

## Happy International Dance Day!

- Park your phones
- Grab your calculators

Quiz Thu.  
(Tentative)  
Test May 8<sup>th</sup>

**Did you take the AP Survey????**

Introduction to Polar Coordinates

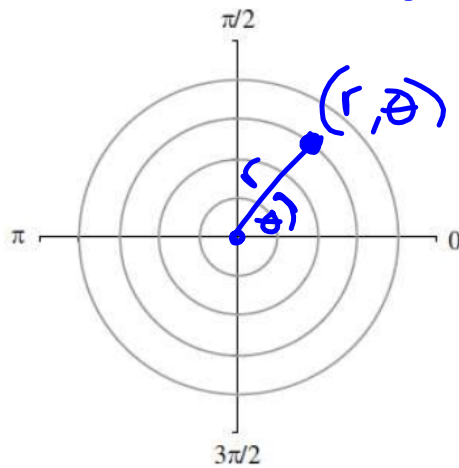
If we list the position of an object based on its distance away and its angle, then we are using a polar coordinate system.

$r = \text{radius}$

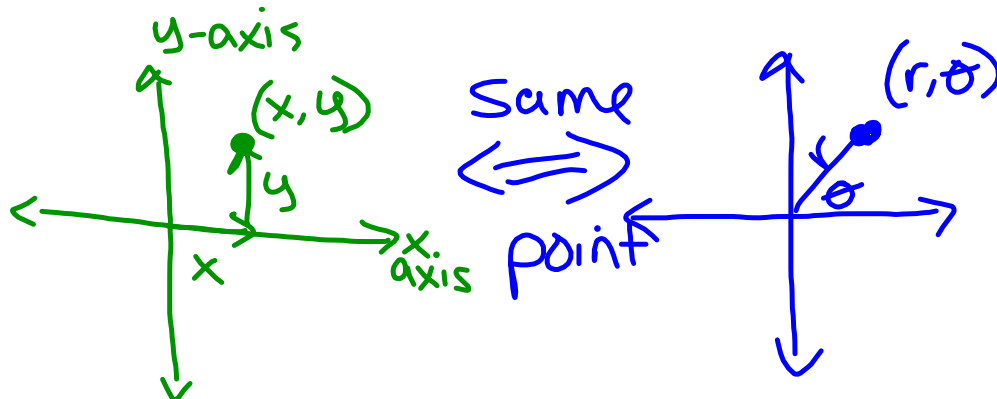
A fixed point  $O$  is called the pole (origin) and the polar axis is the horizontal ray to the right of the pole.

Coordinates will be given as  $(r, \theta)$

$r = \text{radius}$   
 $\theta = \text{angles}$   $\left\{ \begin{array}{l} \text{degrees} \\ \text{radius} \end{array} \right.$



GPS systems and land surveying are just two examples of why we need polar coordinates.



Plot the following points, then create three other polar representations for that point.

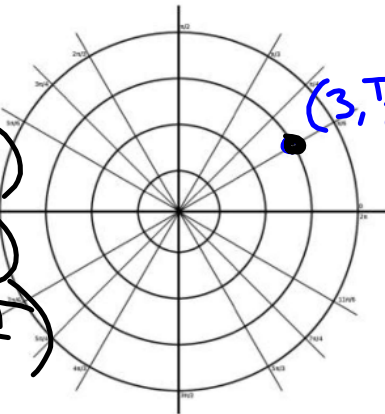
$(r, \theta)$

- 1. angle 1st
- 2. count "r" radius

- 1.  $(3, -\frac{11\pi}{6})$
- 2.  $(-3, \frac{7\pi}{6})$
- 3.  $(-3, -\frac{5\pi}{6})$

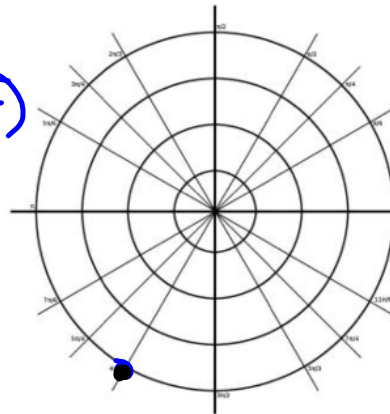
$(3, \frac{\pi}{6})$

$\theta = \frac{\pi}{6}$   
 $r = 3$



$(4, -\frac{2\pi}{3})$

negative angle

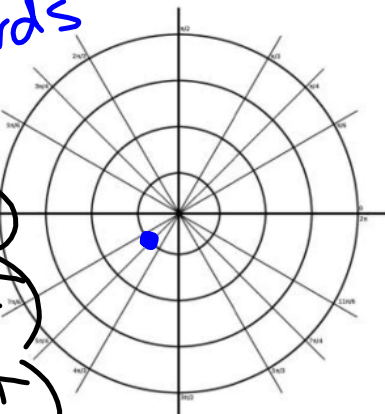


- 1.  $(4, \frac{4\pi}{3})$
- 2.  $(-4, \frac{\pi}{3})$
- 3.  $(-4, -\frac{5\pi}{3})$

negative radius  
 $r = -1$

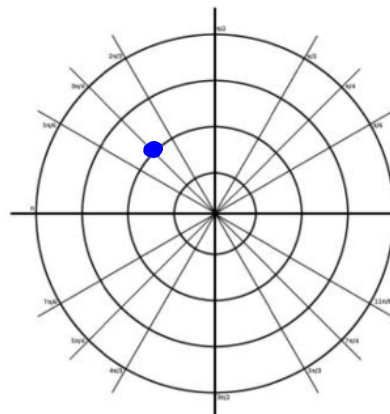
count backwards

$(-1, \frac{\pi}{4})$



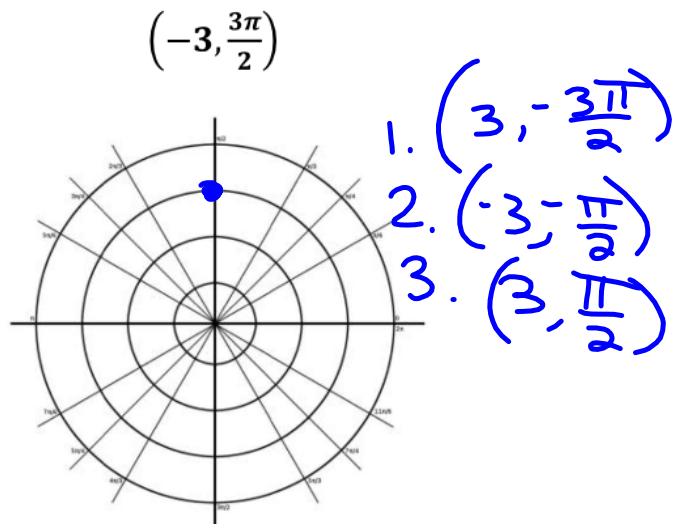
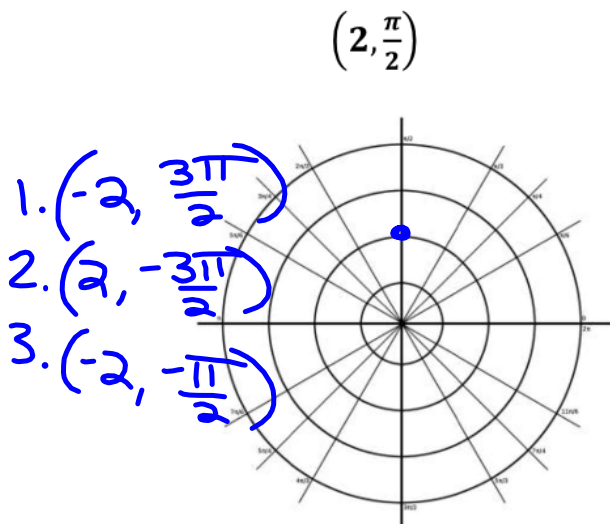
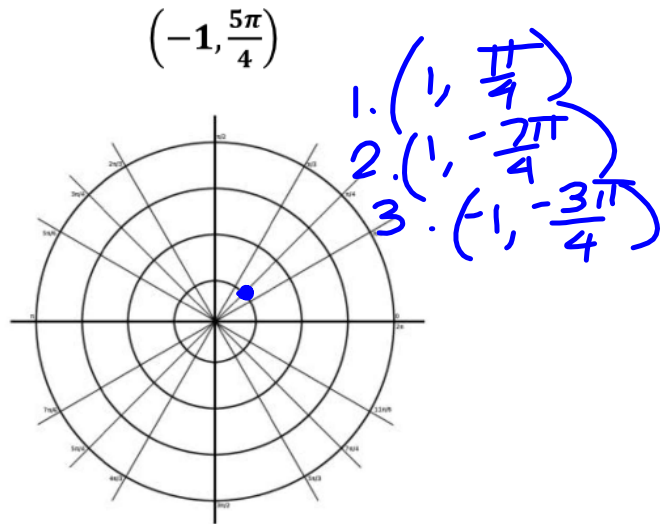
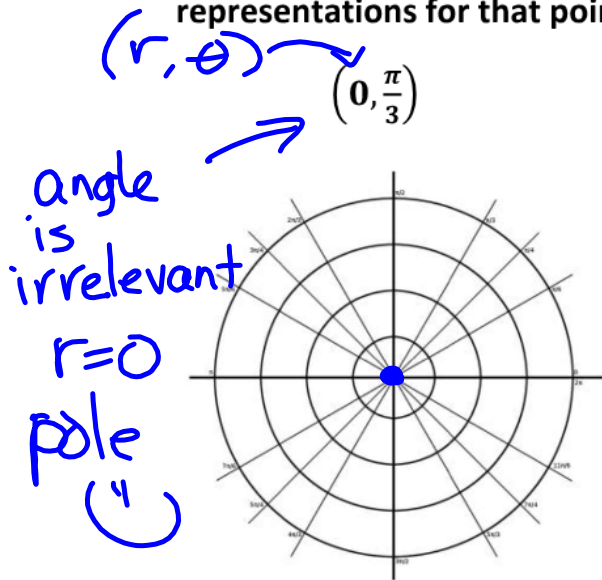
- 1.  $(1, \frac{5\pi}{4})$
- 2.  $(-1, -\frac{7\pi}{4})$
- 3.  $(1, -\frac{3\pi}{4})$

$(-2, \frac{7\pi}{4})$



- 1.  $(2, \frac{3\pi}{4})$
- 2.  $(2, -\frac{5\pi}{4})$
- 3.  $(-2, -\frac{\pi}{4})$

Plot the following points, then create three other polar representations for that point.



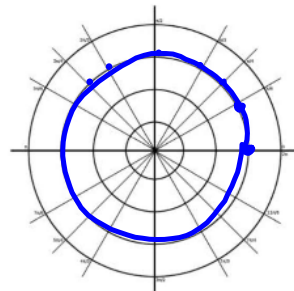
+ radius  $\Rightarrow$  on the line

- radius  $\Rightarrow$  across from the line

Basic Polar Graphs

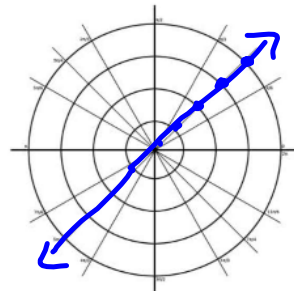
Graph  $r = 3$

Circle, centered at the pole



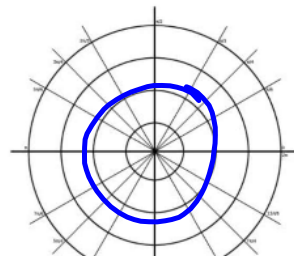
Graph  $\theta = \frac{\pi}{4}$

Line, with angle  $\frac{\pi}{4}$



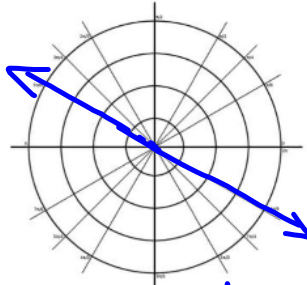
Graph  $r = 2$

Circle



Graph  $\theta = -\frac{7\pi}{6}$

Line



$r = \text{constant} \Rightarrow$  circle, centered at pole  
 $\theta = \text{angle} \Rightarrow$  Line, through the pole  
 deg. Radians

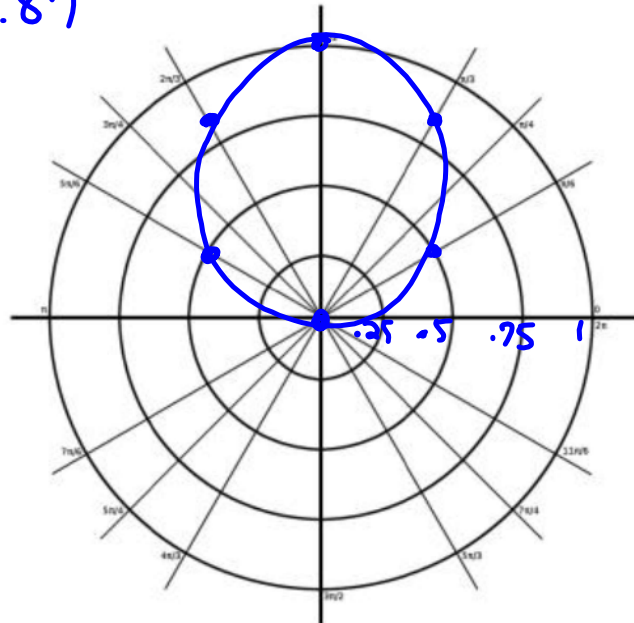
Special Polar Graphs:  $(r, \theta)$

Graph:  $r = \sin\theta$

Mode:  
 - Polar  
 - Radians

Handwritten notes:  
 $\pi/6$   
 $\pi/3$

$\theta$	$r$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$ $(\frac{1}{2}, \frac{\pi}{6})$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2} \approx .87$
$\frac{\pi}{2}$	-1
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{5\pi}{6}$	$\frac{1}{2}$
$\pi$	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$
$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$\frac{3\pi}{2}$	-1
$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$\frac{11\pi}{6}$	$-\frac{1}{2}$
$2\pi$	0



Window:

$\theta_{min} = 0$

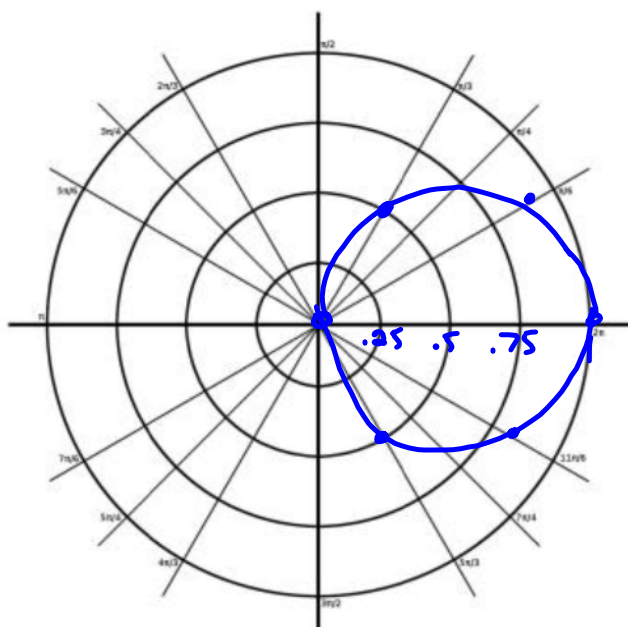
$\theta_{max} = 2\pi \approx 6.28$

Tbl setup

$\Delta Tbl = \pi/6$

Graph  $r = \cos\theta$  ↖ circle

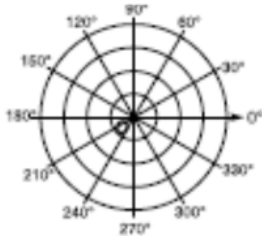
$\theta$	$r$
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-\frac{1}{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$
$\pi$	-1
$\frac{7\pi}{6}$	.
$\frac{4\pi}{3}$	.
$\frac{3\pi}{2}$	.
$\frac{5\pi}{3}$	.
$\frac{11\pi}{6}$	.
$2\pi$	.



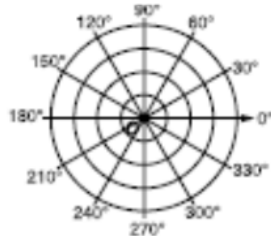
Graphing Polar Coordinates Practice

name \_\_\_\_\_

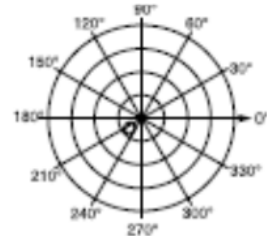
$(3, 45^\circ)$



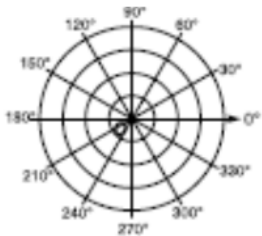
$(-2, 60^\circ)$



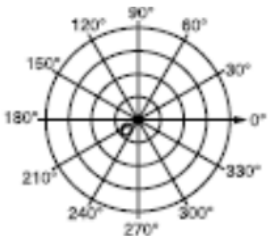
$(4, 225^\circ)$



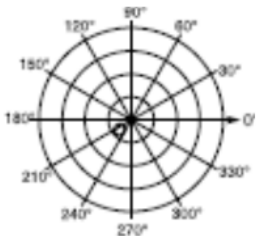
$(-3, 315^\circ)$



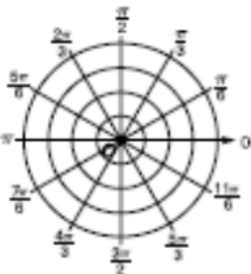
$(-2, -300^\circ)$



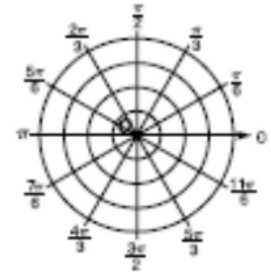
$(4, 150^\circ)$



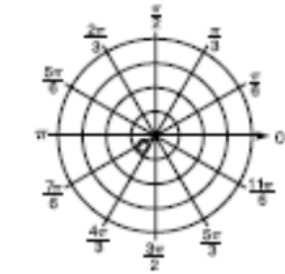
$(2, \frac{\pi}{3})$



$(3, \frac{2\pi}{3})$



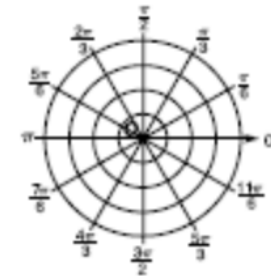
$(-4, -\frac{\pi}{6})$



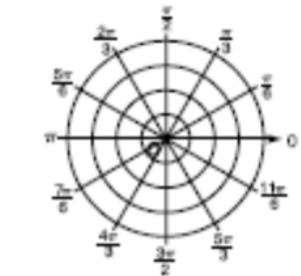
$(-2, \frac{5\pi}{6})$



$(4, \frac{7\pi}{4})$



$(-3, \frac{7\pi}{6})$





Graphing Polar Coordinates Practice

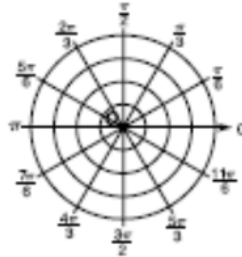
name \_\_\_\_\_

Graph each polar equation and describe its shape.

$$r = 3$$



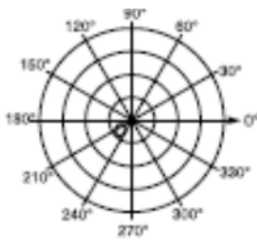
$$\theta = \frac{11\pi}{6}$$



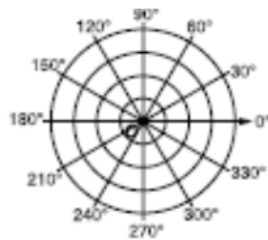
$$\theta = \frac{\pi}{6}$$



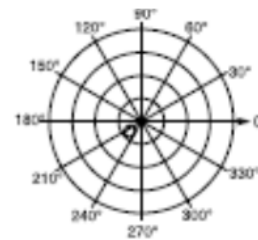
$$\theta = 120^\circ$$



$$r = 2$$



$$\theta = -150^\circ$$



For the following coordinate points: Graph the point and then rewrite them in three different ways

$$\left(-2, \frac{5\pi}{6}\right)$$



$$\left(4, \frac{7\pi}{4}\right)$$

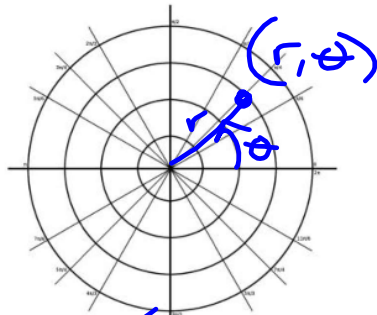


$$\left(-3, \frac{7\pi}{6}\right)$$



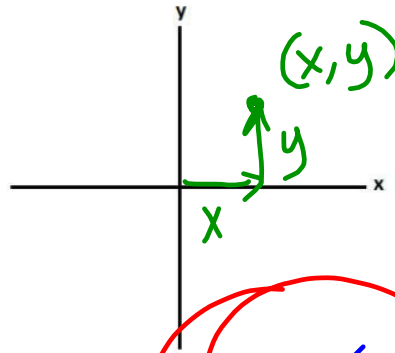
Polar Coordinates

$(r, \theta)$



Rectangular Coordinates

$(x, y)$



conversion Formulas

$$\tan(\theta) = \frac{y}{x}$$

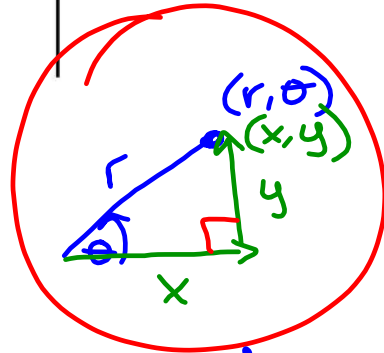
$$x = r \cdot \cos\theta$$

$$y = r \cdot \sin\theta$$

Pythagorean Identity:

$$x^2 + y^2 = r^2$$

circle  
☺



$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

Polar  
 $(r, \theta)$

↑  
Radians  
Degrees

Same  
point

Rectangular  
 $(x, y)$

Quadrants  
are same

$$x = r \cdot \cos \theta$$

$$y = r \cdot \sin \theta$$

$$\tan \theta = \frac{y}{x}$$

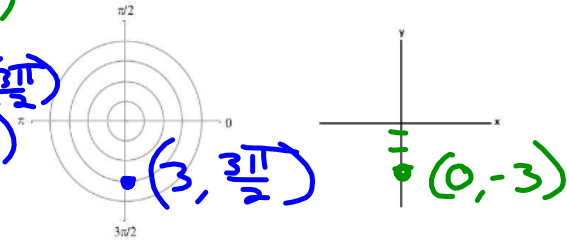
Convert the following polar coordinates to rectangular - Then graph both points:

polar  $\rightarrow$   $(3, \frac{3\pi}{2})$

$(0, -3)$

$$x = 3 \cos(\frac{3\pi}{2}) = 3 \cdot 0 = 0$$

$$y = 3 \sin(\frac{3\pi}{2}) = 3(-1) = -3$$

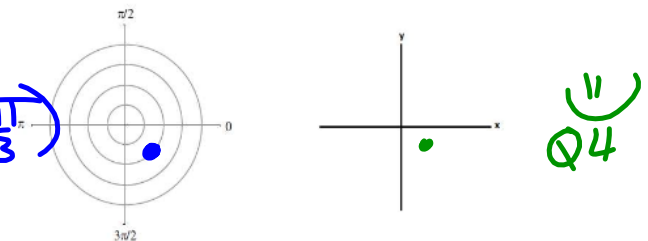


$(-2, \frac{2\pi}{3})$

$(1, -\sqrt{3})$

$$x = -2 \cos(\frac{2\pi}{3}) = -2 \cdot (-\frac{1}{2}) = 1$$

$$y = -2 \sin(\frac{2\pi}{3}) = -2 \cdot \frac{\sqrt{3}}{2} = -\sqrt{3} \approx -1.7$$

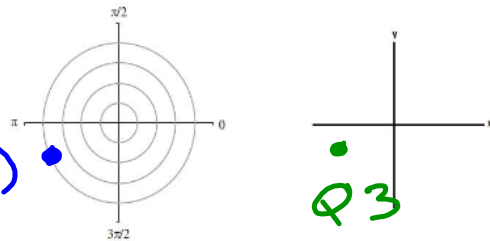


$(-4, \frac{\pi}{6})$

$(-2\sqrt{3}, -2)$

$$x = -4 \cos(\frac{\pi}{6}) = -4 \cdot \frac{\sqrt{3}}{2} = -2\sqrt{3}$$

$$y = -4 \sin(\frac{\pi}{6}) = -4 \cdot \frac{1}{2} = -2$$

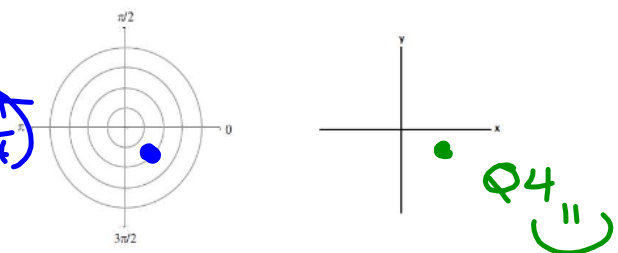


$(2, -\frac{\pi}{4})$

$\frac{7\pi}{4}$

$$x = 2 \cos(-\frac{\pi}{4}) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$$

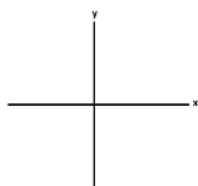
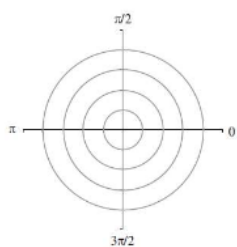
$$y = 2 \sin(-\frac{\pi}{4}) = 2 \cdot (-\frac{\sqrt{2}}{2}) = -\sqrt{2}$$



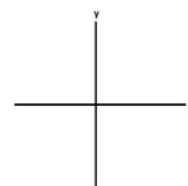
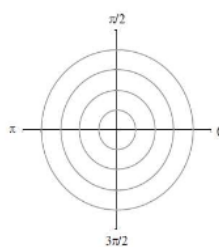
$(\sqrt{2}, -\sqrt{2})$

Try to convert these polar points into rectangular without using the formulas:

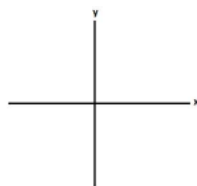
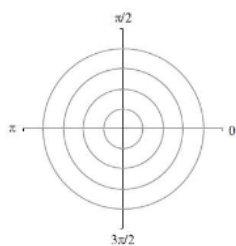
Convert  $(3, \frac{\pi}{2})$  to rectangular



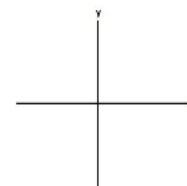
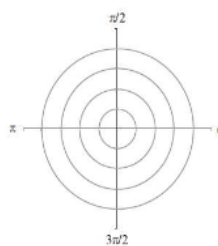
Convert  $(-2, \frac{\pi}{2})$  to rectangular



Convert  $(-4, 2\pi)$  to rectangular



Convert  $(2, \pi)$  to rectangular



Practice: Find the rectangular coordinates of each point with the given polar coordinates.

1.  $(6, 120^\circ)$

2.  $(-4, 45^\circ)$

3.  $(3, \frac{7\pi}{6})$

4.  $(-2, \frac{5\pi}{6})$

5.  $(-5, \frac{4\pi}{3})$

6.  $(3, \frac{\pi}{2})$