. $(-2,1.26^R)$

$x=-2\cos(1.26)=-.611$

$y=-2\sin(1.26)=-1.904$

x y

$\boxed{(-.61,-1.9)}$

Convert the equations from polar to rectangular or rectangular to polar.

13. $x^2+(y-3)^2=9$

$x^2+y^2-6y+9=9$

$x^2+y^2-6y=0$

$r^2-6r\sin\theta=0$

$\frac{r^2=6r\sin\theta}{r}$

$\boxed{r=6\sin\theta}$

14. $r=6\sin(\theta)$ mult by r

$r^2=6r\sin\theta$

$x^2+y^2=6y$

$x^2+y^2-6y+\_\_=0+\_\_$

$x^2+y^2-6y+9=0+9$

$\boxed{x^2+(y-3)^2=9}$

15. $r=8$ mult by r

$r^2=8r$

$x^2+y^2=8(8) \leftarrow$ b/c r=8

$\boxed{x^2+y^2=64}$

16. , 17. , 18. on the next page

16. $r = \dfrac{8}{(1+\sin\theta)}$ Parabola

$r(1+\sin\theta) = 8$

$r + r\sin\theta = 8$

$\sqrt{x^2+y^2} + y = 8 \rightarrow \sqrt{x^2+y^2} = 8 - y \rightarrow x^2+y^2 = (8-y)^2$

$x^2+y^2 = 64 - 16y + y^2$

$x^2 = 64 - 16y$

$\dfrac{x^2 - 64 = -16y}{-16}$

Vertex $(0,4)$

$\boxed{-\dfrac{1}{16}x^2 + 4 = y}$

17. $3x - y = 10$

$3r\cos\theta - r\sin\theta = 10$

$r(3\cos\theta - \sin\theta) = 10$

$\boxed{r = \dfrac{10}{3\cos\theta - \sin\theta}}$

18. $r = \dfrac{7}{3\cos\theta - 8\sin\theta}$

$3r\cos\theta - 8r\sin\theta = 7$

$\boxed{3x - 8y = 7}$ (Standard Form of a Line)