# Happy National Apple Pie Day!

- Park your phones
- Grab your calculators

***Introduction to Polar Coordinates***

r=radius

If we list the position of an object based on its distance away and its angle, then we are using a **polar coordinate system.**

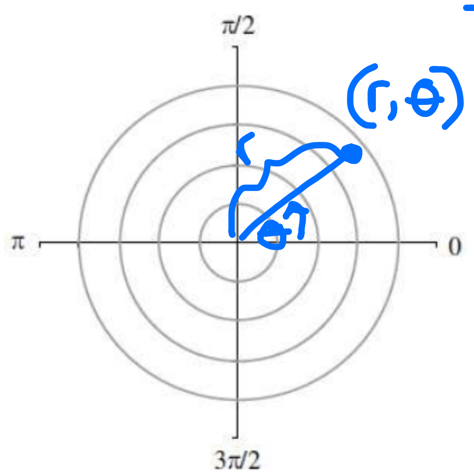
$\theta$

A fixed point O is called the **pole** (origin) and **the polar axis** is the horizontal ray to the right of the pole.

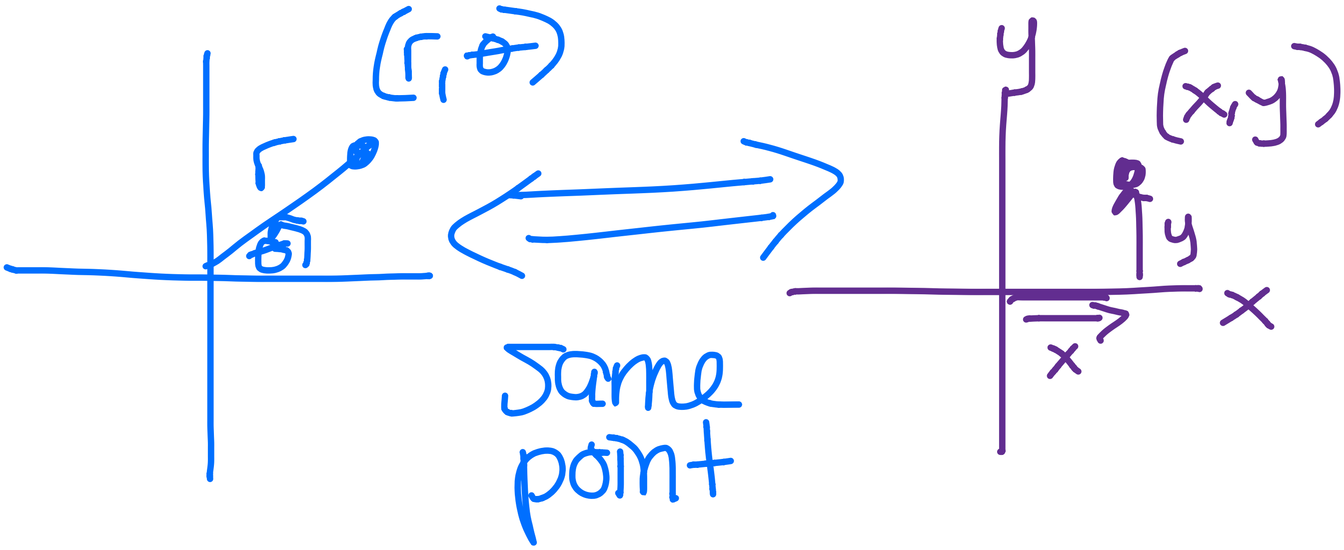
$\boxed{\text{Coordinates will be given as } (r, \theta)}$

r = radius

$\theta$ = angle




**GPS systems and land surveying are just two examples of why we need polar coordinates.**



Plot the following points, then create three other polar representations for that point.

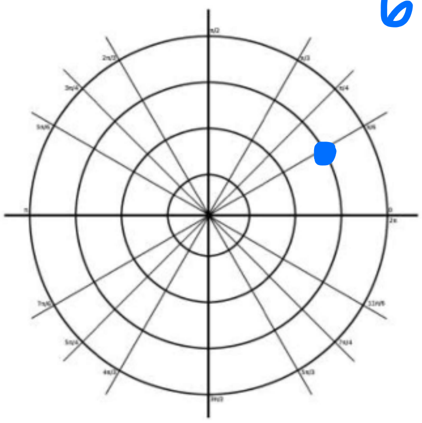
$(r, \theta)$

1. angle
2. radius

$\left(3, \frac{\pi}{6}\right)$

$r = 3$

$\theta = \frac{\pi}{6}$



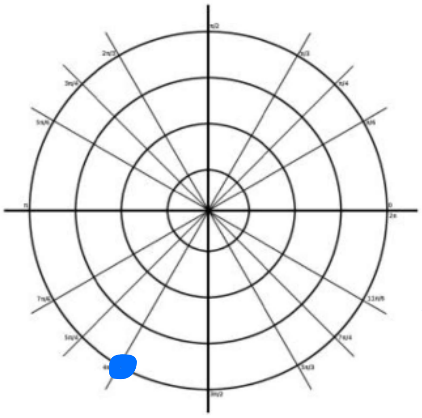
$\left(-3, \frac{7\pi}{6}\right)$

$\left(3, -\frac{11\pi}{6}\right)$

$\left(-3, -\frac{5\pi}{6}\right)$

$\left(4, -\frac{2\pi}{3}\right)$

negative angle



$\left(-4, \frac{\pi}{3}\right)$

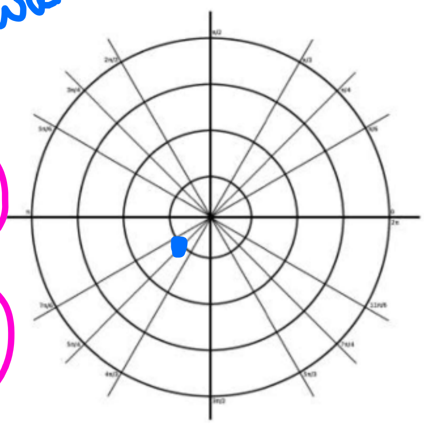
$\left(4, \frac{4\pi}{3}\right)$

$\left(-4, -\frac{5\pi}{3}\right)$

$\left(-1, \frac{\pi}{4}\right)$

negative radius

count backwards

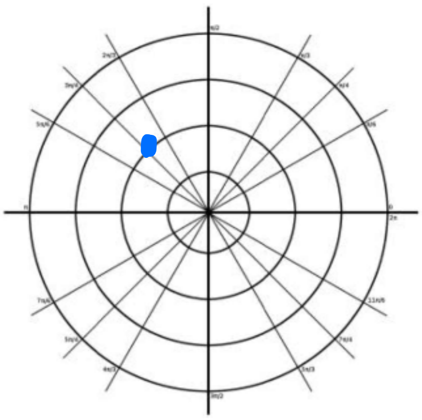


$\left(1, -\frac{3\pi}{4}\right)$

$\left(-1, -\frac{7\pi}{4}\right)$

$\left(1, \frac{5\pi}{4}\right)$

$\left(-2, \frac{7\pi}{4}\right)$



$\left(2, \frac{3\pi}{4}\right)$

$\left(2, -\frac{5\pi}{4}\right)$

$\left(-2, \frac{-\pi}{4}\right)$

+ radius $\Rightarrow$ on the line

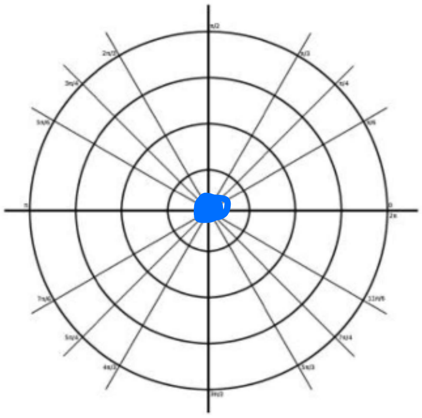
− radius $\Rightarrow$ across from the line

**Plot the following points, then create three other polar representations for that point.**

$\left(0, \frac{\pi}{3}\right)$

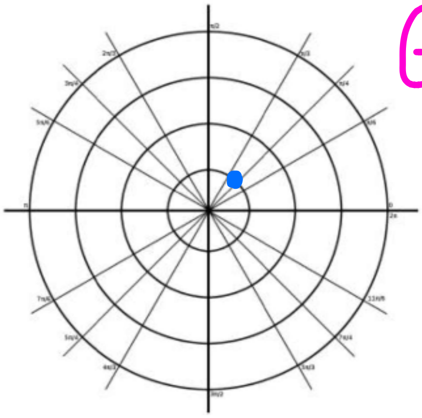
angle is irrelevant
$r=0$

angle $0^\circ$
+ polar axis



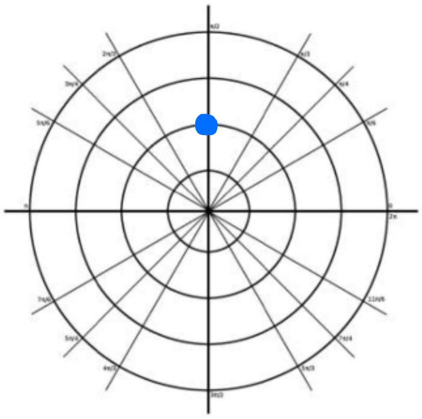
$\left(-1, \frac{5\pi}{4}\right)$

$\left(1, \frac{\pi}{4}\right)$
$\left(-1, -\frac{3\pi}{4}\right)$
$\left(1, -\frac{7\pi}{4}\right)$



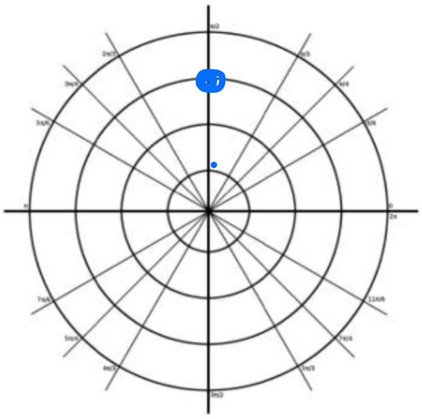
$\left(2, \frac{\pi}{2}\right)$

$\left(2, -\frac{3\pi}{2}\right)$
$\left(-2, -\frac{\pi}{2}\right)$
$\left(-2, \frac{3\pi}{2}\right)$



$\left(-3, \frac{3\pi}{2}\right)$

$\left(3, \frac{\pi}{2}\right)$
$\left(-3, -\frac{\pi}{2}\right)$
$\left(3, -\frac{3\pi}{2}\right)$

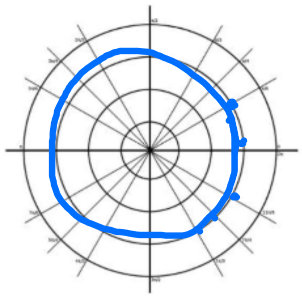


***Basic Polar Graphs***

**Graph r = 3**

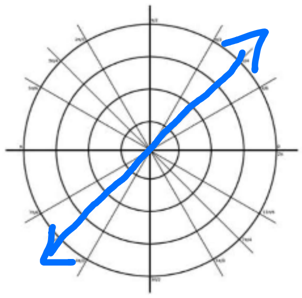
* Centered at the pole

Circle w/radius 3



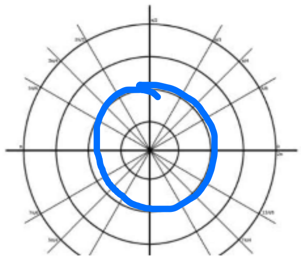
**Graph $\theta = \frac{\pi}{4}$**

Line w/angle $\frac{\pi}{4}$



**Graph r = 2**

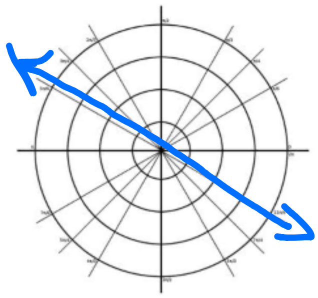
Circle



**Graph $\theta = -\frac{7\pi}{6}$**

$-\frac{7\pi}{6}$

Line



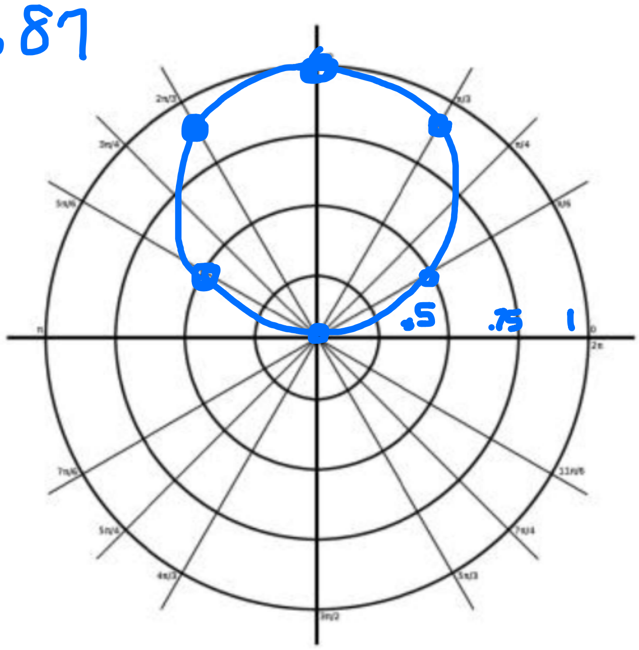
$(r, \theta)$

Mode:
Polar
Radians

**Special Polar Graphs:**

Graph: $r = sin\theta$

| θ | r |
|---|---|
| $0$ | $0$ |
| $\frac{\pi}{6}$ | $1/2$ |
| $\frac{\pi}{3}$ | $\sqrt{3}/2 \approx .87$ |
| $\frac{\pi}{2}$ | $1$ |
| $\frac{2\pi}{3}$ | $\sqrt{3}/2$ |
| $\frac{5\pi}{6}$ | $1/2$ |
| $\pi$ | $0$ |
| $\frac{7\pi}{6}$ | $-1/2$ |
| $\frac{4\pi}{3}$ | $-\sqrt{3}/2$ |
| $\frac{3\pi}{2}$ | $-1$ |
| $\frac{5\pi}{3}$ | $-\sqrt{3}/2$ |
| $\frac{11\pi}{6}$ | $-1/2$ |
| $2\pi$ | $0$ |



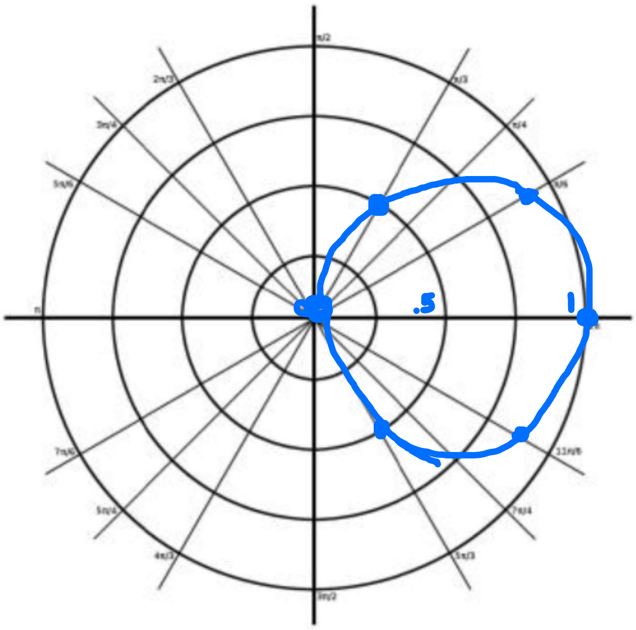

Zoom 7 trig

Window
$\theta_{min} = 0$
$\theta_{max} = 2\pi$

Tbl set up $\Delta Tbl \frac{\pi}{6}$

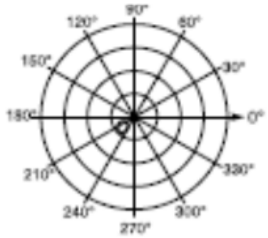
Graph $r = cos\theta$

| θ | r |
|---|---|
| 0 | 1 |
| $\frac{\pi}{6}$ | $\sqrt{3}/2$ |
| $\frac{\pi}{3}$ | $1/2$ |
| $\frac{\pi}{2}$ | 0 |
| $\frac{2\pi}{3}$ | $-1/2$ |
| $\frac{5\pi}{6}$ | $-\sqrt{3}/2$ |
| $\pi$ | $-1$ |
| $\frac{7\pi}{6}$ | |
| $\frac{4\pi}{3}$ | |
| $\frac{3\pi}{2}$ | |
| $\frac{5\pi}{3}$ | |
| $\frac{11\pi}{6}$ | |
| $2\pi$ | |

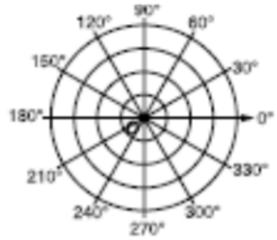


**Graphing Polar Coordinates Practice** **name___________________**

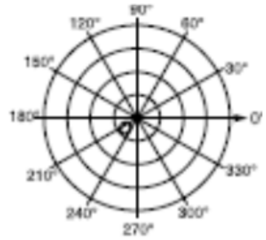
$(3, 45°)$



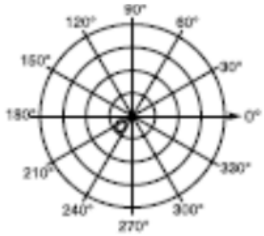
$(-2, 60°)$



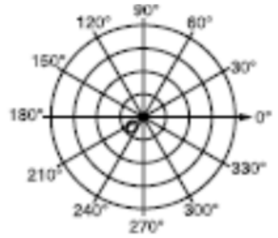
$(4, 225°)$



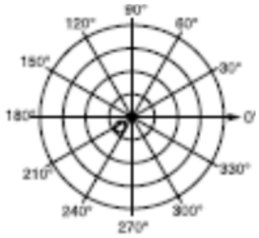
$(-3, 315°)$



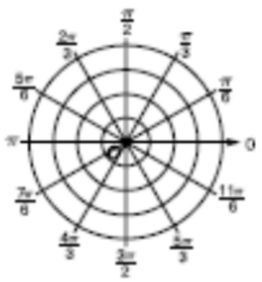
$(-2, -300°)$



$(4, 150°)$



$\left(2, \frac{\pi}{3}\right)$



$\left(3, \frac{2\pi}{3}\right)$

$\left(-4, -\frac{\pi}{6}\right)$

$\left(-2, \frac{5\pi}{6}\right)$

$\left(4, \frac{7\pi}{4}\right)$

$\left(-3, \frac{7\pi}{6}\right)$

**Graphing Polar Coordinates Practice** **name__________________**

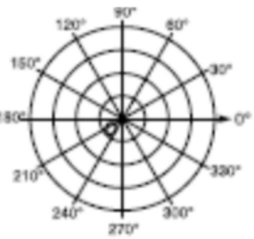
**Graph each polar equation and describe its shape.**

$r = 3$

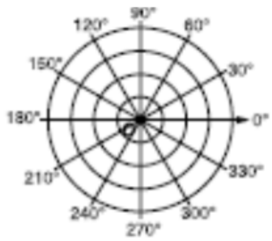
$\theta = \frac{11\pi}{6}$

$\theta = \frac{\pi}{6}$

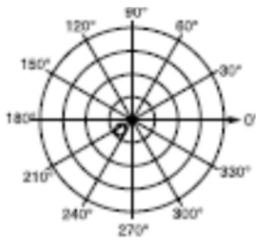
$\theta = 120°$



$r = 2$



$\theta = -150°$



**For the following coordinate points: Graph the point and then rewrite them in three different ways**

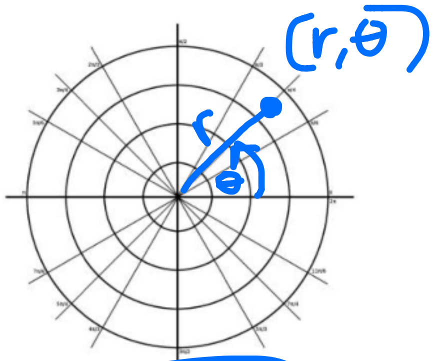
$\left(-2, \frac{5\pi}{6}\right)$

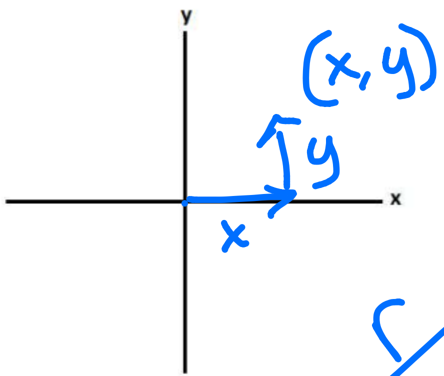
$\left(4, \frac{7\pi}{4}\right)$

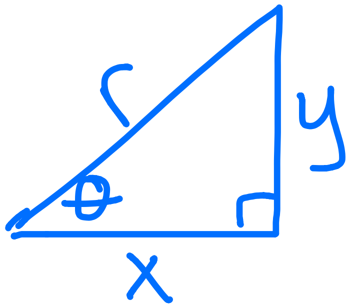
$\left(-3, \frac{7\pi}{6}\right)$

**Polar Coordinates** **Rectangular Coordinates**

$(r,\theta)$ $(x,y)$




$\text{Tan}(\theta) = \frac{y}{x}$

$x = r \cdot \cos\theta$

$y = r \cdot \sin\theta$

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

**Pythagorean Identity:**

$x^2 + y^2 = r^2$

Circle

Polar $(r,\theta)$ $\Longleftrightarrow$ same point Rectangular $(x,y)$

**Convert the following polar coordinates to rectangular - Then graph both points:**

$(r, \theta)$

$\left(3, \frac{3\pi}{2}\right)$

$x = 3 \cdot \cos \frac{3\pi}{2}$

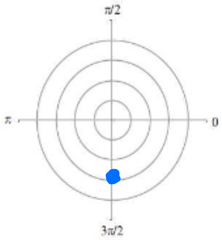
$x = 3 \cdot 0$

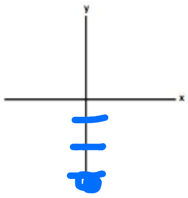
$x = 0$

$y = 3 \sin \frac{3\pi}{2}$

$= 3 \cdot -1$

$y = -3$




$(0, -3)$

$r \quad \theta$

$\left(-2, \frac{2\pi}{3}\right)$

$x = -2 \cos \frac{2\pi}{3}$

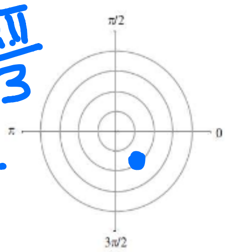
$= -2\left(-\frac{1}{2}\right)$

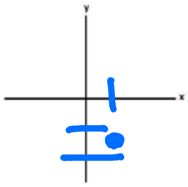
$x = 1$

$y = -2 \sin \frac{2\pi}{3}$

$y = -2 \cdot \frac{\sqrt{3}}{2}$

$y = -\sqrt{3}$




$(1, -\sqrt{3})$

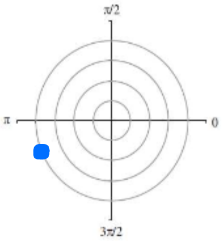
$\left(-4, \frac{\pi}{6}\right)$

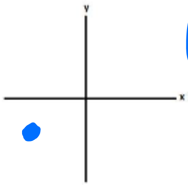
$x = -4 \cos \frac{\pi}{6}$

$x = -2\sqrt{3}$

$y = -4 \sin \frac{\pi}{6}$

$y = -2$




$(-2\sqrt{3}, -2)$

$\left(2, -\frac{\pi}{4}\right)$

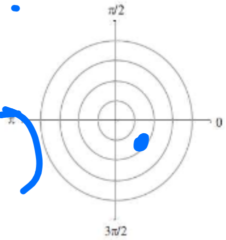
$\frac{7\pi}{4}$

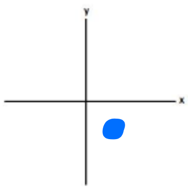
$x = 2 \cos\left(\frac{7\pi}{4}\right)$

$x = \sqrt{2}$

$y = 2 \sin\left(\frac{7\pi}{4}\right)$

$y = -\sqrt{2}$

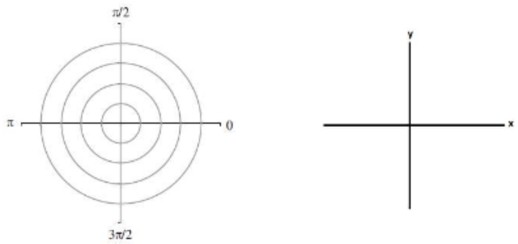


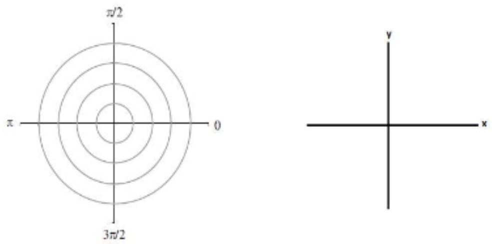

$(\sqrt{2}, -\sqrt{2})$

**Try to convert these polar points into rectangular without using the formulas:**

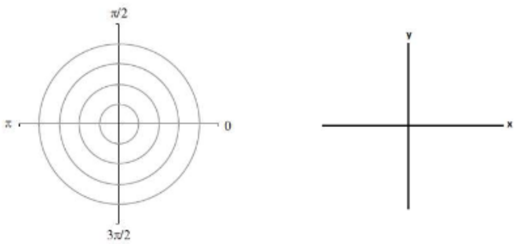
**Convert $(3, \frac{\pi}{2})$ to rectangular**



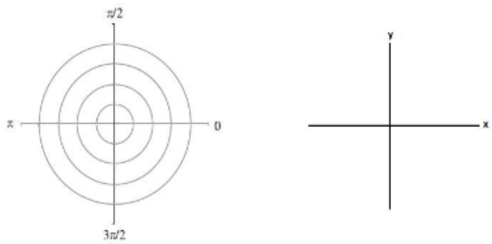

**Convert $(-2, \frac{\pi}{2})$ to rectangular**




**Convert $(-4, 2\pi)$ to rectangular**




**Convert $(2, \pi)$ to rectangular**




**Practice: Find the rectangular coordinates of each point with the given polar coordinates.**

1. $(6,120°)$
2. $(-4,45°)$
3. $\left(3, \frac{7\pi}{6}\right)$
4. $\left(-2, \frac{5\pi}{6}\right)$
5. $\left(-5, \frac{4\pi}{3}\right)$
6. $\left(3, \frac{\pi}{2}\right)$