## Partial Fraction Decomposition

Simplify: $\left(\frac{x-4}{x-4}\right)\frac{8}{x+1} - \frac{5}{x-4}\left(\frac{x+1}{x+1}\right)$

$$\frac{8x-32}{(x-4)(x+1)} + \frac{-5x-5}{(x-4)(x+1)}$$

$$\frac{3x-37}{(x-4)(x+1)} = \boxed{\frac{3x-37}{x^2-3x-4}}$$

The fractions $\frac{8}{x+1}$ and $\frac{5}{x-4}$ are called partial fractions.

We want to take the rational function $\frac{P(x)}{Q(x)}$ and break it into its partial fractions. This process is called partial fraction decomposition.

Goal

$$\frac{3x-37}{x^2-3x-4} = \frac{8}{x+1} - \frac{5}{x-4}$$

1. Divide if improper: If N(x)/D(x) is an improper fraction, divide the denominator into the numerator to obtain
$$\frac{N(x)}{D(x)} = polynomial + \frac{N_1(x)}{D(x)}$$
$\frac{N_1(x)}{D(x)}$ is the function we need to decompose

2. *Factor the denominator:* Completely factor the denominator into linear factors and/or quadratic factors

3. Based on the number and types of factors, we can set up a sum of fractions. See the table below.

| Factor in denominator | | Term in partial fraction decomposition |
|---|---|---|
| $ax+b$ | ← Distinct Linear | $\frac{A}{ax+b}$ |
| $(ax+b)^k$ | ← Repeated Linear | $\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$ |
| $ax^2+bx+c$ | ← Distinct Quad. | $\frac{Ax+B}{ax^2+bx+c}$ |
| $(ax^2+bx+c)^k$ | ← Repeated Quadratic | $\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$ |

1.) Factor Denominator

2) make a decision "type"

**Distinct Linear Factors**

**Set-up the partial fractions, but do not solve.**

A & B are constants

$$\frac{x-2}{x^2-x-12} = \frac{A}{x+3} + \frac{B}{x-4}$$

$(x+3)(x-4)$

Distinct Linear

$$\frac{5x-8}{x^3+3x^2+2x} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x+1}$$

$x(x^2+3x+2)$

$x(x+2)(x+1)$

$$\frac{x^2+6}{x^2+2x-8} = \frac{A}{x+4} + \frac{B}{x-2}$$

$(x+4)(x-2)$

$$\frac{5x-2}{x^3-5x^2+6x} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x-2}$$

$x(x^2-5x+6)$

$x(x-3)(x-2)$

$$** \frac{2x+3}{x^4-36} = \frac{Ax+B}{x^2-6} + \frac{Cx+D}{x^2+6}$$

$(x^2-6)(x^2+6)$

Distinct Quad

**Repeated Linear Factors – Setup the partial fractions but do not solve.**

$$\frac{3x-41}{(x^2+6x+9)(x-2)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-2}$$

$(x+3)(x+3)(x-2)$

$(x+3)^2(x-2)$

repeated Linear

$$\frac{7x-20}{(x-4)(x-4)(x-4)(x+2)} = \frac{A}{(x+2)} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2} + \frac{D}{(x-4)^3}$$

$(x-4)^3(x+2)$

$$\frac{3x^2+5}{(x^2-8x+16)(x+1)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x+1}$$

$(x-4)^2(x+1)$

**Distinct Quadratic Factors – Setup the partial fractions but do not solve**

$$\frac{x-42}{x^4+7x^2+12} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+4}$$

$(x^2+3)(x^2+4)$

$$\frac{4x^2+1}{x^3+5x} = \frac{A}{x} + \frac{Bx+C}{x^2+5}$$

$x(x^2+5)$

Partial Fraction Decomposition Name:______________________

Find the partial fraction decomposition for each...do not solve for the constants, just set them up!

1) $\frac{3x-1}{x^2-x}$ Answer:______________

2) $\frac{-4x+3}{x^2-9}$ Answer:______________

3) $\frac{-7x^2-4x-14}{(x^2+1)(x-2)}$ Answer:______________

4) $\frac{-3x-29}{(x-7)(x+3)}$ Answer:______________

5) $\frac{11x-7}{(2x+1)(x-2)}$ Answer:______________

6) $\frac{7x^2-16x+36}{x^4-16}$ Answer:______________

7) $\frac{21-5x-3x^2}{(x+3)(x^2+4x+6)}$ Answer:______________

8) $\frac{x}{16x^4-1}$ Answer:______________

9) $\frac{-x^2+13x-5}{(x+1)(x^2+5)}$ Answer:______________

10) $\frac{x^3}{(x+2)^2(x-2)^2}$ Answer:______________