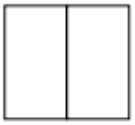
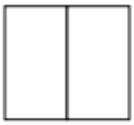
Pre-Calculus
Optimization Problems

Fencing Problems

1. A farmer has 480 meters of fencing with which to build two animal pens with a common side as shown in the diagram. Find the dimensions of the field with the maximum area. What is the maximum area?



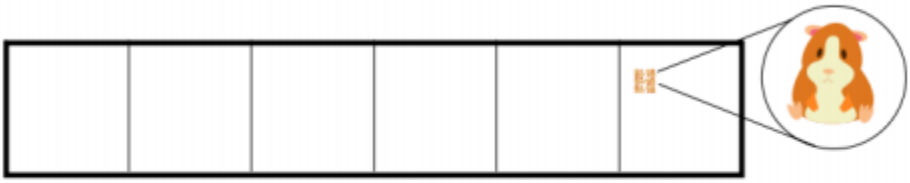
2. Max plans to build two side-by-side identical rectangular pens for his pigs that will enclose a total area of 216 square feet. What is the minimum length of fencing he will need? What are the dimensions of the total enclosure?



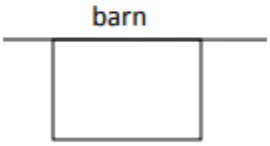
3. Build a rectangular pen with three parallel partitions using 500 feet of fencing. What overall dimensions will maximize the total area of the pen? What is the maximum area?

4. You are to fence a rectangular area. The fencing for the left and right sides costs $20 per foot and the fencing for the front and back sides costs $30 per foot. Find the dimensions of the rectangular area that result in the least cost, if the area inside the fencing is to be 3200 square feet. What is the cost? What are the dimensions?

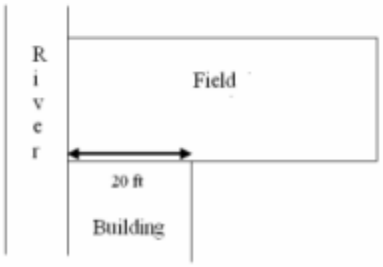
5. Your dream of becoming a hamster breeder has finally come true. You are constructing a set of rectangular pens in which to breed your furry friends. The overall area you are working with is 60 square feet, and you want to divide the area up into six pens of equal size as shown below. The cost of the outside fencing is $10 a foot. The interior fencing costs $5 a foot. You wish to minimize the cost of the fencing. Find the minimum cost and the overall dimensions of the enclosure.



6. A rectangular lettuce patch, 480 square feet in area, is to be fenced off against rabbits. Find the least amount of fencing required if one side of the land is already protected by a barn. What are the dimensions?

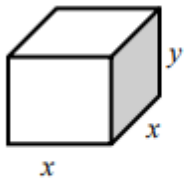


7. A rectangular field as shown is to be bounded by a fence. Find the dimensions of the field with maximum area that can be enclosed with 1000 feet of fencing. You can assume that fencing is not required along the river and the building. What is the max area?



Boxes (Rectangular Prisms)

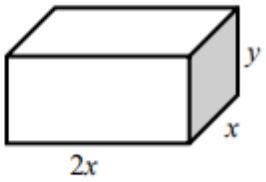
1. An open-top rectangular box with square base is to be made from 48 square feet of material. What dimensions will result in a box with the largest possible volume? What is the volume?



2. An open-top rectangular box with square base is to be made from 1200 square cm of material. What dimensions will result in a box with the largest possible volume? What is the volume?

3. A closed-top rectangular container with a square base is to have a volume 300 in^3. The material for the top and bottom of the container will cost \$2 per in^2, and the material for the sides will cost \$6 per in^2. Find the dimensions of the container of least cost. What is that cost?

4. An open-top box will be constructed with material costing \$7 per square meter for the sides and \$13 per square meter for the bottom. The dimensions of the bottom are to have its length equal to twice its width (see diagram). Find the dimensions of the box of largest volume than can be built with at most \$300 of materials. What is the volume?



5. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10/ft^2 and the material used to build the sides cost \$6/ft^2. If the box must have a volume of 50ft^3 determine the dimensions that will minimize the cost to build the box.

6. A pool with a square bottom is to have a volume of 2000 cubic feet. The owners plan to use a fancy tile to complete the pool. The sides of the pool will cost \$80 per square foot and the bottom of the pool will cost \$40 per square foot. Find the pool dimensions that will minimize the cost of construction.

7. Find the dimensions of the least expensive rectangular box which is three times as long as it is wide and which holds 100 cubic centimeters of water. The material for the bottom costs 7¢ per cm^2, the sides cost 5¢ per cm^2 and the top cost 2¢ per cm^2.

Answers:

Fencing:

1. overall dimensions: 80m x 120m; Maximum Area = $9600m^2$
2. overall dimensions: 12 feet x 18 feet; Minimum Fencing: 72 feet
3. overall dimensions: 50 feet x 125 feet; Maximum Area = 6,250 ft^2
4. overall dimensions: 46.188 x 69.282; Minimum Cost = $5542.56
5. overall dimensions: 5.164 feet x 11.619; Minimum Cost = $464.76
6. dimensions: 15.492 x 30.984; Minimum Fencing: 61.968 ft
7. dimensions: 255 feet x 510 feet; Maximum Area = 130,050 ft^2

Boxes:

1. $x = 4$ ft. and $y = 2$ ft, Volume = 32 ft^3
2. $V = 4000$ cm^3; box dimensions are 20 x 20 x 10 cm.
3. The cost is minimized when the dimensions are 9.655x9.655x3.218. The cost is $1118.60.
4. Dimensions: 1.96 x 3.92 x 2.43 m; Maximum Volume = 18.68 m^3.
5. Dimensions: 1.882 x 5.646 x 4.705 ft; Minimum Cost = $637.60
6. Dimensions are 20 x 20 x 5 feet; Minimum Cost is $48,000.
7. Dimensions are 2.912 x 8.736 x 3.931 cm, Minimum Cost = $6.87